Topological Rotation Symmetry-Based Wavelength Allocation for Entanglement Distribution Networks

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Abstract: We propose a wavelength allocation scheme based on topological rotation symmetry for entanglement distribution networks, reducing the number of wavelength channels required for N users from the order of $O(N)$ to $O(\sqrt{N})$.

1. Introduction

The construction of quantum networks is an important milestone in the field of quantum communications. Although there have been many field demonstrations of quantum key distribution (QKD) networks [1], most of them are based on trusted nodes with potential security risks. To avoid such risks, a framework of fully-connected networks based on quantum entanglement has been proposed [2], enabling real-time communication between any two users. To facilitate the construction of large-scale fully-connected entanglement distribution networks (EDNs) for multiple users, two solutions have been proposed. The first solution involves the production of high-dimensional quantum states [3], but it is susceptible to the environment in complex medium species, resulting in high experimental difficulty. The second solution relies on the wavelength division multiplexing (WDM) technique, which enables multi-user communication through multiplexing and bipartite entanglement [4]. Entanglement distribution among users is commonly implemented over optical fibers [2,4]. Given that the wavelength resources are precious and limited, how to design a efficient wavelength allocation (WA) scheme to fulfill the wavelength requirements of multiple users in an EDN without wavelength conflicts is a challenging problem.

In the traditional WA scheme without subnet division for EDNs, the number of wavelength channels required for N users is of the order of $O(N^2)$ [2]. With subnet division, the associated number of wavelength channels required can be reduced to the order of $O(N)$ [4]. However, this still hinders the scaling of WDM to multiple users, due to the limitation of available wavelength resources. To address this problem, we propose a topological rotation symmetry-based WA (TRS-WA) scheme for EDNs, reducing the order of wavelength channels required to $O(\sqrt{N})$.

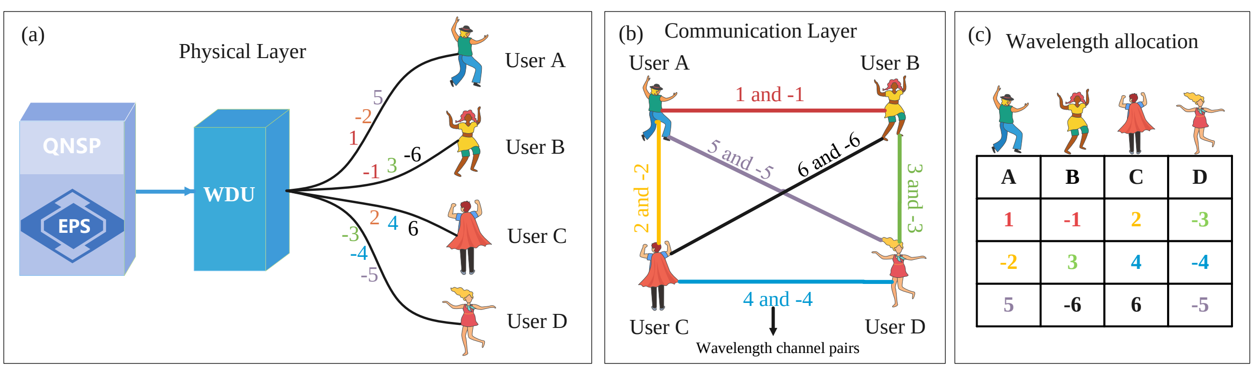

Fig. 1. Illustration of (a) the physical layer and (b) the communication layer of the EDN framework, as well as (c) the traditional WA scheme.

2. EDN Framework

As depicted in Fig. 1, the EDN framework consists of the physical layer and the communication layer. Figure 1(a) shows an example of multiple users in the physical layer of the EDN. The components of the physical layer can be categorized into three types, i.e., the quantum network service provider (QNSP), the wavelength distribution unit (WDU), and users. The QNSP provides a frequency-correlated entangled photon source (EPS). The WDU firstly divides the entangled bandwidth into different wavelength channels, and then allocates the wavelength channels to distant users using optical multiplexers and beam splitters (BSs) according to the WA scheme. Finally, quantum communication among users becomes feasible by relying on the establishment of quantum entanglement.

Figure 1(b) illustrates the wavelength channel pairs used between different users when there are four users in the EDN. Different colors indicate different wavelength channel pairs. Quantum entanglement exists in the form of a pair of entangled photons. When the entangled bandwidth is divided into multiple wavelength channels, only a pair of wavelength channels (e.g., 1 and -1) symmetrically centered around the central wavelength exhibits the entanglement properties. To achieve fully connected communication among four users, 6 pairs of wavelength channels (i.e., 1 and -1, 2 and -2, 3 and -3, 4 and -4, 5 and -5, 6 and -6) are required.

Figure 1(c) exemplifies how the wavelength channels are rationally allocated among four users by the traditional WA scheme. Among these 12 wavelength channels, each user receives three channels multiplexed over a single fiber. Eventually, the EPS continuously distributes six pairs of entangled photons between four different users, allowing every two users to share a pair of photons with each other. When there are N users, a total of $N(N-1)/2$ wavelength channel pairs are required to achieve the full connection among them.

3. TRS-WA Scheme for EDNs

In order to construct a large-scale fully-connected EDN through a limited number of wavelength channel pairs, we propose a TRS-WA scheme. The TRS-WA scheme can fully exploit the topological rotation symmetry to reduce the required number of wavelength channel pairs. As shown in Fig. 2(a), the traditional WA scheme and our proposed TRS-WA scheme are exemplified on the left and right, respectively. Note that each user has to use a different wavelength channel to establish the quantum entanglement with the other users. In contrast to the traditional WA scheme, which requires six wavelength channel pairs to satisfy the requirements of six users to establish quantum entanglement, the TRS-WA scheme requires only three wavelength channel pairs.

When the number of users is N, the TRS-WA scheme requires only $N-1$ wavelength channel pairs, where each wavelength channel is multiplexed $N/2$ times. When N is odd, $N/2$ is a decimal number and is not feasible. In this case, adding one additional wavelength channel pair allows each wavelength channel to be multiplexed $(N-1)/2$ times, making it feasible. As shown in Fig. 2(b), when there are five users, we first construct the topology of six users and then remove one user, which effectively meets the needs of five users. This means that when N is odd, the number of wavelength channel pairs required for N users and $N+1$ users are consistent. On the basis of the topological rotation symmetry, we further include the subnet division approach in the TRS-WA scheme, as illustrated in Fig. 2(c). The subnet division approach uniformly divides all users into several parts, that is, dividing a large network into several small networks for facilitating WA management. Specifically, the TRS-WA scheme is implemented according to the number of users in the subnet.

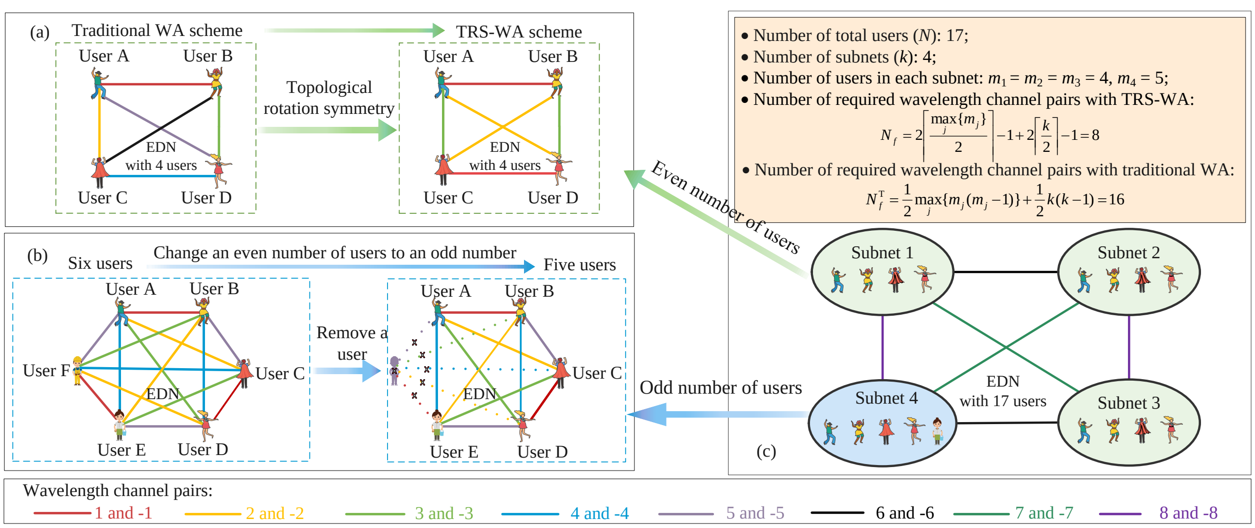

Fig. 2. (a) Comparison between traditional WA scheme and the TRS-WA scheme with even number of users; (b) The TRS-WA scheme with odd number of users; (c) An example of TRS-WA scheme in an EDN with 17 users.

4. Evaluation and Discussion

We compare the proposed TRS-WA scheme against the benchmark 1 (i.e., the traditional WA scheme [2]) or benchmark 2 (i.e., the traditional WA scheme [4]) in four cases. 1) case I: no subnets are divided and the TRS-WA scheme is used between users; 2) case II: the TRS-WA scheme is only used between subnets; 3) case III: the TRS-WA scheme is only used within subnets; 4) case IV: the TRS-WA scheme is used both within and between subnets.

The simulation results are shown in Figs. 3(a)-(e). From Fig. 3(a) we can observe that the TRS-WA scheme can reduce the required number of wavelength channel pairs by 99% relative to the benchmark 1, when there are 150 users without dividing subnets. In different cases compared in Figs. 3(b), 3(c), and 3(d), when 150 users are divided into 10 subnets, the reduction in the number of wavelength channel pairs required by the TRS-WA scheme versus the benchmark 2 can reach 27%, 65%, 92%, respectively. And when 40 users are divided into 10 subnets, the associated reduction can reach 79%, 9%, 88%, respectively.

Taking the two magenta lines in Figs. 3(b) and 3(c) as an example, we find that in case II, the TRS-WA scheme exhibits greater advantages with a small number of users, while in case III, our approach shows greater advantages with a large number of users. This is because in case II, when the number of subnets is the same and the number of users is small, the number of wavelength channel pairs required between subnets is more in the total number. In case III, when the number of subnets is the same and the number of users is large, the number of wavelength channel pairs required within the subnet accounts for more in the total number. Figure 3(d) demonstrates that when the TRS-WA scheme is applied throughout the EDN, it maintains a significant advantage regardless of the number of users.

Figure 3(e) illustrates the variation in the required number of wavelength pairs with the number of users under different cases, where the users are divided into five subnets. Compared to the benchmark, the number of wavelength channel pairs for the proposed TRS-WA scheme can be reduced by 1.8%, 94.5%, and 96.2% in cases II, III, and IV, respectively, when the number of users is 150. In case II, the performance improvement is relatively small, because only the WA between subnets is improved, while the number of wavelength channel pairs required within subnets accounts for a larger part of the total. In addition, if the same wavelength channel is multiplexed too many times, using BSs will reduce the effective entanglement probability between users (see Fig. 3(f)). However, this issue does not arise when using optical switches, which contributes to the realization of scalable EDNs.

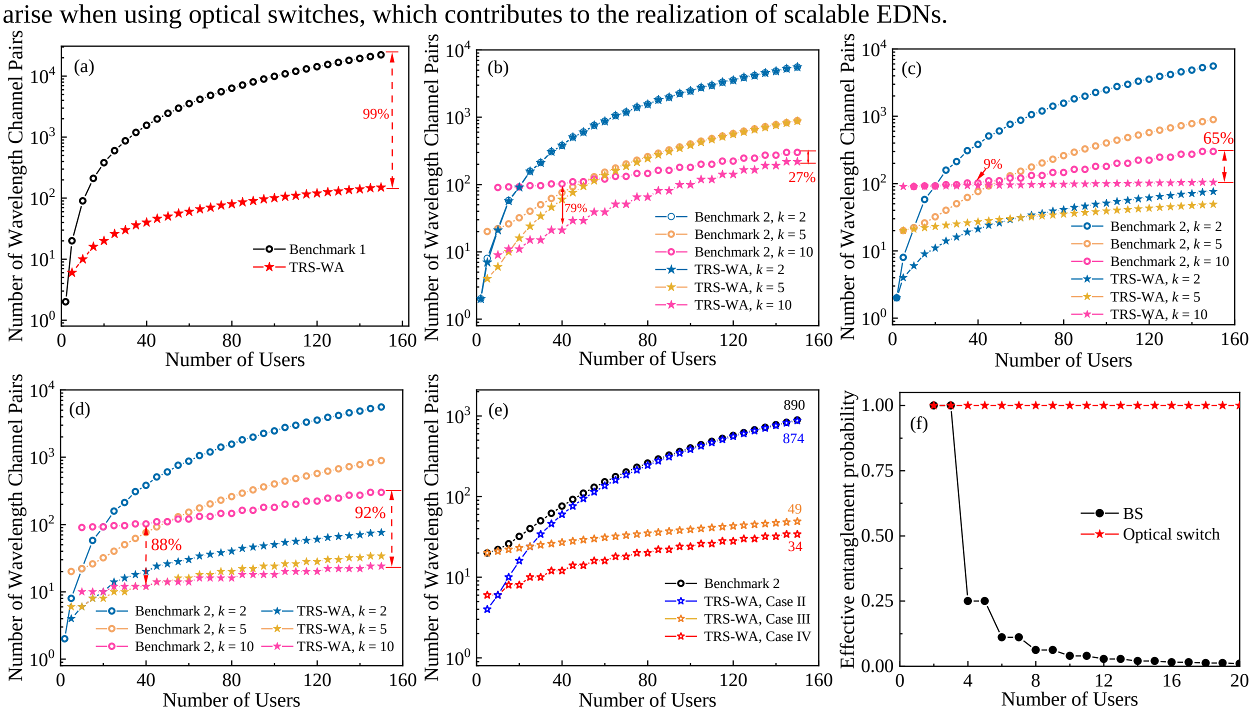

Fig. 3. (a) The TRS-WA scheme vs. benchmark 1 under case I. The TRS-WA scheme vs. benchmark 2 under (b) case II, (c) case III, (d) case IV, and (e) $k = 5$. (f) Effective entanglement probability with BS vs. optical switch.

5. Conclusion

In this work, we present a TRS-WA scheme for EDNs. Simulation results indicate that the reduction in the number of wavelength channel pairs required by the TRS-WA scheme relative to the benchmark can reach 99%.

References

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