

A 92% Complexity Reduction of Low-Latency Multi-Group Precoding Scheme based on Björck Sequences

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Abstract: We present a multi-group precoding scheme based on Björck sequences, achieving a tradeoff between complexity and BER performance. Experiments at ~200 Gb/s demonstrate that the proposed approach outperforms OCT while reducing complexity by 92%. © 2024 The Author(s)

1. Introduction

With emerging technologies like telemedicine, vehicles to everything, and online financial transactions, cloud and edge data centers will inevitably serve as the central infrastructure for the deployment of latency-sensitive real-time applications [1]. Their coverage range is typically a few kilometers. Thus, Intensity-modulation direct detection (IMDD) systems are the preferred solution because of their low cost. With a strong demand for high capacity and low latency, various schemes have been proposed to achieve better communication performance [2-5]. Post-equalization schemes can enhance BER performance but require additional training sequences, which reduce spectral efficiency [2]. Utilizing hollow core fiber allows propagation at nearly light speed in a vacuum and lower chromatic dispersion, achieving reduced latency and increased capacity. However, this requires reinstallation of new optical fibers [3]. Bit-power loading alleviates fading by sensing the channel state but has high complexity and requires channel state information (CSI) feedback, causing additional communication overhead [4]. Precoding schemes avoid the aforementioned issues by adding extra encoding and decoding matrix operations at the transmitter and the receiver, respectively, to enhance performance [5]. However, this inevitably introduces additional computational complexity and latency. In particular, the complexity increases with (i) the number of subcarriers and (ii) the number of complex multiplications needed. This signal processing-induced latency impacts user experience and poses potentially high risks to user safety in applications such as telesurgery and self-driving vehicles.

In this paper, we propose for the first time using Björck sequences to realize a low-complexity precoding scheme without any CSI feedback or training sequences. A ~200Gb/s experiment shows that our proposed scheme outperforms the orthogonal circulant matrix transform (OCT) precoding scheme, one of the simplest schemes among conventional methods, with ~50% lower complexity. Note that the computational complexity increases with the matrix dimension, namely, the number of subcarriers in OFDM systems. Therefore, we further propose a multi-group Björck precoding scheme where the subcarriers are divided into multiple groups with smaller precoding matrices applied individually to each group, enabling the sharing of the precoding module across groups and the reduction of computation delay. Our experiments demonstrate that the proposed approach can achieve a complexity reduction of approximately ~91.54% compared to the OCT scheme at a comparable BER performance, providing an attractive tradeoff between complexity and BER performance.

2. Principle

We start with an $N \times N$ circulant matrix, \mathbf{C} , generated by circularly right-shifting a sequence $(c_0, c_1, \dots, c_{N-1})$:

$$\mathbf{C} = [c_0, c_1, \dots, c_{N-2}, c_{N-1}; c_{N-1}, c_0, c_1, \dots, c_{N-2}; \dots; c_1, c_2, \dots, c_{N-1}, c_0] \quad (1)$$

At the transmitter, data precoding is accomplished using matrix \mathbf{C} . We consider data blocks before and after precoding, denoted as \mathbf{A} and \mathbf{B} , respectively, where $\mathbf{B} = \mathbf{C}\mathbf{A}$. The core of precoding involves matrix multiplication that introduces processing delays. The number of complex multiplications significantly impacts processing latency. To mitigate complexity, we strive to devise a novel precoding matrix with minimal complex elements. An extensive literature review reveals that Björck sequences and their cyclic shifted variations exhibit the required characteristics. The construction of Björck sequences is delineated as follows [6]:

Let p be a prime number. If $p \equiv 1 \pmod{4}$, the Björck Sequences of length p is:

$$b_p[k] = \exp(i\theta_p(k)), \quad \forall k = 0, 1, \dots, p-1 \text{ where } \theta_p(k) = \left(\frac{k}{p}\right) \arccos\left(\frac{1}{1+\sqrt{p}}\right) \quad (2)$$

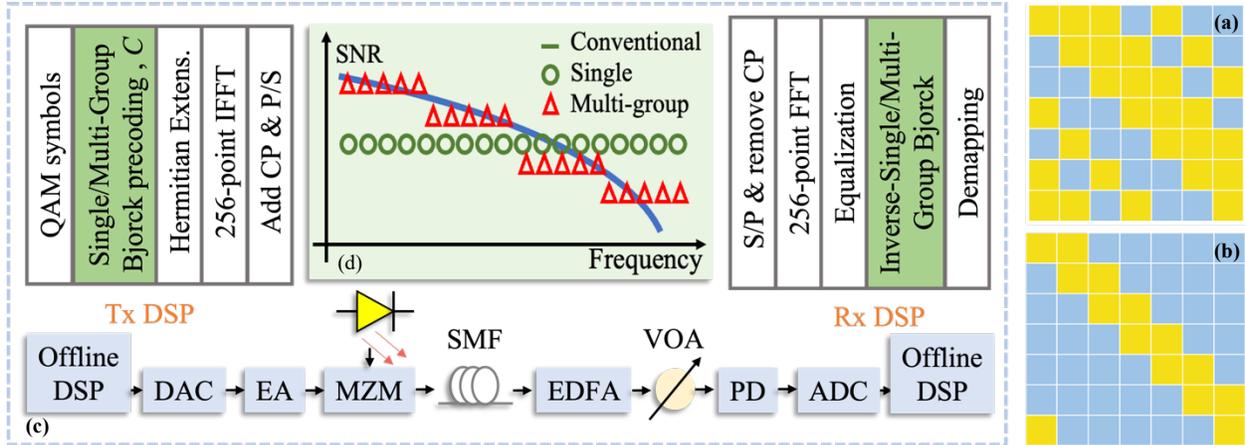


Fig. 1. Precoding matrix schematic diagram used for (a) OCT, and (b) proposed scheme. (c) The diagram and experimental setup of the proposed single/multi-group Björck precoding scheme. (d) SNR distribution under different schemes.

$$\text{where } \left(\frac{k}{p}\right) = \begin{cases} 0, & \text{if } k \equiv 0 \pmod{p} \\ 1, & \text{if } k \not\equiv 0 \pmod{p} \text{ is a square } \pmod{p} \\ -1, & \text{if } k \not\equiv 0 \pmod{p} \text{ is not a square } \pmod{p} \end{cases} \quad (3)$$

If $p \equiv 3 \pmod{4}$, the Björck Sequences of length p is defined as:

$$b_p[k] = \begin{cases} \exp(i\theta_p(k)), & \text{if } \left(\frac{k}{p}\right) = -1, \quad k = 0, 1, \dots, N-1 \\ 1, & \text{otherwise} \end{cases} \quad \theta_p(k) = \arccos\left(\frac{1-p}{1+p}\right) \quad (4)$$

For example, when $p = 7$, the Björck sequence is $(1, 1, 1, \exp(i\theta), 1, \exp(i\theta), \exp(i\theta))$, $\theta = \arccos(-3/4)$. The corresponding precoding matrix is graphically depicted in Fig. 1(a), where real and complex numbers are represented in yellow and blue, respectively. It is evident that the proportion of complex numbers in the proposed scheme is less than the OCT scheme illustrated in Fig. 1(b). Björck sequences exhibit a superior ambiguity function compared to Frank-Zadoff-Chu sequences, rendering them suitable for radar applications, especially in the presence of high Doppler shifts. On the other hand, in communication systems, their constant amplitude and perfect autocorrelation properties make them suitable for synchronization [6].

Furthermore, as another attribute to the excessive computation complexity is the large-scale precoding matrices, we propose a multi-group Björck precoding scheme to further alleviate this issue. In this approach, payload subcarriers are divided into l groups, each consisting of m subcarriers. Subsequently, lower-dimensional precoding matrices are used to individually equalize each group, resulting in a substantial reduction in complexity.

3. Experimental setup and results

The experimental setup is illustrated in Fig. 1(c). The number of modulated subcarriers, denoted as M , is set to 107, and the sampling rate of the arbitrary waveform generator (AWG) (Keysight M8199A) is 256 GSa/s. These parameters are varied to accommodate the requirement of a prime length for the multi-group Björck sequences while maintaining the bit rate constant. The generation of the time-domain signal involves several steps, including single/multi-group Björck precoding, Hermitian extension, a 256-point Inverse Fast Fourier Transform (IFFT), cyclic prefix (CP) insertion, and parallel-to-serial (P/S) conversion. Subsequently, the signal is amplified by an electrical amplifier (EA) and is employed to modulate a continuous-wave (CW) laser operating at 1550 nm using a Mach-Zehnder modulator (MZM). The optical signal is launched into a 1-km single-mode fiber (SMF). At the receiver side, the optical signal undergoes amplification through an erbium-doped fiber amplifier (EDFA) preamplifier. A variable optical attenuator (VOA) is utilized to regulate the received optical power. The signal is subsequently detected by a photodetector (PD) and sampled by a 256-GSa/s oscilloscope (Keysight UXR0592AP).

We first investigate the impact of data rates on the conventional OFDM, OCT precoding, and the proposed scheme, under the bandwidth limitation of the back-to-back (B2B) system configuration. The BER performance at various data rates is depicted in Fig. 2(a). The results demonstrate that, across different data rates, the proposed scheme consistently achieves superior or comparable performance compared to OCT. Most importantly, the proposed scheme exhibits significantly fewer complex multiplications, resulting in lower signal processing latency at the transmitter and receiver side. This is further illustrated in Fig. 3(a), the proposed scheme reduces the number of complex multiplications by approximately ~50% compared to OCT, across various numbers of payload subcarriers.

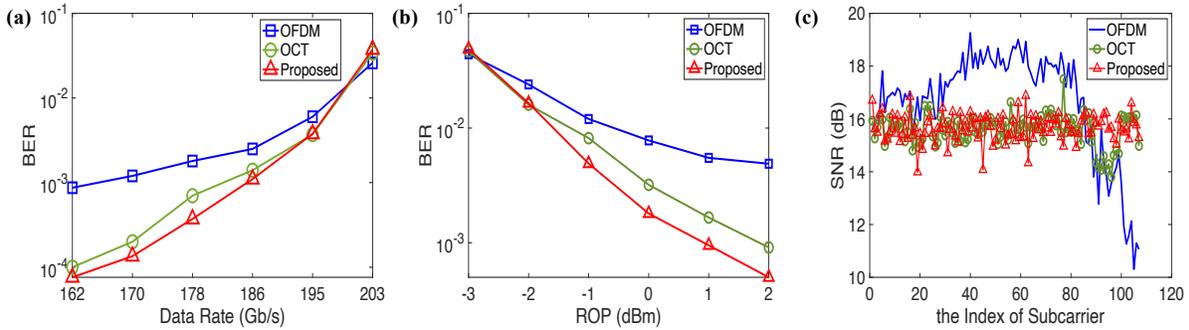


Fig. 2. BER versus (a) the data rates with the B2B scenario, and (b) the ROP with 1-km fiber under different precoding schemes with 146-Gb/s data rates. (c) The SNR versus the index of subcarrier using different precoding schemes.

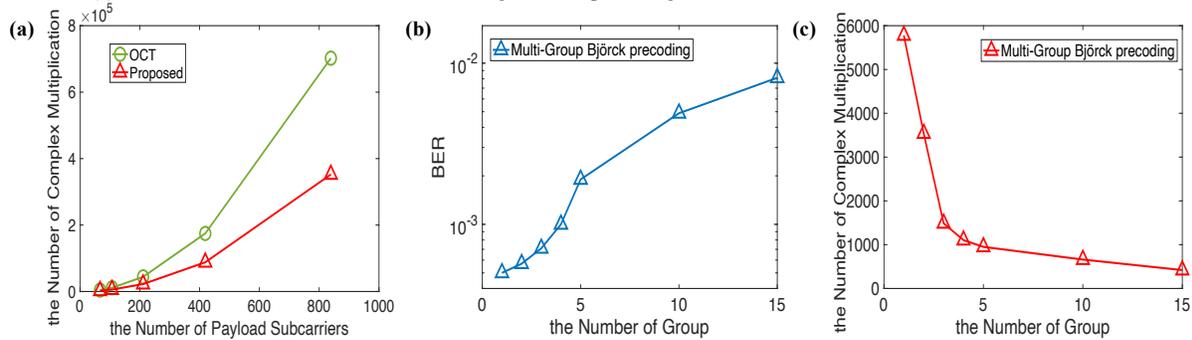


Fig. 3. (a) the complexity analysis of proposed single group Björck precoding. (b) BER, and (c) the number of complex multiplication versus the number of groups using multi-group Björck precoding.

This reduction is attributed to the inherently lower complexity of the proposed Björck precoding scheme. Fig. 2(b) presents the BER performance of different schemes at various received optical power (ROP) after 1 km of fiber transmission. The proposed scheme retains its advantage of achieving better performance with lower complexity. This advantage is derived from its more evenly distributed signal-to-noise ratio (SNR) across different subcarriers, as illustrated in Fig. 2(c).

To further minimize the additional processing latency introduced by precoding, we explore the multi-group Björck precoding scheme, which re-distributes the SNR under a smaller range of subcarriers as shown in Fig. 1(d). In Fig. 3(b) and 3(c), we examine the BER performance and the complexity of the multi-group Björck precoding scheme for different numbers of groups at 146 Gb/s. It is observed that an increase in the number of groups leads to further complexity reduction, at the cost of BER degradation. When considering an acceptable penalty in BER performance (BER < FEC Threshold), a substantial overall complexity reduction of nearly ~91.54% can be achieved.

4. Conclusion

In this paper, we proposed a low-complexity single-group Björck precoding scheme, which achieves superior BER performance compared to traditional approaches while reducing complexity by ~50%. Furthermore, we introduce the multi-group Björck precoding scheme, which achieves a tradeoff between complexity and BER performance. Under acceptable BER degradation, this approach can yield a substantial complexity reduction of ~91.54%, compared to that of the OCT scheme. *This work was supported in part by HKSAR GRF 14219322.*

5. Reference

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