

Distortion Characterization and Performance Estimation of Time-interleaved DAC and ADC Based on the Measurement of Nonlinear Noise Spectrum

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Abstract: Unlike many nonlinear devices, the time-interleaved DAC and ADC can be characterized by simple notch method accurately due to their unique nonlinear mechanism. By constructing equivalent model with measured noise spectrum, nonlinear system Q is estimated with 0.2-dB accuracy. © 2024 The Author(s)

1. Introduction

To match the 100 Gbaud-beyond optical communications, high-speed digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) usually employ the interleaved structures which multiplex many low-speed sub-converters into a high-speed one in time-domain [1,2] or frequency-domain [3]. Such converters have complicated nonlinear distortions so that the widely used effective number of bits and signal-to-noise-and-distortion ratio are not able to represent their performance accurately [4,5]. Since the distortions of converters limit the communication system performance, practical measurement and accurate characterization of them are needed.

Orthogonal component, which is orthogonal to the best linear approximation of a nonlinear system and accurately predicts performance degradation caused by nonlinearity, could be obtained by the orthogonal decomposition (OD) [6,7]. Inconveniently, OD is difficult to be widely used as it needs extremely accurate measurement, and expensive instruments such as high-speed arbitrary waveform generator and digital storage oscilloscope (DSO) [7]. Simple notch method [8] adds notch(es) on the input signal spectrum and measures the increment of output power at notched frequency by a spectrum analyzer. It is easy to be implemented and has large notch depth. However, many nonlinearities including the nonlinearity of non-time-interleaved DAC can't be measured by simple notch correctly when the stimulus is non-Gaussian [9]. The reason is that simple notch changes the probability distribution function (PDF) of the stimulus and changes the nonlinear noise thereafter. Probability-maintained (PM) notch generation method proposed in [9] is to find a special symbol sequence maintaining both frequency notch(es) and the same PDF as actual communication signal. It measures the in-band nonlinear noise accurately and practically. However, the quantization limits the notch depth as well as the dynamic range of PM notch measurement, in particular when the stimulus is in low-order format such as PAM4, PAM2 [10]. Is there any practical and accurate method for characterizing the nonlinear distortion in TI-DAC and TI-ADC with sufficient dynamic range?

In this paper, we find that simple notch method can accurately measure the nonlinearity of TI-DAC and TI-ADC with sufficient dynamic range, regardless of the input signal formats. The reason is the special nonlinear distortion mechanism of TI-converters. Based on the measured in-band noise spectrum, the Q performance can be estimated with 0.2 dB accuracy. Due to the special mechanism, the nonlinear distortion in TI-converters can't be modeled or compensated by Volterra series.

2. Distortion Characterization of TI-DAC and TI-ADC

Fig. 1(a) shows the experimental platform composed of a commercial-available 64 GSa/s 8-bit TI-DAC and an 80 Gsa/s 8-bit TI-ADC embedded in a DSO. The Tx and Rx digital signal processing (DSP) flows are shown concurrently. To obtain the actual in-band nonlinear noise [6], i.e. orthogonal component $y_o(t)$, 601-tap OD is implemented to calculate the best linear approximation $y_c(t) = \sum g_k x(t-k)$ of output signal $y(t)$ and the rest part $y_o(t) = y(t) - y_c(t)$, as shown in Fig. 1(b).

The notch methods, which are more practical than OD, are also implemented to measure the in-band noise directly, as illustrated in Fig. 1(c). Taking PAM8 as an example, PM notch only achieves 25 dB depth, whereas simple notch signal can maintain a notch depth greater than 40 dB after 8-bit quantization, as shown in Fig. 1(d), which provide sufficient dynamic range for measurement. The measured noise-to-power ratios (NPRs) of six in-band frequencies are plotted in Fig. 1(e). Experimental results show that even if the actual stimulus is not Gaussian, NPRs of simple notch are similar with both orthogonal NPRs and PM NPRs. The consistence of simple notch measured noise and

orthogonal component also can be observed from the spectra overlapping in Fig. 1(f). Such consistence is quite different from the phenomena in conventional nonlinear device where the nonlinear distortion strongly depends on input signal PDF and simple notch method fails [9].

Is the nonlinear distortion of TI-system insensitive to the signal PDF so that simple notch functions correctly? To answer this question, we investigate the signals with seven different modulation formats, including PAM2, PAM4, PAM8, Gaussian signal, and probabilistic constellation shaping (PCS) based on PAM8 with information entropies of 1.5-bit, 2-bit, and 2.5-bit under Maxwell-Boltzmann distribution. Fig. 1(g) shows the NPRs of different formats fitted together and verifies that the nonlinearity of TI-system is insensitive to the signal PDF.

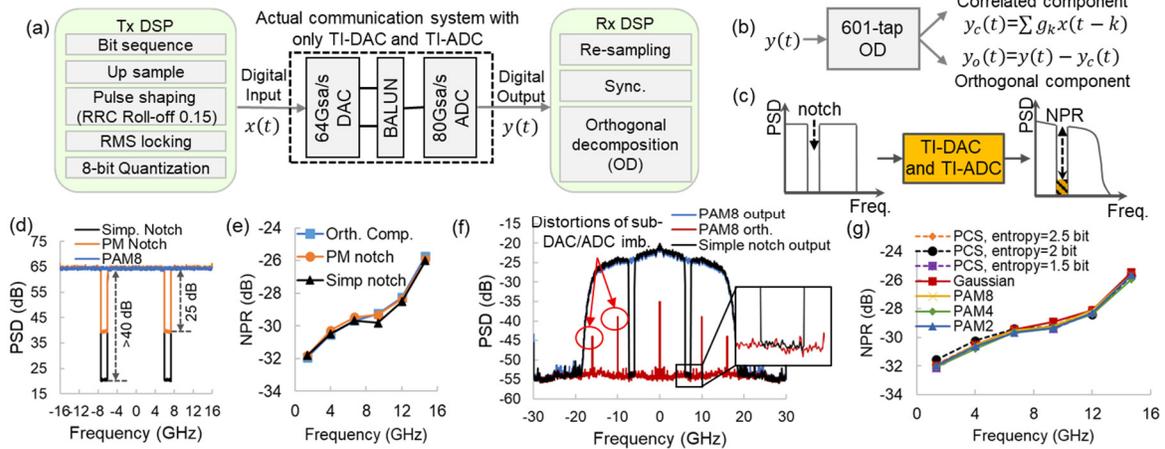


Fig. 1 (a) Experimental setup and Tx/Rx DSP of actual communication system with only TI-DAC and TI-ADC. (b) Schematic of orthogonal decomposition. (c) Schematic of notch method. (d) Power spectrum density (PSD) of PAM8, simple notch, and PM notch. (e) The measured NPRs of PM notch signal and simple notch signal consist with the orthogonal NPRs. (f) Measured PSDs shows that the bottom of simple notch is overlapped with the spectrum of orthogonal component. (g) The in-band NPRs of all the investigated modulation formats consist with others.

By analyzing the distortion mechanism of TI-system, the independence of nonlinearity on the signal PDF can be explained. Taking a TI-DAC as an example, the output analog signal with sampling rate f_s is interleaved by sub-DAC signals with relatively low sampling frequency f_s/M , as shown in Fig. 2(a). The offset imbalances DC_m , gain imbalances $1 + g_m$, timing error mismatches δ_m among sub-DACs, as well as the imperfections of sub-DACs themselves distort the output signal. Simulations in Fig. 2(b) illustrate the properties of those distortions. In the case that only considering imbalances among sub-DACs, the power of orthogonal component increases linearly along with the power of input digital signal because the offset imbalance is an additive distortion independent on the input signal, and the gain imbalance as well as timing error are multiplicative distortions independent on the input signal PDF. As the nonlinearity of sub-DAC (i.e., integral nonlinearity (INL)) is polynomial and depends on input signal, the nonlinearity increases much faster than the input power does when such nonlinearity of sub-devices dominates.

In the experiment, the relationship between orthogonal power and input power is linear and independent with signal PDF, as shown in Fig. 2(c). This format-independent linear relationship proves that the imbalance among sub-DACs/sub-ADCs is a dominate distortion of TI-system rather than the polynomial sub-device nonlinearity. Another straightforward evidence is the output spectrum of 2-tone stimuli in Fig. 2(d), where the 3rd-order inter-modulation distortions is much smaller than other glitches caused by sub-DAC/ADC imbalance.

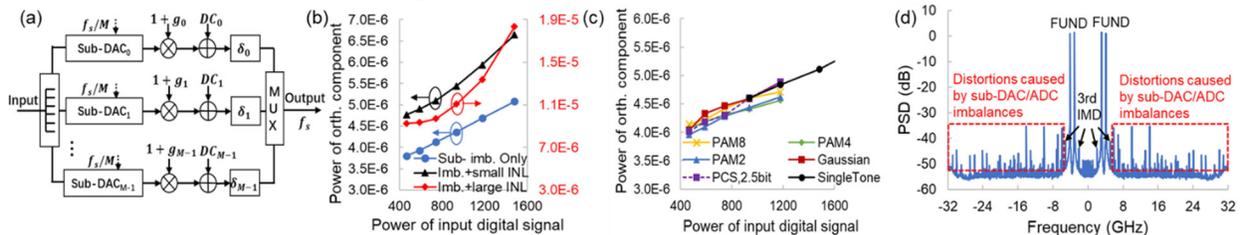


Fig. 2 (a) Schematic figure and distortions of time-interleaved DAC. (b) Relationships between the power of orthogonal component and that of input digital signal of different distortion sources by simulation. (c) Experimental power relationships of all the formats are linear, which reveals that the sub-DAC/ADC imbalances dominate. (d) Output spectrum of two-tone signal.

As sub-DAC/ADC imbalances dominate, the noise induced by TI-DAC and TI-ADC is independent with signal PDF, which is quite different from the nonlinearity in other devices such as non-TI-DAC, driver, modulator, and trans-impedance amplifier (TIA) [9]. Thus, although the simple notch changes the input signal PDF, it does not

change the nonlinear noise. Then, simple notch can be a practical and accurate solution for measuring the in-band nonlinear noise of TI-system.

3. Performance Estimation of TI- System

Based on the measured in-band noise spectrum, the system performance of TI-DAC and TI-ADC could be estimated by an equivalent additive noise model. As shown in Fig. 3(a), same Tx/Rx DSP are used to calculate the Q factors of actual TI-system and equivalent additive noise model with 32 Gbaud PAM8 signal. The equivalent additive noise model uses linear filter and Gaussian-distributed noise with same spectrum as in-band noise to emulate the actual TI-system. Fig. 3(b) shows that the Q factors of equivalent additive noise model are close to the actual Q factors of TI-system, with a root mean square error (RMSE) of 0.2 dB.

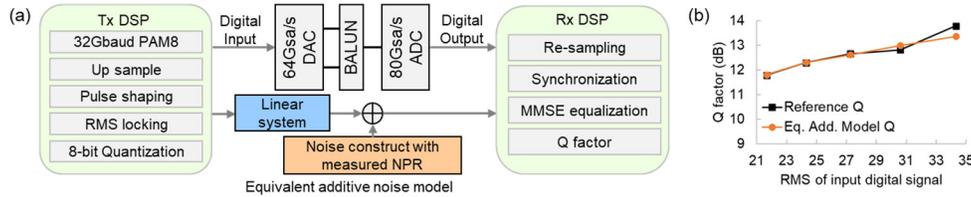


Fig. 3 (a) Equivalent additive noise model, which contains linear system and NPR-constructed random noise, could emulate the actual TI-system, and be used to calculate Q factor. (b) The estimation accuracy of equivalent additive-Gaussian-noise model is 0.2dB.

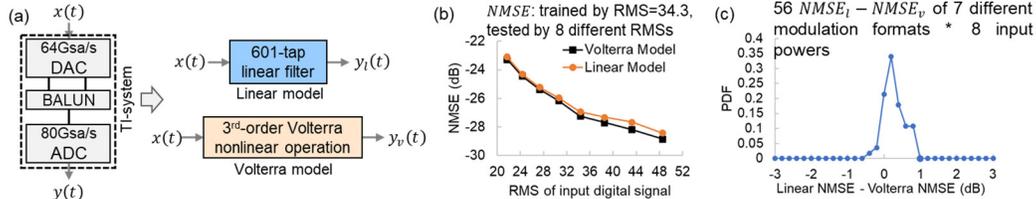


Fig. 4 (a) Schematic figures of actual TI-DAC and TI-ADC system, linear model, and Volterra model. (b) The NMSE of Volterra model has no obvious improvement compared to that of linear model. (c) PDF of 56 differences between linear NMSE and Volterra NMSE.

In addition, Volterra series is widely used to model and compensate the nonlinearity. It is common in dealing with the nonlinearities of driver, modulator, and TIA, and is considered to have better performance than linear model. However, things would go different in TI-system due to its unique nonlinear mechanism. We investigate the normalized mean square errors (NMSEs) of 601-tap linear model and 3rd-order Volterra model with tap number of 601, 11, and 5. The definition of NMSEs of linear and Volterra models are expressed in both Fig. 4(a) and Eq. 1.

$$NMSE_l = \frac{\langle |y(t) - y_l(t)|^2 \rangle}{\langle |y(t)|^2 \rangle}, \quad NMSE_v = \frac{\langle |y(t) - y_v(t)|^2 \rangle}{\langle |y(t)|^2 \rangle} \quad (1)$$

Fig. 4(b) shows that the NMSEs of Volterra model is almost the same as those of linear model. Here, the coefficients are trained by the PAM8 digital output with 34.4 input RMS and tested by the outputs of eight different RMSs. If the Volterra and linear models are trained and tested by seven different modulation formats, the probability distribution of 56 differences between $NMSE_v$ and $NMSE_l$ could be counted and showed in Fig. 4(c). The 0.26-dB averaged value of NMSE differences suggests that the beneficial effects of Volterra model are limited. More importantly, this also indicates that the benefit of Volterra based nonlinear compensation is negligible.

4. Conclusions

In characterizing the nonlinearity of TI-DAC and TI-ADC system, simple notch method performs accurately and practically with large dynamic range, regardless of the input signal format. The main reason is that the dominant nonlinear distortion in TI-DAC/ADC is the imbalance among sub-DAC/ADCs, rather than the nonlinearity of sub-devices themselves. Based on the measured noise spectrum, the Q performance of TI-DAC and TI-ADC system can be estimated with 0.2 dB accuracy. It is also verified that Volterra model is not appropriate for modeling or compensating the nonlinearity of TI-system.

5. References

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