Double-stage Carrier Frequency Offset Estimation Using the Eigenvalue and Scattering Coefficient *b* in the Nonlinear Fourier Transform

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Abstract: We propose a novel method to estimate the carrier frequency offset (CFO) using the eigenvalue and scattering coefficient b in the NFT. Our experiments demonstrate fine CFO estimation below 10 kHz for the proposed method.

1. Introduction

Optical eigenvalue modulation [1] using the inverse scattering transform (IST) [2] is a promising technology for overcoming the Kerr nonlinearity limit in optical fiber communication systems [3]. IST is a well-known nonlinear Fourier transform (NFT). Eigenvalues of the eigenvalue equations associated with the nonlinear Schrödinger equation are invariant during optical fiber propagation, even though the waveform and spectrum are changed by the effects of fiber dispersion and nonlinearity. To increase the transmission capacity, various NFT-based transmission methods using multilevels and eigenvalues have been proposed, e.g., 64-QAM of the nonlinear spectrum [4] and 4096-ary signal transmission based on on–off encoding of 12 eigenvalues [5].

Eigenvalue communication uses a generic digital coherent receiver to obtain complex envelop amplitude for an optical signal, and detect the eigenvalue and scattering coefficients. Carrier frequency offset (CFO) estimation is necessary to estimate and compensate for the wavelength mismatch between the carrier signal and the local oscillator (LO) because CFO induces shifts of eigenvalues and scattering coefficients, which results in symbol errors. Recently, several CFO estimation methods based on NFT have been proposed [6], [7]. We proposed an eigenvalue-based estimation [6], which could cover the wide CFO range of ± 5 GHz, however, the estimation accuracy was approximately 1 MHz. Although the method based on coefficient b can potentially achieve high accuracy, the estimation range is limited, and the method has not been experimentally validated [7]. An estimation method that can simultaneously achieve a high accuracy estimation and cover a wide range CFO for high multilevel eigenvalue modulation has yet to be researched.

In this study, we propose a CFO estimation method that combines the eigenvalue-based and coefficient b-based estimation methods. In the proposed method, a CFO is roughly estimated in the eigenvalue domain at the first stage. At the second stage, a fine CFO estimation is performed using coefficient b. A fine estimation of 10 kHz covering a wide range of CFO of ± 5 GHz was achieved in the simulation. In addition, we experimentally demonstrated a fine estimation accuracy below 10 kHz using the proposed method.

2. Double-stage CFO Estimation Method

Fig. 1 illustrates the proposed CFO estimation method. The method consists of two stages: 1) an estimation based on the eigenvalue; 2) an estimation based on coefficient b. In the eigenvalue transmission, we transmit soliton pulses, which are converted by inverse NFT corresponding eigenvalue ζ and nonlinear spectrum $q_d(\zeta)$ (or b) [3]. Bit data is encoded into states of the eigenvalue or $q_d(\zeta)$ (or b). Then, a receiver detects the eigenvalue or $q_d(\zeta)$ (or b), which is decoded into bit data. In this study, we consider the soliton transmission that has an eigenvalue.

First, we explain the eigenvalue-based CFO estimation at the first stage [6]. In the eigenvalue transmission system, the frequency shift of the received soliton pulse appears as a shift of the real part of eigenvalue $\mathrm{Re}[\zeta]$ in the eigenvalue domain [3]. The relationship between the frequency shift Δf of the time domain signal $u(T)$ and the eigenvalue shift is expressed as follows: $u(T)\exp(-i2\pi\Delta f t_0 T) \Leftrightarrow \zeta - \pi\Delta f t_0$, where t_0 is the base time for signal generation, which satisfies $T = t/t_0$ for real time t. Therefore, the CFO can be estimated by calculating the difference in the real part of the eigenvalue between the transmitted and received soliton pulses. When the real parts of the eigenvalues of the transmitted pulses are zero ($\zeta_\mathrm{t} = 0$), the CFO is calculated by referring only to the eigenvalues of the received pulses. Another approach is to use pilot pulses that have a known eigenvalue ζ_t. The eigenvalue-based estimation can treat a wide range of the CFO, such as ± 5 GHz, however, the estimation accuracy is limited by the conditions of the transceiver and soliton pulse.

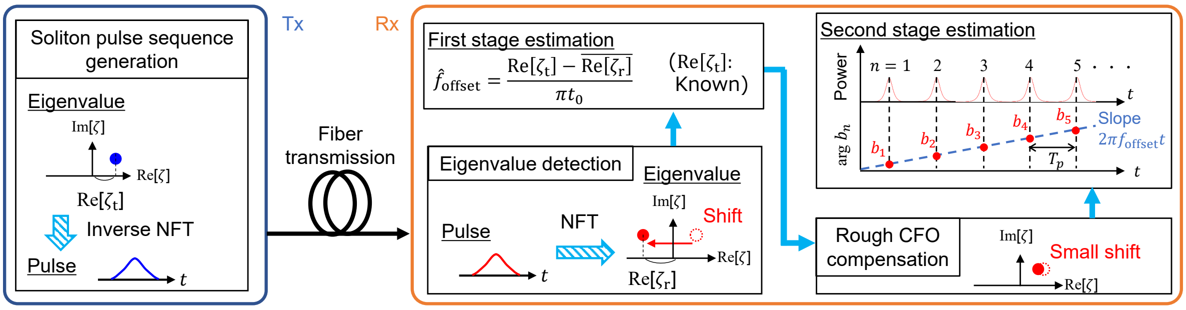

Fig. 1: Overview of the double-stage CFO estimation (proposed method).

In this study, we employ a coefficient b-based estimation after a rough CFO compensation with the eigenvalue-based estimation. In the b-based estimation, a soliton pulse sequence is received, and each b for each soliton pulse is detected. Note that the phase of the soliton component corresponds to the argument of b (arg b). Therefore, the remaining CFO at the second stage can be estimated by

$$\hat{f}_{\text{offset}} = \frac{1}{2\pi T_p (N-1)} \sum_{n=1}^{N} (\arg b_n - \arg b_{n-1}), \tag{1}$$

where T_p and N represent the pulse duration and the number of pulses, respectively. A phase unwrapping process is required when we trace a transition of arg b. Therefore, an applicable CFO range of the b-based estimation is $|\hat{f}_{\text{offset}}| < 1/(2T_p)$. Combining the b-based method with the first stage estimation, we obtain a wide-range and high-accuracy CFO estimation.

3. Simulations

We investigated the characteristics of the proposed CFO estimation method through simulations. The simulation model is shown in Fig. 2. CFO estimation was performed using a fundamental soliton pulse with the eigenvalue of $\zeta = 0.5i$ (Re[ζ] = 0). At the transmitter, the eigenvalue was converted into a soliton pulse and transmitted as an optical pulse train using an IQ modulator. The optical pulses were received by a coherent receiver for the back-to-back configuration. A CFO and phase noise were added on the received pulse. The linewidth of the laser was set to 1 kHz, which induced the phase noise assuming a Wiener process model [8]. Eigenvalues were detected from the received pulses using the Fourier collocation method [9]. At the receiver, CFO was estimated using both methods: the eigenvalue-based [6] and the proposed method. We used 32 pulses for each estimation. The absolute value of the difference between the estimated and actual CFO was defined as the estimation error for evaluating the estimation accuracy. The estimation accuracy was evaluated depending on the pulse duration T_p and the number of sampling points per pulse N_s (sampling rate $R_s = N_s/T_p$). The base time t_0 was set to 200 ps.

Fig. 3(a) shows the estimation error for $T_p = 3.2$ ns and $N_s = 64$ ($R_s = 20$ GSa/s) while depending on the CFO value. The estimation error for the eigenvalue-based method has changed with the period of the fast Fourier transform ($1/T_p = 312.5$ MHz) because we used the Fourier collocation method for the eigenvalue detection. The maximum estimation error was approximately 1 MHz. The coefficient b-based method achieved a fine estimation error below 10 kHz in the CFO range from -161 to 161 MHz, which corresponded to $\pm 1/(2T_p)$. However, the estimation could not work for $|\text{CFO}| > 1/(2T_p)$. On the other hand, the proposed method achieved a consistently fine estimation with an error below 10 kHz over a CFO of ± 5 GHz.

Fig. 3(b) shows the contour chart of the estimation error of the eigenvalue-based and the proposed methods depending on the pulse duration T_p and sampling points N_s (sampling rate R_s). The CFO was set to a fixed value of 120 MHz. For the eigenvalue-based method, the estimation error increased when T_p or R_s were small. This is because the eigenvalue was not detected precisely when the edges of the soliton pulse and spectrum were eliminated by the limited condition. When T_p and R_s are sufficiently large, such as $T_p = 6.4$ ns and $R_s = 20$ GSa/s, the estimation error was approximately 100 kHz. For the proposed method, the estimation error significantly increased when $R_s < 2.5$ GSa/s. This is because the first stage estimation in the eigenvalue domain did not work well. When the sampling rate was 5 GSa/s or more, a fine estimation below 100 kHz was achieved even for $|\text{CFO}| > 1/(2T_p)$ with a large T_p. For $T_p = 6.4$ ns and $R_s = 20$ GSa/s, a estimation accu-

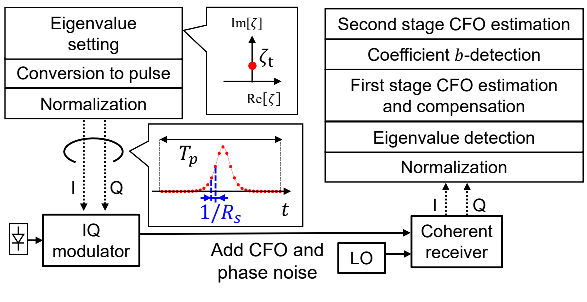

Fig. 2: Simulation model.

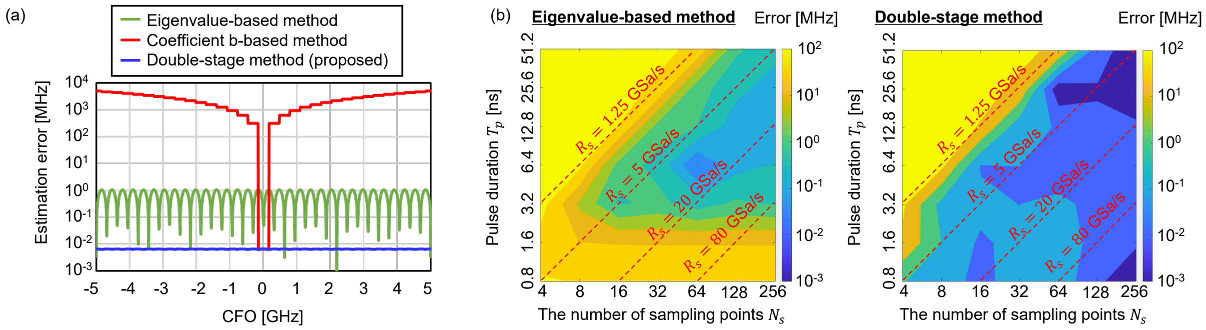

Fig. 3: Simulation results: (a) CFO dependence and (b) contour chart of the estimation error.

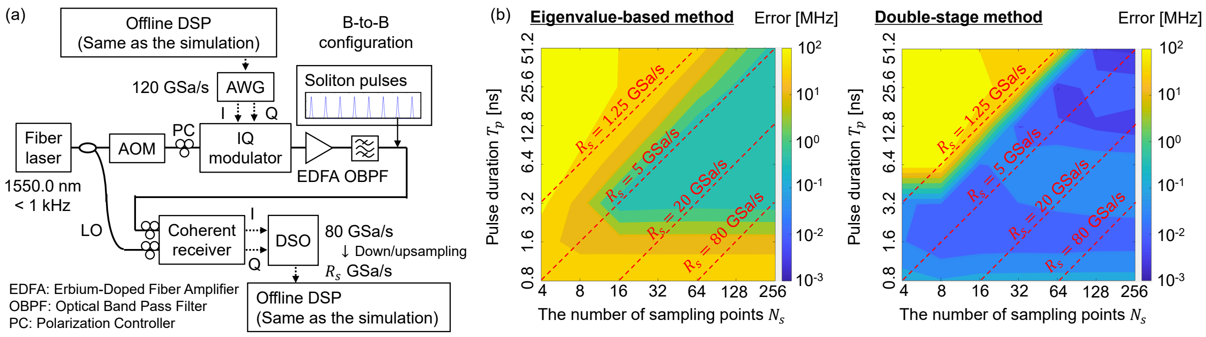

Fig. 4: (a) Experimental setup. (b) Contour chart of the estimation error in the experiment.

racy below 10 KHz was obtained. From the results, we demonstrated that the proposed method can achieve more precise and stable estimation accuracy for a wide range of the pulse duration T_p and sampling rate R_s.

4. Experiments

We investigated the CFO estimation accuracy of the proposed method through experiments for back-to-back configuration. Fig. 4(a) shows the experimental setup. A shared lightwave source with a linewidth < 1 kHz was used for the signal light and LO to provide a static CFO. An acoustic optical modulator (AOM) introduced a static frequency shift of 119.995 MHz on the signal measured using the periodogram method with a long measurement time. An optical soliton pulse was generated using an arbitrary waveform generator (AWG) and an IQ modulator. The optical soliton pulse was received by a coherent receiver, and A/D conversion was performed using a digital storage oscilloscope (DSO) at 80 GSa/s.The CFO was estimated using the eigenvalue-based and the proposed double-stage methods. The other parameters and digital signal processing (DSP) for the pulse generation and CFO estimation were identical to those in the simulation. The estimation accuracy was evaluated varying the pulse duration T_p and sampling points N_s (sampling rate R_s).

Fig. 4(b) shows the contour chart of the estimation error. Compared with the simulation, a similar trend of the estimation error was obtained in the experiments. For the eigenvalue-based method, the estimation error was approximately 1 MHz for $T_p > 3.2$ ns and $R_s > 5$ GSa/s. When we used the proposed method, a better estimation accuracy below 100 kHz was achieved for $T_p > 1.6$ ns and $R_s > 5$ GSa/s. When $T_p = 12.8$ ns and $R_s = 20$ GSa/s, we achieved a fine estimation accuracy below 10 kHz in the experiment.

5. Conclusions

We propose a novel CFO estimation method based on a double-stage estimation using the eigenvalue and the scattering coefficient b. The proposed method achieve a fine CFO estimation below 10 kHz in the experiment.

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