Low-Complexity SD-FEC based on Channel-Polarized Multistage Codes for Data Center Networks

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Abstract: We propose channel-polarized multistage codes (CP-MSC) using multiple SD-FEC codes with different overheads. 0.08-dB net coding gain improvement is obtained by 21% total-overhead CP-MSC over conventional low-complexity SD-FEC code at the same overhead and complexity. © 2024 The Author(s)

1. Introduction

The demand for intra-data center networks (DCN) is increasing due to the growth of Internet traffic [1]. To economically increase the transmission capacity, 800LR have been discussed [2], which is 800Gbps coherent optical systems from 2 to 10 km. 800LR adopts a soft-decision forward error correction (SD-FEC), which consists of 21%-overhead (OH) concatenated code with short block length BCH codes with a low-complexity SD decoder (SDD) and KP4 code. The slight net coding gain (NCG) improvement drastically improves a substantial required optical signal-to-noise ratio (SNR) in practical optical transmission systems that have implementation penalties, such as bandwidth limitations [3]. However, a low-complexity SDD with short-block length codes limits the NCG improvement [4,5]. Our previous works proposed channel-polarized MLC (CP-MLC), which transforms multiple original channels into different capacity channels and utilizes SDD only on an unreliable channel [6]. 12%-OH CP-MLC with short-block length BCH and KP4 codes improves decoding performance and complexity compared with BCH-KP4 concatenated code [7,8]. On the other hand, CP-MLC with a high total OH as 21%-OH cannot efficiently achieve the KP4 BER threshold due to the error floor caused by the BER of reliable bits that comes at the cost of reducing SDD overly.

In this study, we propose a channel-polarized multistage code (CP-MSC) using multiple SD-FEC with different OHs for next-generation intra-DCN applications. The proposed CP-MSC intensively corrects the unreliable bits using strong SD-FEC with bypassing SD-FEC on the reliable bits, while suppressing of the error floor at the KP4 BER threshold even under high total OH. The proposed CP-MSC with a total 21%-OH achieves a 0.08-dB NCG improvement over the conventional low-complexity SD-FEC code in 800LR at the same OH and complexity.

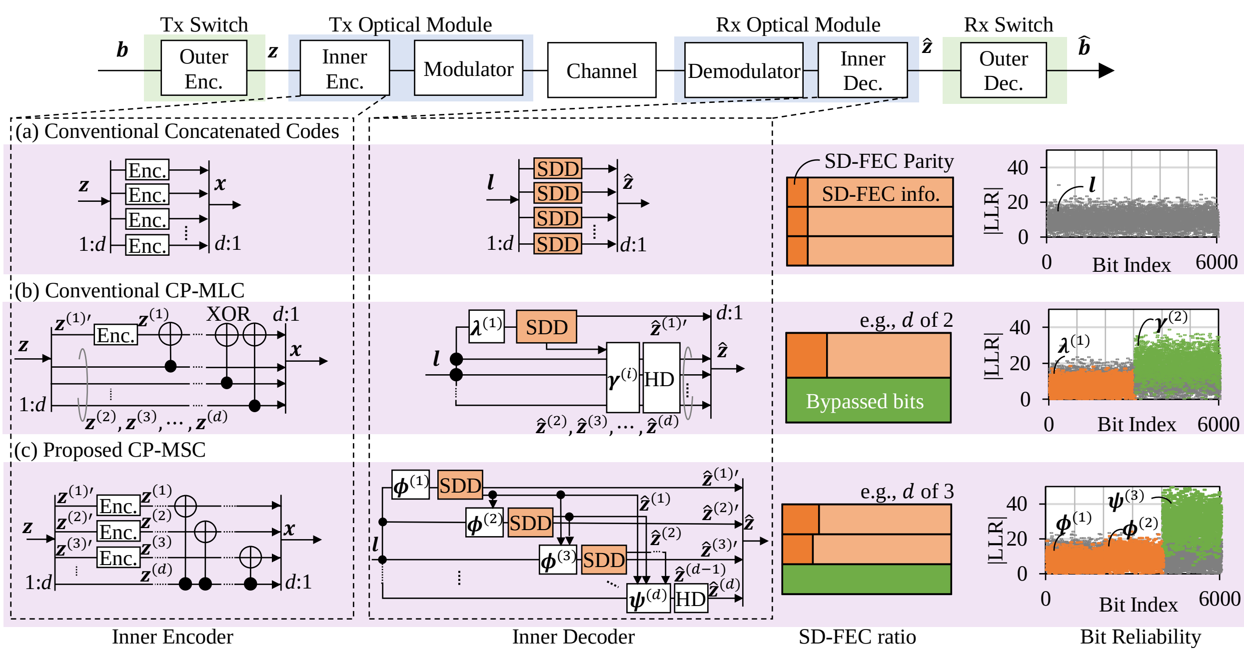

Fig.1. Encoder/decoder, SD-FEC ratio, and bit reliability of (a) conventional SD-FEC codes, (b) CP-MLC and (c) CP-MSC. Bit reliabilities are given by the amplitude of decoder-input LLR at pre-FEC BER of 1.31×10^{-2} under BPSK and AWGN channels.

2. System model for conventional concatenated code, CP-MLC, and CP-MSC

Figure 1 (a) shows the system model, the SD-FEC ratio in the entire FEC frame, and the bit reliability using conventional concatenated codes. An outer encoder outputs the codeword, which is encoded by inner codes on the optical module side. The receiver's inner SD decoder (SDD) restores the codeword, and the outer decoder then calculates information bits. The outer code used on the modern switch side is a KP4 code, which consists of (544,514) Reed-Solomon codes and achieves a pre-FEC BER of 2.2×10^{-4} at a post-FEC BER of 10^{-15}. Here, (n, k) codes are denoted by the block length as n, information length k. The BCH-KP4 concatenated codes with ordered statistical decoding (OSD) achieves a pre-FEC BER of 1.22×10^{-2} at a post-FEC BER of 10^{-15} under 16 quadrature amplitude modulation (QAM) [9].

Figure 1 (b) shows the configuration for CP-MLC, which makes the bit reliability uneven, assigns SD-FEC only to unreliable bits, and bypasses reliable bits. CP-MLC first encodes the partial bits of the outer codeword $\boldsymbol{z}^{(1)\prime}$ into the SD-FEC codeword $\boldsymbol{z}^{(1)}$ and then calculates $\boldsymbol{x} = (\boldsymbol{z}^{(1)} \oplus \boldsymbol{z}^{(2)} \oplus \cdots \oplus \boldsymbol{z}^{(d)}, \boldsymbol{z}^{(2)}, \cdots, \boldsymbol{z}^{(d)})$ by exclusive OR (XOR) for each bit. On the decoder side, SDD restores $\hat{\boldsymbol{z}}^{(1)}$ by using the unreliable log-likelihood ratio (LLR) $\boldsymbol{\lambda}^{(1)} = f(\boldsymbol{l}^{(1)}, \boldsymbol{l}^{(2)}, \cdots, \boldsymbol{l}^{(d)}) = \boldsymbol{l}^{(1)} \boxplus \boldsymbol{l}^{(2)} \boxplus \cdots \boxplus \boldsymbol{l}^{(d)}$, where $a \boxplus b = \mathrm{sign}(a)\mathrm{sign}(b)\min(|a|, |b|)$ and $\mathrm{sign}(a) \coloneqq a/|a|$ are defined [10]. The decoder finally calculates the reliable LLR $\boldsymbol{\gamma}^{(i)} = g(\boldsymbol{l}^{(i)}, f(\boldsymbol{l}^{(1)}, \boldsymbol{l}^{(2)}, \cdots, \boldsymbol{l}^{(i-1)}, \boldsymbol{l}^{(i+1)}, \cdots, \boldsymbol{l}^{(d)}), \hat{\boldsymbol{z}}^{(1)})$ and performs the hard decision (HD) $\hat{\boldsymbol{z}}^{(i)} = 0.5(1 - \mathrm{sign}(\boldsymbol{\gamma}^{(i)}))$, where $g(a, b, z) \coloneqq a + (-1)^z b$ is defined.

Figure 1 (c) show the configuration of the proposed CP-MSC. The encoder calculates inner codewords $\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}, \cdots, \boldsymbol{z}^{(d-1)}$, and then outputs $\boldsymbol{x} = (\boldsymbol{z}^{(1)} \oplus \boldsymbol{z}^{(d)}, \boldsymbol{z}^{(2)} \oplus \boldsymbol{z}^{(d)}, \cdots, \boldsymbol{z}^{(d)})$. On the decoder side, the $\hat{\boldsymbol{z}}^{(1)}, \hat{\boldsymbol{z}}^{(2)}, \cdots, \hat{\boldsymbol{z}}^{(d-1)}$ is sequentially decoded by using LLR $\boldsymbol{\phi}^{(i)} = f\left(\boldsymbol{l}^{(i)}, \boldsymbol{\psi}^{(i)}\right)$, where $\boldsymbol{\psi}^{(1)} = \boldsymbol{l}^{(d)}$ and $\boldsymbol{\psi}^{(i+1)} = g(\boldsymbol{\psi}^{(i)}, \boldsymbol{l}^{(i)}, \hat{\boldsymbol{z}}^{(i)}))$. The decoder finally performs $\hat{\boldsymbol{z}}^{(d)} = 0.5(1 - \mathrm{sign}(\boldsymbol{\psi}^{(d)}))$. CP-MSC uses the feedback information for $\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}, \cdots, \boldsymbol{z}^{(i-1)}$ to decode the $\boldsymbol{z}^{(i)}$, while CP-MLC uses only the feedback information $\boldsymbol{z}^{(1)}$. Thus, the bypassed bit reliability can be raised at the cost of increasing the SD-FEC ratio $(d-1)/d$ compared to CP-MLC, as shown in Fig.1 (b) and (c). As well as, CP-MLC, CP-MSC bypasses the SDD on reliable bits, thus unreliable bits can be corrected intensively using SD-FEC with high OH. CP-MSC provide to flexibly adjust the SD-FEC ratio and bit reliability with the target of KP4 BER threshold by changing d.

3. Method for decoding complexity evaluation

OSD first selects the k bit position with a lower amplitude of LLR and transforms the original code space into a code space with unreliable parity bits by using Gaussian elimination (GE). The test patterns (TP) are generated by flipping some bits of the transformed information bits of the new code space. Next, all-candidate codewords are calculated from each TP, by reconstructing parity bits using the encoder. Note that the OSD does not use the HD decoder. Finally, the OSD outputs an estimated codeword, which is a candidate codeword with a minimum Euclidean distance from the LLR. GE and parity reconstruction have the number of binary operations $n \cdot \min(k, n-k)^2$ and $k(n-k)$ [11], respectively. The complexity orders of GE and parity reconstruction are $O(nk^2)$ and $O(TP \cdot k(n-k))$, respectively, which are larger than that of sorting, codeword estimation, $\boldsymbol{\phi}^{(i)}$, and $\boldsymbol{\psi}^{(i)}$, which are $O(n\, log\, n)$, $O(TP \cdot n)$, $O(dn)$ and $O(dn)$, respectively. We evaluate the relative decoding complexity ξ for conventional SD-FEC codes and CP-MSC as $G + t$ and $((d-1)G + \sum_{i=1}^{d-1} t^{(i)})/d$, respectively, because the complexity of the decoder is dominated by GE and parity reconstruction. Here, $G = n \cdot \min(k, n-k)^2/(k(n-k)) = n(n-k)/k$ and $t^{(i)}$ are the relative complexity of GE and the number of TP, respectively.

4. Simulation results and discussion

Table 1 show that our constructed BCH-KP4 concatenated codes for 800LR [9], CP-MLC, and the proposed CP-MSC with a d of 3 using BCH/KP4 codes, where the total OH is approximately 21%. Here, component code of middle reliable channel outputs $\boldsymbol{z}^{(2)}$. Figure 2 shows that the block error rate (BLER) of the short-block-length (127,113)-, (127,106)-, and (127,99)-BCH code using OSD with TP of 114, 1546, and 21700, respectively, almost asymptote to the MLD performance under binary phase shift keying (BPSK) and additive white Gaussian noise (AWGN) channels. We can approximate maximum decoding performance of CP-MLC by using OSD with these TPs on each component BCH code. The decoding performance of BCH-KP4 concatenated codes using OSD with TP of 111 [9], which achieved a pre-FEC BER of 1.22×10^{-2}, is expected to be asymptotic to MLD because the (126,110)-BCH code is a shortened and double-extended code based on the (127,113)-BCH code [2].

Figure 3 shows the decoding performance of conventional CP-MSC under AWGN channel, 16QAM signals, and bit-interleaved coded modulation (BICM) [13], where CP-MSC has TPs $t^{(1)} + t^{(2)}$ of 207, 387, 727, 967, 1477, 3437, 5982, and 23246. For comparison, we also show the decoding performance of 22.8%-OH CP-MLC with a d of 2 with TPs of >20000. CP-MLC cannot efficiently fall below the KP4 BER threshold due to the error floor caused by the

Table. 1. FEC code configuration for concatenated codes in 800LR and proposed CP-MSC under 16QAM.

Code	Total-OH [%]	Pre-FEC BER (Max.)	NCG (Max.) [dB]	Inner codes			
				Unreliable ch.	OH [%]	Middle-reliable ch.	OH [%]
Concatenated code	21.2%	1.22×10^{-2} [9]	10.53	(126,110)-BCH	14.5%	-	
Proposed CP-MSC ($d = 3$)	21.5%	1.29×10^{-2} (1.38×10^{-2})	10.61 (10.72)	(127,99)-BCH	28.3%	(127,106)-BCH	19.8%
CP-MLC	22.8%	-	-	(127,92)-BCH	38.0%	-	

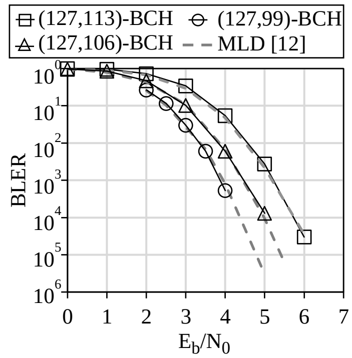

Fig. 2. BLER for OSD and MLD under BPSK. Note that BLER for (127,113)-BCH is reference for (126,110)-BCH.

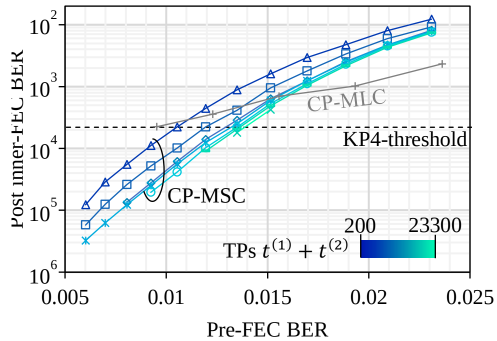

Fig. 3. Post-inner FEC BER for CP-MSC for each TPs and CP-MLC under 16QAM.

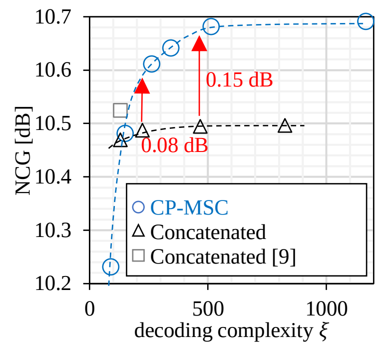

Fig. 4. NCG (16QAM) versus decoding complexity ξ of concatenated codes and CP-MSC.

BER of the reliable bits, despite increasing OH. It can be seen that in the CP-MSC, the error floor of the bypass bit was significantly lowered due to the feedback of $\boldsymbol{z}^{(2)}$ and $\boldsymbol{z}^{(3)}$. CP-MSC achieves a pre-FEC BER of about a maximum of 1.38×10^{-2} at a post-FEC BER of 10^{-15}.

Figure 4 shows the NCG and decoding complexity ξ of the conventional SD-FEC code and CP-MSC under 16-QAM with BICM. NCG was calculated from the SNR corresponding to pre-FEC BER. CP-MSC improves decoding performance by 0.08 and 0.15 dB at ξ of about 220 and 470, respectively, when the NCGs were compared with the conventional concatenated code at the same complexity. Since conventional SD-FEC codes may be saturated at a pre-FEC BER of 1.22×10^{-2}. CP-MSC requires a large number of TPs for performance convergence because the OH of the component BCH codes is large, while the high-complexity GE is reduced in the entire FEC frame due to the SD-FEC ratio of 2/3. It is possible to intensively correct the unreliable bits using SD-FEC codes with high OH without changing the total OH, similar to CP-MLC with low OH [8].

5. Conclusion

We proposed a CP-MSC using multiple SD-FEC codes with different OHs, which can realize a low-complexity SD-FEC code with a high OH of 21% on a short-block length regime by changing the bit-reliability and SD-FEC ratio flexibly. The proposed CP-MLC intensively corrects the only unreliable bits using strong SD-FEC and bypasses the other bits, with suppressing of the error floor at the KP4 BER threshold even under high total OH. Our simulation results show the CP-MSC improves the NCG by 0.08 and 0.15 dB compared to 800LR BCH-KP4 concatenated codes at decoding complexity of about 220 and 470, respectively.

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