Role of Frequency-Resolved SNR in Entropy-Loading DMT Systems: Rate Comparison and Simplified Options

Peiji Song, Di Che*

Nokia Bell Labs, Murray Hill, NJ 07974, United States

* *di.che@nokia-bell-labs.com*

Abstract: We verify that frequency-resolved SNR is crucial to approach the capacity of a discrete-multitone system with entropy loading (EL), and propose several methods to reduce the complexity of EL while keeping the SNR information to minimize the rate penalty. © 2024 The Author(s)

1. Introduction

The data rate per wavelength going beyond 200Gb/s is crucial to satisfy the demand of the advent 1.6T era [1, 2]. It may be inevitable to push future short-reach transceivers beyond their bandwidth limit, considering it becomes more challenging to scale the component's bandwidth linearly with the capacity demand. Discrete-multitone (DMT) is a promising technology to make full use of the bandwidth resource with frequency selective characteristics [3] because it can adapt the modulations per subcarrier based on the estimated signal-to-noise ratio (SNR) profile. The information theory has suggested that DMT uses the well-known water filling to approach the channel capacity. As the most popular algorithm approximating water filling, bit loading (BL) assigns various discrete bit levels to subcarriers [4]. Built on probabilistic constellation shaping (PCS) [5], entropy loading (EL) can load continuous entropies exactly matching the estimated SNR and thus go beyond BL with the shaping gain. The EL-based DMT has been proven as a capacity-approaching modulation and was demonstrated to outperform both the BL-based DMT and maximum likelihood sequence estimation (MLSE) assisted pulse-amplitude-modulation (PAM) (see Fig. 18 in Ref. [6]) in a bandwidth-limited 200G system. Nonetheless, implementing EL in the DMT system is too complicated as it needs a pair of distribution matchers (DMs)/inverse DMs and a pair of encoders/decoders for every subcarrier. As an alternative approach, the orthogonal matrix precoding-based scheme is a popular research path for DMT [7, 8], which aims to achieve a uniform signal-to-noise-ratio (SNR) profile for all the subcarriers. This allows us to use only one modulation format, known as uniform entropy loading (UEL) [9], to reduce the implementation complexity of EL-based DMT significantly. However, it is unclear if the simple UEL can approach the capacity of a DMT system like EL, since no apple-to-apple comparison has been made between the subcarrier-wise EL and the orthogonal matrix precoding-based UEL.

In this work, we demonstrate that UEL cannot approach the capacity due to the loss of frequency-resolved SNR information during the precoding process. Consequently, we propose several methods to simplify the sophisticated per-subcarrier EL algorithm, which can take advantage of the frequency-resolved SNR to minimize the gap to the system capacity while greatly reducing the implementation complexity. We perform a comprehensive achievable information rate (AIR) comparison among the EL per subcarrier basis, the orthogonal matrix precoding-based UEL, and several simplified EL proposals in a 200Gb/λ-class intensity modulation-direct detection (IM-DD) transmission experiment.

2. Principles

We first introduce the basic principle of UEL. Without loss of generality, we select the discrete Fourier transform (DFT) as the orthogonal matrix for precoding/decoding in this work. At the transmitter, we get $\boldsymbol{X}_i=\boldsymbol{C}\times\boldsymbol{S}_i$ with an $N\times N$ DFT matrix $\boldsymbol{C}$, where $\boldsymbol{X}_i$ and $\boldsymbol{S}_i$ are the i-th symbols after and before orthogonal matrix precoding. After IM-DD transmission and zero-forcing equalization at the receiver, the equalized signal can be written as [10] $\boldsymbol{Y}_i=\boldsymbol{S}_i+\boldsymbol{C}^{-1}\boldsymbol{H}^{-1}\boldsymbol{Z}_i+\boldsymbol{C}^{-1}\boldsymbol{H}^{-1}\boldsymbol{N}_i$, where $\boldsymbol{H}$, $\boldsymbol{Z}_i$, and $\boldsymbol{N}_i$ represent the channel coefficients, the ISI/ICI vector, and the noise vector, respectively. The blue curve in Fig. 1(a) illustrates the experimental channel SNR curve in this work without precoding, which has a frequency roll-off characteristic. Due to the diversity role of the orthogonal matrix $\boldsymbol{C}$, the ISI/ICI $\boldsymbol{Z}_i$ and noise $\boldsymbol{N}_i$ will be evenly spread into the equalized signal $\boldsymbol{Y}_i$, generating a uniform SNR profile, as shown by the green curve in Fig. 1(a). As a result, we can then perform UEL to match the uniform SNR among all subcarriers.

Another way of simplifying the EL per subcarrier basis is subcarrier grouping (SCG). SCG simply divides the entire subcarrier set into L groups ($L\geqslant1$), where each group is assigned the same entropy H_i (i=1, 2, ⋯, L), which only treats one format instead of a variety of formats, thus greatly reducing the number of required DMs/inverse DMs and encoders/decoders. Intuitively, the capacity of EL-SCG increases with L because the system changes into the EL-based DMT when L equals the number of effective subcarriers. Fig. 1(b)−(c) illustrates the principle of the SCG,

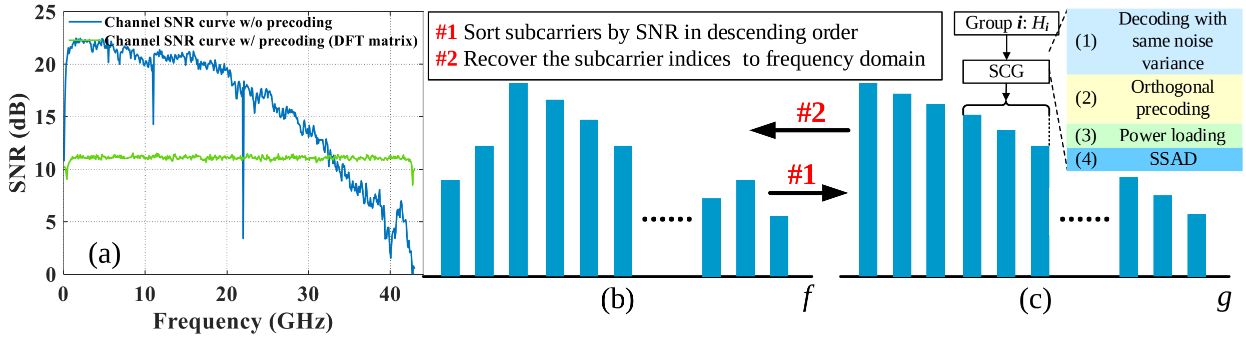


Fig. 1. (a) The SNR profiles with and without DFT matrix, (b)-(c) the schematic diagram of SCG with several decoding schemes.

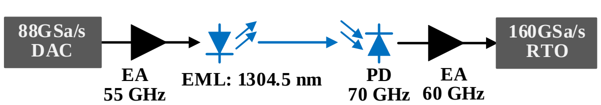
where each bar denotes one subcarrier, and the bar height corresponds to the estimated SNR. To deal with the wide-range SNR variation among subcarriers, as schematically shown in Fig. 1(b), a crucial step is to sort the subcarriers based on their SNR. This minimizes the SNR difference within each group of subcarriers, as presented in Fig. 1(c). Despite the subcarrier sorting, the SNR difference within a group of subcarriers may be non-neglectable when the number of groups (L) is small. We choose a baseline, noted as scheme (1), to evaluate various EL-SCG schemes - it just assumes the same noise variance within each group of subcarriers, which is applied for the decoding within each group. To alleviate the issue of SNR difference inside a group, we propose scheme (2) which uses the orthogonal matrix precoding per group basis to flatten the SNR inside each group to uniform the noise variations for decoding. Furthermore, to avoid the loss of frequency-resolved SNR information within each subcarrier group, we also propose 2 simple schemes that utilize the SNR per subcarrier in either the encoding or decoding process while keeping the same format per group: (3) power loading, which requires the detailed frequency-resolved SNR information feedback to the transmitter, is used to flatten the SNR inside each group, and (4) subcarrier-wise SNR-assisted decoding (SSAD), where each subcarrier uses its own noise for decoding. Assuming a length-m bit sequence $\boldsymbol{b} = b_{m-1}b_{m-2}\dots b_0$ is mapped to a symbol $c = c(\boldsymbol{b})$, the LLR of the j-th bit of a received symbol r is defined as $LLR_j = \log\left[P(b_j = 0|r)/P(b_j = 1|r)\right] = \log\left[p(r|b_j = 0)/p(r|b_j = 1)\right]$, where P is the probability and p is the probability density function (pdf). If the noise of symbols follows a Gaussian distribution with variance σ^2, p is expressed as

$$p\left(r\middle|b_j = i\right) = \sum_{\boldsymbol{b}:\, b_j=i} \frac{p(c(\boldsymbol{b}))}{2\pi\sigma^2} \exp\left(-\frac{\|r-c(\boldsymbol{b})\|^2}{2\sigma^2}\right),$$

where $p(c(\boldsymbol{b}))$ is the priori probability of a constellation point $c(\boldsymbol{b})$. A crucial point when calculating the LLR is the noise variance (σ^2), which can be estimated from the equalized and transmitted symbol vector $\boldsymbol{r}$ and $\boldsymbol{c}$ with length n,

$$\hat{\sigma}^2 = (\boldsymbol{r}\boldsymbol{r}^H - \boldsymbol{c}\boldsymbol{c}^H)/n.$$

The LLR can be directly used for hard decision (HD) where the j-th bit is judged to 0 or 1 based on the sign of LLR_j, or be sent to a soft decision (SD) forward-error-detection (FEC) decoder. In particular, HD will be a simple and practical case for IM-DD applications, and in SSAD or EL, the LLR-based HD with a frequency-resolved SNR is like finding a unique decision boundary for each subcarrier, but the usage of SCG makes the SSAD greatly reduce the complexity of EL. It should be noted that the conventional UEL described above is a particular case of (2) when L=1. For the four simplified methods, (3) and (4) directly utilize the frequency-resolved SNR for decoding, while (1) and (2) do not use it. Compared with scheme (3), the SSAD does not need to feedback the SNR to the transmitter, thus simplifying the operation. We will study their performance in detail next.



TxDSP	RxDSP
EL-SCG (BER Target)	Resample (160GSa/s)
QAM mapping	Serial-to-parallel
Pre-equalization	CP removal
Hermitian symmetry	1024-point FFT
1024-point IFFT	1-tap equalization
8-point cyclic prefix	Symbol decision
Parallel-to-serial	BER/hGMI

Fig. 2. Experimental setup and DSP flowchart. hGMI: *GMI* under binary HD-FEC decoding [11].

3. Experimental setup and results

Figure 2 shows the experimental setup and digital signal processing (DSP) flow chart. The OFDM signal has a DFT size of 1024, the effective number of subcarriers of 500, and the length of CP be 8. The combined bandwidth limits from the DAC and the EML results in a colored end-to-end channel SNR, as shown by the blue curve in Fig. 1(a). We evaluate all signals in the back-to-back transmission without considering the influence of dispersion. The receiver comprises a 70-GHz photodiode (PD), a 60-GHz EA, and a 160-GSa/s real-time oscilloscope (RTO). We use

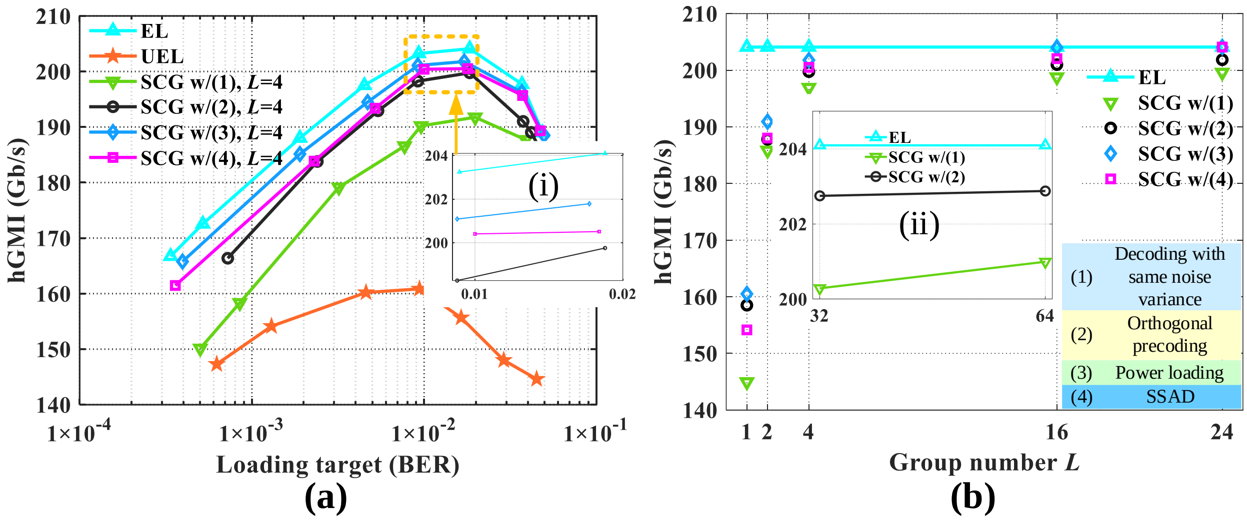


Fig. 3. The AIR comparisons for different schemes (a) with respect to various BERs, and (b) at BER=2×10^{-2} with respect to different group numbers Ls.

bit-error-ratio (BER) as the loading target and select the generalized mutual information (GMI) under the binary HD decoding (hGMI) as the AIR metric [11], calculated as $hGMI = H(X) - log_2(|\chi|) \cdot H_2(\varepsilon)$,where $|\chi|$ is the size of the modulation format, $H(X)$ with $X\in\chi$ is the entropy of X, ε is the BER, and H_2 is the binary entropy function.

Figure 3 depicts the AIR comparisons for different schemes in the DMT system, where the group number L is set to 4 of the proposed methods in Fig. 3(a). It can be found from Fig. 3(a) that UEL [9] exhibits a huge hGMI penalty compared to both EL and all EL-SCG schemes, especially at the capacity-approaching region, indicating that orthogonal matrix precoding-based DMT is inferior, and it is critical to maintain the frequency-resolved SNR information in an EL system to avoid significant rate plenty. Encouragingly, the proposed methods provide a close-to-capacity AIR of EL with a group number as small as L=4. Moreover, since scheme (3) and (4) use the frequency-resolved SNR information, they outperform both scheme (1) and (2) that discard the SNR variance within each group as shown in the inset (i) of Fig. 3(a), and achieve the rate closer to the EL-based DMT. This further verifies the role of frequency-resolved SNR information in maximizing the AIR.

Due to the fact that all schemes reach their maximum capacity at BER around 2×10^{-2}, Fig. 3(b) plots the AIR comparisons concerning different group number Ls around this BER target. We can see from Fig. 3(b) that EL-SCG exhibits an hGMI gap to EL, but the gap quickly narrows down at larger Ls. Besides, the performance of SCG decoding with (3) and (4) shows almost no penalty compared with EL when L=16 and 24, respectively, leading to approximately 96.8% and 95.2% reduced complexity. Considering the gap between (3) and (4) is tiny, and (4) without requiring the original SNR information at the transmitter, scheme (4), which we call SSAD, can be a potential scheme. In contrast, the hGMI gap still exists for SCG decoding with (1) and (2) even if L=64, as presented in inset (ii), due to the lack of frequency-resolved SNR information within each group of subcarriers.

4. Conclusions

EL-based DMT is a capacity-approaching technique for IM-DD systems whose superiority is backed by subcarrier-SNR-aware adaptive modulations. We prove that the attempt of simplifying EL by discarding the frequency-resolved SNR (like UEL) induces a non-neglectable rate penalty. We proposed several simplified methods that reduce the complexity of EL while keeping the SNR information to minimize the rate penalty.

5. References

[1] X. Pang *et al*., "200 Gbps/lane IM/DD technologies for short-reach optical interconnects," J. Lightw. Tech. 38, 2, (2020).
[2] D. Che *et al*., "200-Gb/s direct modulation of a 50-GHz class laser with advanced digital modulations," J. Lightw. Technol. 39, 3, (2021).
[3] J. Armstrong, "OFDM for optical communications," J. Lightw. Tech. 27, 3 (2009).
[4] Q. Yang *et al*., "Bit and power loading for coherent optical OFDM," IEEE Photon. Technol. Lett. 20, 15, (2008).
[5] D. Che *et al*., "Squeezing out the last few bits from band-limited channels with entropy loading," Opt. Express. 27, 7, (2019).
[6] D. Che *et al*., "Modulation Format and Digital Signal Processing for IM-DD Optics at Post-200G Era." J. Lightw. Tech. Early access.
[7] F. Li *et al*., "Transmission of 100-Gb/s VSB DFT- spread DMT signal in short-reach …," IEEE Photon. J. 7, (2015).
[8] S. Karabetsos *et al*., "DFT-spread DMT modulation for 1-Gb/s transmission rate over …," IEEE Photon. Technol. Lett. 24, 10, (2012).
[9] X. Chen et al., "Adaptive Uniform Entropy Loading for SSB- DMT Systems," J. Lightw. Tech. 37, 23, (2019).
[10] Z. Fan et al., "Orthogonal Circulant Multiplexing based on the Chirp-Like Polyphase Sequence in …," OFC'2021, Th4D.1.
[11] G. Böcherer, "Achievable rates for probabilistic shaping," arXiv:1707.01134.