Mode Coupling in Optical Fibers

Luca Palmieri
Department of Information Engineering, Unviersity of Padova, Via Gradenigo 6B, 35131 Padova, Italy
luca.palmieri@unipd.it

Abstract: Mode coupling plays a crucial role in spatial-division-multiplexed transmission systems. This paper review and explores new approaches to modelling and characterization of mode coupling in multicore and multimode fibers. © 2024 The Author(s)

1. Introduction

Spatial division multiplexing (SDM) is the widely accepted solution to cope with the steady increase of capacity demand in optical fiber communication systems [1,2]. Different technologies are needed to implement an SDM system, including optical fibers, multiplexers, amplifiers, and DSP algorithms. Among these, optical fibers play a key role influencing the development of the others. SDM can be implemented using either few-mode fibers (FMF) or multi-core fibers (MCF). MCF can be further distinguished in uncoupled-core (UC) and coupled-core (CC), depending on whether the cores can be treated as almost independent transmission channels or not. Commonly, UC-MCFs have single-mode cores, but there is an interest also in UC-MCF with few-mode cores [2].

Independently of the kind of fiber, transmission across an SDM link can be described in the frequency domain by $\mathbf{a}_{\text{out}}(\omega) = \mathbf{F}(\omega)\mathbf{a}_{\text{in}}(\omega)$, where $\mathbf{a}(\omega)$ is the vector of complex spectra of the M guided modes (counting also polarization modes), and $\mathbf{F}(\omega)$ is the $M \times M$ generalized Jones matrix. The matrix $\mathbf{F}$ and its dependence on frequency set the modal delays (MD) and the mode-dependent loss (MDL) of the link, the two main parameters that limit the capacity and impact the complexity of the transmission system [3]. Important as they are, these parameters describe the fiber as a whole, and do not provide physical insight on what is causing them. The origin of these propagation distortions lays in the way in which the modes interact locally along the fiber. This is described by the coupling matrix $\mathbf{K}(z)$, which controls how the transmission matrix $\mathbf{F}$ varies along the fiber, according to $d\mathbf{F}/dz = -j[\boldsymbol{\beta} + \mathbf{K}(z)]\mathbf{F}(z)$ [4], where $\boldsymbol{\beta}$ is the diagonal matrix of the propagation constants. Therefore, modelling and measuring mode coupling in optical fiber resorts to modelling and measuring the matrix $\mathbf{K}(z)$, or some associated quantity. This work reviews main approaches and new perspectives in mode coupling characterization.

2. Modelling mode coupling

Theoretical modelling of mode coupling is performed through the well established coupled mode theory (CMT), according to which the elements of the matrix $\mathbf{K}(z)$ can be calculated as

$$K_{\mu,\nu}(z) \approx \iint_A \mathbf{E}_\mu^*(\mathbf{x}_t)\left[\varepsilon(\mathbf{x}_t,z) - \bar{\varepsilon}(\mathbf{x}_t)\right]\mathbf{E}_\nu(\mathbf{x}_t)d\mathbf{x}_t\,, \tag{1}$$

where $\mathbf{E}_{\mu,\nu}$ are the electric fields of the modes, ε and $\bar{\varepsilon}$ are the dielectric tensors describing the perturbed fiber and the ideal reference one, respectively, $\mathbf{x}_t$ is the vector of transverse coordinate, and A is the fiber cross section. While the values of the above integral depend on the applied perturbations, some properties have general validity. Note that (1) holds only in the absence of losses, thus it cannot described MDL. A neat generalization to include MDL is possible [5], but not considered here for the sake of simplicity and clarity. Reciprocity of the fiber (which is always guaranteed unless the fiber is exposed to a strong magnetic field) implies that $\mathbf{K}$ is symmetric [4].

While in general the coupling matrix is complex, it can be shown that in the very common case of weakly guiding fibers the imaginary part of $\mathbf{K}$ is nonzero only if the fiber is twisted (or exposed to a very intense magnetic field) [4]. Given that usually telecommunication fiber are not strongly twisted (neither they are exposed to strong magnetic fields), a fairly accurate model can be based on a real coupling matrix. Twist induces on the fiber a torsional stress that increases radially and couples the transverse components of the field with the longitudinal one. The resulting effect is a rotation of the mode both in polarization and spatial orientation. Remarkably, however, this rotation occurs in a direction that is opposite to the one of the twist and by an amplitude proportional to about $g \approx 0.07$ that of the twist [6]. This occurs both in FMFs and in MCFs. Figure 1(a) shows for example measurements performed on a core of a twisted UC-MCF. The MCF was laid straight and twisted by different numbers of turns in a point between two fixed positions. The angle of the birefringence orientation was measured for each applied twist by standard polarization sensitive reflectometry [7]. The graph reports this measured angle and the fact that the traces match the applied number of turns confirms the value of g predicted by the theory.

All the other sources of perturbation impact only on the real part of $\mathbf{K}$; among these, bending is of great practical interest. Bending induces a complex stress field made of a longitudinal and a transverse component [9]. Owing to

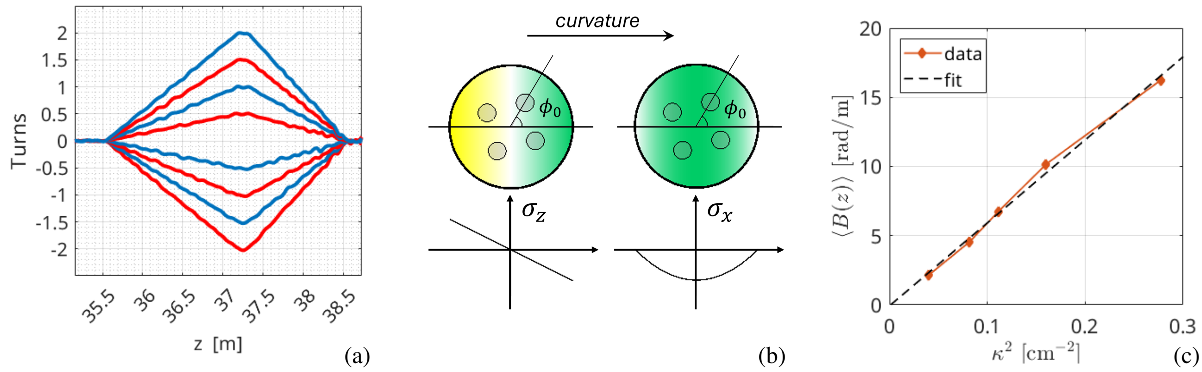

Fig. 1: (a) Angle of birefringence measured as a function of distance along a core of a UC-MCF. The fiber was fixed at 35.5 m and 38.5 m, and twist at 37.25 m. (b) Schematic representation of the stress fields induced by bending. (c) Dependence on the curvature κ of $\langle B(z)\rangle$, a quantity related to the bending birefringence by a complex expression (see [8] for more details). Markers: experimental values; solid line: theoretical model.

symmetry considerations, the longitudinal component has no effect on single-mode fibers. Differently, on FMFs it induces strong coupling between modes of different orders, such as for example the $LP_{0,1}$ and the $LP_{1,1}$ [9]. The longitudinal component of the stress is immaterial also in MCFs, despite the cores are not on the fiber axis. In MCFs, however, the effect of bending depends on the fiber orientation (Fig. 1(b)). It can be shown that in UC-MCF, the birefringence of the core is $\beta_{SMF}[1-(R_C/R_{cl})^2\cos^2\phi_0]$, where R_{cl} is the cladding radius, R_C is the distance of the core center from the fiber axis, ϕ_0 its orientation with respect to the bending direction, and β_{SMF} is the bending birefringence experienced by a single-mode fiber in the same conditions [8]. The verification of this model is hampered by the fact that the cores' orientations along the fiber are in general unknown; nevertheless, fig. 1(c) shows a promising preliminary experimental result.

Besides the stress effects considered above, another source of mode coupling are geometrical deformations with respect to the ideal shape. Despite these can still be described by (1), the accuracy is hindered by the fact that the mode of the reference fiber does not respect the boundary conditions of the perturbed one. A more accurate approach based on the theory transformation optics (TTO) has been recently proposed [5]. The method uses TTO to convert the geometric deformation in material anisotropy and then describes the effects of this anisotropy in terms of CMT. This process induces however both dielectric and magnetic anisotropy; therefore, the coupled mode equation becomes $d\mathbf{F}/dz = -j[\boldsymbol{\beta}+\mathbf{K}(z)+\mathbf{C}(z)]\mathbf{F}(z)$, where the elements of $\mathbf{C}(z)$ are given by an expression similar to (1) involving, however, magnetic fields and magnetic tensors [5].

3. Measuring mode coupling

Experimental characterization of mode coupling can be performed in a few different ways. For example, Ref. [10] analyze the power coupling among modes, and Ref. [11] measure the average MD accumulated along the link. Nevertheless, strictly speaking, the complete characterization of mode coupling requires the measurement of the matrix $\mathbf{K}(z)$ along the fiber. This task is however quite formidable. Distributed mode coupling characterization is based on the analysis of the Rayleigh backscattered light described by $\mathbf{b}(\omega)=\mathbf{R}(\omega)\mathbf{a}_{\text{in}}(\omega)$, where [4]

$$\mathbf{R}(\omega)=j\int_0^L \tilde{\mathbf{B}}(z,\omega)\left\{e^{-j\boldsymbol{\beta}z}\mathbf{S}(z)e^{-j\boldsymbol{\beta}z}\right\}\tilde{\mathbf{F}}(z,\omega)dz\,, \qquad S_{\mu,\nu}(z)\approx\iint_S \delta\varepsilon(\mathbf{x}_t,z)\,\mathbf{E}^*_\mu(\mathbf{x}_t)\mathbf{E}_\nu(\mathbf{x}_t)d\mathbf{x}_t\,, \tag{2}$$

where L is the fiber length, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$ represent forward and backpropagation in a reference frame where the phase delays of the modes are compensated, $\mathbf{S}$ is the Rayleigh backscattering matrix with elements $S_{\mu,\nu}$, and $\delta\varepsilon$ describes the random fluctuations of silica permittivity due to its amorphous nature. The elements of $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{B}}$ vary on the scale of the beat lengths between spatial modes and between polarization modes; these go from fractions of millimeters to several meters. Differently, the factor in curly braces gives raise to terms like $\sigma_{\mu,\nu}(z)=\exp[-j(\beta_\nu+\beta_\mu)z]S_{\mu,\nu}(z)$, which vary on the scale of the wavelength. Note that these factors are originated when mode μ backscatters light on mode ν (or vice versa). Owing to the randomness of $\delta\varepsilon$, the quantities $\sigma_{\mu,\nu}$ are random processes known as *Rayleigh signatures* [4]. Since $\beta_\mu=\omega n_\mu/c_0$, and interesting feature of Rayleigh signatures is that a variation in β_μ is equivalent to a spectral shift. Therefore, cross-correlating the spectrum of two different signatures, reveals the difference between the involved propagation constant [12].

As an example, Fig. 2(a) shows the spectral shift measured along a short section of a 2-mode fiber: the correlation peak is associated to the beating between $LP_{0,1}$ and $LP_{1,1}$ modes. According to (2), we should expect that a FMF with 2 mode groups generates 3 signatures given by (i) the $LP_{0,1}$ scattering on itself, with propagation constant $2\beta_1$, (ii) the $LP_{1,1}$ scattering on itself, with propagation constant $2\beta_2$, and (iii) the $LP_{0,1}$ scattering on

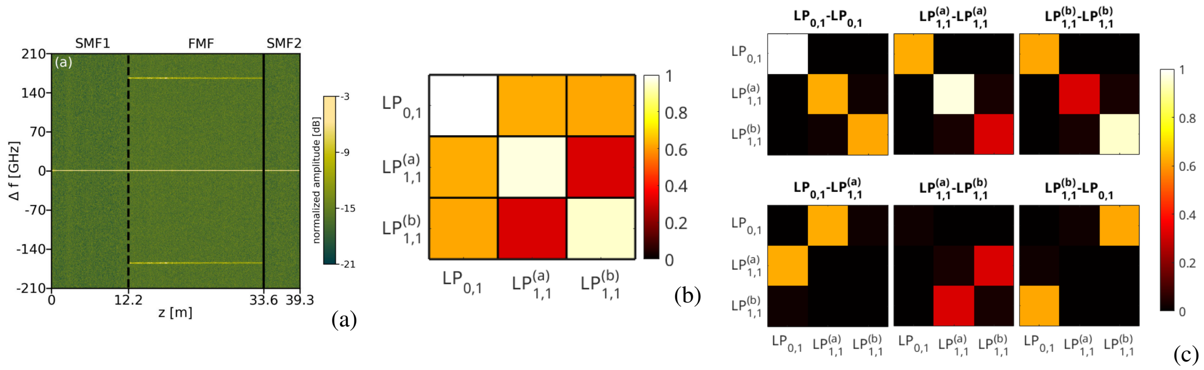

Fig. 2: (a) Spectral correlation between signatures performed on a 2-mode FMF. The spectral shift at 166 GHz (and its negative replica) indicate a modal birefringence of about 1.27×10^{-3} [12]. (b) Intensity of the backscattering from one mode on to another. (c) Cross-variances of signatures: the title of each tile indicates the reference signature; each pixel of the tiles indicate the cross-variance of the reference with the corresponding signature.

the $LP_{1,1}$ (and vice versa), with propagation constant $\beta_1 + \beta_2$. While the spectral correlation between the first two signatures is characterized by a frequency shift proportional to $|\beta_1 - \beta_2|$, the spectral correlation of these two with the third should give a peak at $|\beta_1 - \beta_2|/2$, which is however missing. The presence of this peaks requires however two conditions. The first is that one mode scatter on to the other, which depends on the standard deviation of the overlap integrals in (2) [4]. Figure 2(b) shows these standard deviations numerically evaluated on a 2-mode step-index fiber; indeed, there is a non-negligible scattering between modes [13]. The second condition is that the different signatures are correlated. The fact that the coefficients $S_{\mu,\nu}(z)$ are generated by the same random quantity $\delta\varepsilon$ suggests that there is correlation; yet, the presence of different mode fields can fade it away. This is confirmed in fig. 2(c), where the value of $\langle S_{\mu,\nu} S_{\mu',\nu'} \rangle$ for any two couples of modes are shown. It is evident that signatures of the kind $S_{\mu,\mu}$ do not correlate with signatures $S_{\mu',\nu'}$ with $\mu' \neq \nu'$. This explains the lack of the peak at $|\beta_1 - \beta_2|/2$.

This analysis can be applied to fiber with more modes, including MCFs; nevertheless, the complexity scales up quickly. The interpretation of the signatures spectral correlations requires an *a priori* knowledge of the statistical properties of the scattering matrix **S**, which can be obtained by numerical analysis. As it will be shown with more detail in an extended version of this work, despite all these difficulties the analysis of the Rayleigh signatures, including the analysis of their auto-correlation (not discussed here for the lack of space), can provide a deep insight on the origin of mode coupling in SDM fibers.

This research was partly funded by the Italian Ministry for University (MIUR, PRIN 2017 – project FIRST).

References

1. R.-J. Essiambre, R. Ryf, N. Fontaine, and S. Randel, "Space-Division Multiplexing in Multimode and Multicore Fibers for High-Capacity Optical Communication," IEEE Photonics J. **5**, 0701307–0701307 (2013).
2. B. J. Puttnam et al., "Space-division multiplexing for optical fiber communications," Optica **8**, 1186–1203 (2021).
3. K.-P. Ho and J. Kahn, "Linear Propagation Effects in Mode-Division Multiplexing Systems," J. Light. Technol. **32**, 614–628 (2014).
4. L. Palmieri et al., "Rayleigh-Based Distributed Optical Fiber Sensing," Sensors **22**, 6811 (2022).
5. G. Guerra et al., "Unified Coupled-Mode Theory for Geometric and Material Perturbations in Optical Waveguides," J. Light. Technol. **40**, 4714–4727 (2022).
6. S. Piccirilli, L. Palmieri, M. Santagiustina, and A. Galtarossa, "Experimental analysis of twist-induced mode coupling in few mode fibers," in *Opt. Fiber Commun. Conf. 2016 Pap. Th1J3,* (Optical Society of America, 2016), p. Th1J.3.
7. D. Orsuti et al., "Multi-core Fibers as a Technological Platform for Distributed Twist Sensing," in *Opt. Fiber Sens.*, (Optica Publishing Group, 2023), pp. Th6–27.
8. M. Cappelletti et al., "Preliminary analysis of polarization effects in bent uncoupled-core multicore fibers," in *Opt. Fiber Sens.*, (Optica Publishing Group, 2023), pp. Th6–73.
9. L. Palmieri, "Coupling mechanism in multimode fibers," in *Photonics West OPTO,* vol. 9009 (SPIE, San Francisco (US-CA), 2014), pp. 90090G–90090G–9.
10. M. Bsaibes et al., "Modal dependence of Rayleigh scattering coefficients based on the bidirectional OTDR technique," Opt. Lasers Eng. **171**, 107795 (2023).
11. S. Ohno et al., "Distributed spatial mode dispersion measurement along strongly coupled multicore fibers based on the correlation analysis of Rayleigh backscattering amplitudes," Opt. Express, OE **25**, 29650–29658 (2017).
12. R. Veronese, A. Galtarossa, and L. Palmieri, "Distributed Characterization of Few-Mode Fibers Based on Optical Frequency Domain Reflectometry," J. Light. Technol. **38**, 4843–4849 (2020).
13. Z. Wang et al., "Rayleigh scattering in few-mode optical fibers," Sci. Rep. **6**, 35844 (2016).