Effect of Modal Dispersion on the Nonlinear Interference Noise in SDM Transmissions

C. Lasagni[1*], P. Serena[1], A. Bononi[1], A. Mecozzi[2] and C. Antonelli[2]

[1] *Department of Engineering and Architecture, University of Parma, 43124, Parma, Italy*
[2] *Department of Physical and Chemical Sciences, University of L'Aquila, 67100, L'Aquila, Italy*
[*] *chiara.lasagni@unipr.it*

Abstract: We review the effects of spatial mode dispersion and differential mode group delay on the nonlinear interference noise in space-division multiplexed systems based on few-mode fibers with weak linear coupling between mode groups.

1. Introduction

Few-mode fibers (FMFs) enable space-division multiplexing (SDM) supporting the propagation of groups of quasi-degenerate modes [1]. Although such fibers provide more pathways, the groups interact both linearly and nonlinearly as they propagate along the fiber. While linear coupling depends on the properties of the fiber, the nonlinear coupling depends on the properties of the propagating signals as well. In this context, mode dispersion has been shown to play an important role in setting the strength of the nonlinear interaction [2]. In FMFs, mode dispersion manifests both as a systematic differential mode group delay (DMGD) between non-degenerate groups, due to the differences in their group velocities, and as random spatial mode dispersion (SMD) within each quasi-degenerate mode group, due to random perturbations. In the presence of weak coupling among the groups, the former accumulates linearly with distance, while the latter grows proportionally to the square root of distance.

The presence of mode dispersion makes the analysis of nonlinear effects in SDM systems more involved than in single-mode systems, and the use of perturbative models [2–4] represents a viable simple approach. Similarly to single-mode fiber transmissions [5, 6], these models treat the fiber nonlinear effects as an additive nonlinear interference noise (NLIN). In this work, we review the effects of DMGD and SMD on the NLIN variance in FMF transmission, exploiting the ergodic Gaussian noise (GN) model introduced in [2, 7].

2. Intra- and inter-group NLIN model

With no loss of generality, we consider an FMF supporting two groups of quasi-degenerate modes labeled as a and b, with $2N_a$ and $2N_b$ polarization modes, respectively. Under perturbative assumptions and neglecting the effects of weak inter-group linear coupling on the Kerr effect, the NLIN on symbol $a_{\mathbf{i}}$ in group a can be written as [7]:

$$n_{\mathbf{i}} = \underbrace{-j \sum_{\mathbf{k},\mathbf{m},\mathbf{n}\in\mathcal{S}} a^*_{\mathbf{k}} a_{\mathbf{m}} a_{\mathbf{n}} \mathcal{X}_{\mathbf{kmni}}}_{\text{intra-group}} \underbrace{- j \sum_{\mathbf{k},\mathbf{m},\mathbf{n}\in\mathcal{S}'} b^*_{\mathbf{k}} b_{\mathbf{m}} a_{\mathbf{n}} \mathcal{X}'_{\mathbf{kmni}}}_{\text{inter-group}}, \qquad \mathbf{k} = \left[\underset{\text{time}}{k_1}, \underset{\text{frequency}}{k_2}, \underset{\text{polarization}}{k_3} \right] \tag{1}$$

where $\mathcal{X}_{\mathbf{kmni}}$, $\mathcal{X}'_{\mathbf{kmni}}$ express the intra- and inter-group four wave mixing (FWM) combinations, i.e., inside and between groups, respectively. Each bold index, as $\mathbf{k}$ in Eq. (1), is a three-element vector indicating a channel access per time, frequency, and polarization with $\mathcal{S}$, $\mathcal{S}'$ the set of valid combinations. The FWM process among frequencies $\{\omega, \omega+\omega_1, \omega+\omega_2, \omega+\omega_1+\omega_2\}$ is weighted by the following intra- and inter-group link kernels:

$$\eta_{\mathbf{kmni}} = \kappa_{aa}\gamma \int_0^L f(z) e^{-j\Delta\beta z} P^{(a)}_{k_3 m_3}(z) Q^{(a)}_{i_3 n_3}(z) \mathrm{d}z, \quad \eta'_{\mathbf{kmni}} = \kappa_{ab}\gamma \int_0^L f(z) e^{-j\Delta\beta' z} P^{(b)}_{k_3 m_3}(z) Q^{(a)}_{i_3 n_3}(z) \mathrm{d}z, \tag{2}$$

where L is the total link length, $f(z)$ is the power loss/gain function up to propagation distance z, γ is the fiber nonlinearity coefficient for the fundamental mode, and κ_{aa}, κ_{ab} are the intra- and inter-group Manakov nonlinear coefficients [8], respectively. The term $\Delta\beta$ in the intra-group link kernel is the phase-matching coefficient from single-mode literature [5]. On the other hand, the presence of the DMGD yields $\Delta\beta' \triangleq \Delta\beta + \chi_{ab}\cdot\omega_1$ in the inter-group link kernel, where χ_{ab} is the DMGD per unit length between the two mode groups. It is worth noting that with negligible third-order chromatic dispersion $\Delta\beta \simeq \omega_1\omega_2\beta_2$, such that the effect of the DMGD on the inter-group NLIN can be seen as a shift of the link kernel along the frequency ω_2 yielding phase matching at novel spectral components [3, 9, 10] compared with single-mode transmissions.

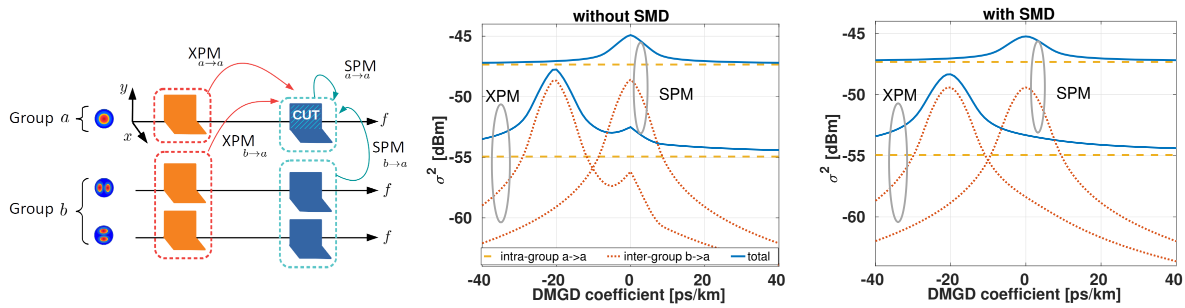

Fig. 1. Left: sketch of intra ($a \to a$) and inter ($b \to a$) group SPM and XPM on the CUT. Center: estimate of SPM and XPM variance vs the DMGD coefficient χ_{ab} in the absence of SMD. Right: $\mathrm{SMD}_a = 0.1\,\mathrm{ps}/\sqrt{\mathrm{km}}$ and $\mathrm{SMD}_b = 10\,\mathrm{ps}/\sqrt{\mathrm{km}}$. Two channels spaced by 150 GHz. 100 km FMF.

The matrices $\mathbf{Q}^{(h)} = [Q_{ij}^{(h)}]$ and $\mathbf{P}^{(h)} = [P_{ij}^{(h)}]$ in Eq. (2), with $h \in (a,b)$, account for the presence of SMD within each group, and are defined as $\mathbf{Q}^{(h)} \triangleq \mathbf{U}_h^{\dagger}(z,\omega)\mathbf{U}_h(z,\omega+\omega_1)$ and $\mathbf{P}^{(h)} \triangleq \mathbf{U}_h^{\dagger}(z,\omega+\omega_1+\omega_2)\mathbf{U}_h(z,\omega+\omega_2)$, with $\dagger$ the transpose-conjugate. The matrices $\mathbf{U}_h$ are independent random unitary matrices accounting for the strong intra-group mode coupling and are frequency-dependent in the presence of SMD. Due to the randomness of the mode coupling process, both link kernels in Eq. (2) are random in nature. Note that, in the absence of SMD, $\mathbf{Q}^{(h)}$ and $\mathbf{P}^{(h)}$ simplify to the identity matrix, such that the intra- and inter-group link kernels become deterministic.

The randomness of the NLIN entailed by SMD jeopardizes the development of simple models for its variance and the product among eight random matrices makes the analysis cumbersome. Focusing on the intra-group NLIN, analytical expressions for the expectation of such a product have been found in [2], leading to a closed-form link kernel. The resulting ergodic GN model [2] provides an estimate of the average value of the variance of the NLIN induced by a group of strongly coupled modes. The same idea has been generalized to the study of the inter-group NLIN [7], where the expectation is limited to the product of four matrices per group only, as a result of the assumption that the coupling matrices are independent in the two mode groups.

3. Numerical results

We investigated the impact of DMGD and SMD on the NLIN of a FMF transmission. The setup consisted of an FMF of length 100 km supporting three spatial modes in two groups, with $N_a = 1$ and $N_b = 2$. The effective area of modes in group a and b was 125 μm^2 and 165 μm^2, respectively, while the cross-group effective area was 250 μm^2 [11]. We assumed mode-independent attenuation of 0.2 dB/km and chromatic dispersion of 17 ps/(nm·km).

First, we considered the transmission of two frequency channels carrying Gaussian distributed symbols at $R =$ 64 Gbaud with spacing $\Delta f = 150$ GHz in each polarization. The dual-polarization channel power was $P = 0$ dBm. Figure 1(left) sketches the self-phase modulation (SPM) and cross-phase modulation (XPM) contributions on the channel under test (CUT) and their relative intra- and inter-group components. Figure 1(center) shows the corresponding variance, which we estimated using the complete model in [2] and [7], respectively, in the presence of the DMGD between the groups, while the SMD was zero inside each group. We note that, while the intra-group effects are independent of the DMGD, the inter-group SPM and XPM variance resonates at different values [9,11]. In particular, the DMGD is beneficial in breaking the phase matching of SPM through a frequency shift of the

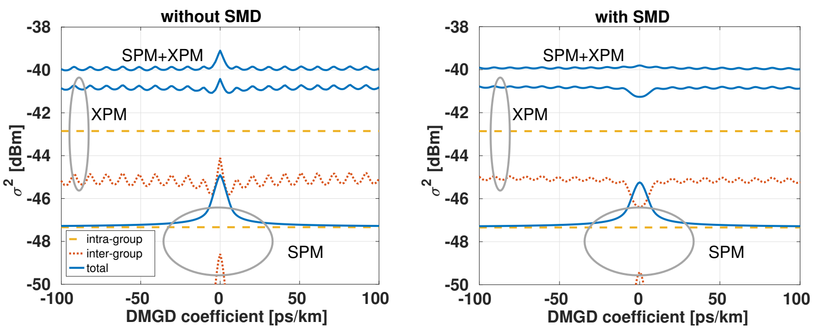

Fig. 2. Same as in Fig. 1 for the central channel of 61 WDM signals spaced by 75 GHz.

kernel outside the channel bandwidth which reduces its variance. On the other hand, the cross-group XPM variance resonates at the value of χ_{ab} that compensates the walk-off of chromatic dispersion, namely $\chi_{ab} = 2\pi\beta_2\Delta f \approx -20.5$ ps/km [4, Eq. (33)]. The presence of a secondary peak around 0 ps/km is due to a cross-polarization modulation (XPolM) contribution, which is less efficient when the link kernel is frequency-shifted by the DMGD. Figure 1(right) shows the effect of $\mathrm{SMD}_a = 0.1\,\mathrm{ps}/\sqrt{\mathrm{km}}$ and $\mathrm{SMD}_b = 10\,\mathrm{ps}/\sqrt{\mathrm{km}}$ within groups a and b, where the smaller value of SMD_a represents the polarization-mode dispersion of the fundamental mode. We note that the SMD mitigates the XPM and SPM variance values at their resonance points and suppresses the XPolM term.

Next, we investigated the transmission of a fully-loaded wavelength division multiplexing (WDM) signal filling the C-band, composed of $N_{\mathrm{ch}} = 61$ channels spaced $\Delta f = 75$ GHz. In this case, the CUT was the central channel in the comb. Due to the wider bandwidth under test, here we included the third order dispersion through a dispersion slope of 0.057 ps/(nm^2·km) for each mode. All the other parameters were unchanged. Figure 2(left) shows that, in the absence of SMD, the inter-group XPM variance exhibits multiple resonances, resulting from the superposition of $N_{\mathrm{ch}} - 1$ XPM contributions similar to the two-channel XPM shown in Fig. 1, with a dominant peak around $\chi_{ab} = 0$ ps/km. In fact, the walk-off is canceled by a different value of DMGD in each contribution to XPM such that peaks appear at multiple values of the DMGD coefficient $\simeq 2\pi\Delta f_k \left(\beta_2 + \frac{1}{2}\beta_3 2\pi\Delta f_k\right)$ ps/km, where $\Delta f_k = f_{\mathrm{CUT}} - f_k$ is the frequency separation between the CUT and the generic channel k. On the other hand, the small XPolM resonance observed in the two-channel scenario remains at $\chi_{ab} = 0$ ps/km in each contribution and it adds up coherently by summing the individual XPM terms. Since even the variance of SPM is maximum in the absence of DMGD, the total NLIN variance has a peak around 0 ps/km, as shown in Fig. 2(left).

Finally, Fig. 2(right) shows that, in the presence of SMD, the XPolM resonance from the inter-group term vanishes, leaving an anti-resonance in the XPM variance around $\chi_{ab} = 0$ ps/km. When combining SPM and XPM, such a minimum is partially equalized by the SPM maximum yielding an NLIN variance that is nearly independent of the DMGD values under test, despite the wild variations observed in the two-channel interaction. Figure 2(right) shows such a smoothing effect of the SMD. Although the inter-group NLIN variance is smaller compared to the intra-group one, its inclusion enhances the total NLIN variance by almost 2 dB.

The inter-group NLIN can be compared to the linear crosstalk to understand its importance at a system level. Treating the linear crosstalk coming from group b as an additive noise, we can write its variance per unit of length as: $\sigma^2_{\mathrm{XT},b} \triangleq \xi_{ab}P$ where ξ_{ab} is the normalized linear inter-group crosstalk coefficient. In the 100-km setup of Fig. 2(right), we obtain an inter-group NLIN variance of -27 dBm at the optimal value $P = 3$ dBm, which is bigger than the linear crosstalk variance accumulated in 100 km when $\xi_{ab} \leq -50$ dB/km.

4. Conclusions

We reviewed the effect of modal dispersion on the variance of the NLIN in few-mode fiber transmissions with weakly-coupled mode groups by accounting for both differential mode group delay and spatial mode dispersion. We characterized the resonances in the NLIN variance caused by DMGD and showed that they can be mitigated by SMD. Our study relied on the ergodic GN model that we recently proposed for the estimation of the NLIN variance in SDM systems with strongly coupled modes and its extension to systems with weakly-coupled groups.

References

1. P. Sillard, K. Benyahya, D. Soma, G. Labroille, P. Jian, K. Igarashi, R. Ryf, N. K. Fontaine, G. Rademacher, and K. Shibahara, "Few-mode fiber technology, deployments, and systems", Proceedings of the IEEE, **110**, 1804–1820 (2022).
2. P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, "The Ergodic GN Model for Space-Division Multiplexing with Strong Mode Coupling", J. Lightw. Technol., **40**, 3263–3276 (2022).
3. G. Rademacher and K. Petermann, "Nonlinear Gaussian Noise Model for Multimode Fibers with Space- Division Multiplexing", J. Lightw. Technol., **34,** 2280–2287, (2016).
4. C. Antonelli, O. Golani, M. Shtaif, and A. Mecozzi, "Nonlinear interference noise in space-division multiplexed transmission through optical fibers", Opt. Express, **25**, 13055–13078 (2017).
5. P. Poggiolini, "The GN model of non-linear propagation in uncompensated coherent optical systems", J. Lightw. Technol., **30**, 3857–3879 (2012).
6. R. Dar, M. Feder, A. Mecozzi, and M. Shtaif, "Properties of nonlinear noise in long, dispersion-uncompensated fiber links", Opt. Express, **21**, 25685–25699 (2013).
7. C. Lasagni, P. Serena, A. Bononi, A. Mecozzi, and C. Antonelli, "Impact of Mode Dispersion on Cross-Phase Modulation in Few-Mode Fiber Transmissions", in *European Conference on Optical Communication* (2023), paper WeD2.2.
8. C. Antonelli, M. Shtaif, and A. Mecozzi, "Modeling of nonlinear propagation in Space-Division Multiplexed fiber-optic transmission", J. Lightw. Technol., **34**, 36–54 (2016).
9. R.-J. Essiambre, M. A. Mestre, R. Ryf, A. H. Gnauck, R. W. Tkach, A. R. Chraplyvy, Y. Sun, X. Jiang, and R. Lingle, "Experimental Investigation of Inter-Modal Four-Wave Mixing in Few-Mode Fibers", IEEE Photon. Technol. Lett., **25**, 539–542 (2013).
10. A. D. Ellis, N. M. Suibhne, F. C. G. Gunning, and S. Sygletos, "Expressions for the nonlinear transmission performance of multi-mode optical fiber", Opt. Express, **21**, 22834–22846 (2013).
11. G. Rademacher, R. S. Luìs, B. J. Puttnam, R. Maruyama, K. Aikawa, Y. Awaji, H. Furukawa, K. Petermann, and N. Wada, "Investigation of Intermodal Nonlinear Signal Distortions in Few-Mode Fiber Transmission", J. Lightw. Technol., **37**, 1273–1279 (2019).