Nonlinear Impairment Scaling in Few-Mode Fiber Transmission Systems with Mode Permutation Technique

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Abstract: Few-mode fiber transmission systems with three states of the permutation strategies are evaluated using a GN model with MDL considered, for the first time. CMP strategy outperforms by 0.3 dB in terms of SNR difference among modes with an MDL of 0.1 dB/km.

1. Introduction

The nonlinear Gaussian noise (GN) model is a sufficiently reliable tool for predicting the nonlinear noise of single-mode fiber (SMF) transmission systems [1-3]. In the past, researchers have made many efforts to improve its accuracy such as an enhanced GN model and extend it to the uncompensated transmission systems based on non-zero dispersion-shifted fibers [4]. With the rapid development of high-capacity space-division multiplexing (SDM) technology, it is also of great significance to generalize the GN model to SDM scenario to evaluate nonlinear impairment scaling in few-mode fibers (FMFs) and multi-core fibers. Recently, the relevant analytical formulas and integral formulas of the few-mode GN model are both proposed [5, 6], considering the strong or weak mode coupling and the coherent accumulation of nonlinear interference (NLI) noise with the span. However, the current few-mode GN model does not include the effect of mode-dependent loss (MDL), one of the most fundamental performance limitations in FMF transmission systems.

Furthermore, the mode permutation technology has been proposed since 2019 [7] and continuously applied to long-haul FMF transmission systems in recent years as an effective way to suppress the accumulation of MDL and mode group delays spreading [7-9]. Till now, careful evaluation on the nonlinear impairment using a GN model has not been carried out for mode-permutation-based SDM systems.

In this work, we improve the few-mode GN model by introducing MDL and study the nonlinear impairment scaling in FMF transmission systems with mode permutation for the first time. It is revealed that MDL increases the difference of nonlinearity between channels, while the mode permutation can dramatically average the NLI noise of each mode similar to the strong coupling. Compared to other permutation strategies, cyclic mode permutation (CMP) strategy can achieve a lowest signal-to-noise ratio difference among modes of <0.3 dB with an MDL of 0.1 dB/km.

2. Theory of nonlinear Gaussian noise model

Nonlinear impairments in FMF transmission systems are caused by intra-mode and inter-mode NLI noise, which are more complex than SMF transmission systems, especially when the mode counts are relatively high. The integral expression of the power spectral density (PSD) $G_{\mathrm{NLI},1span}{}^{(p)}$ in mode p can be expressed as [6]

$$\begin{aligned} G_{\mathrm{NLI},1span}^{(p)}(f_0) = & \overbrace{\tilde{\gamma}_{pppp}^2 \iint_{-\infty}^{\infty} G_{\mathrm{TX}}^{(p)}(f_0+\Delta f_1) G_{\mathrm{TX}}^{(p)}(f_0+\Delta f_2) G_{\mathrm{TX}}^{(p)}(f_0+\Delta f_1+\Delta f_2) \left|\chi\left(\Delta\beta^{(pppp)}\right)\right|^2 d\Delta f_1 d\Delta f_2}^{\text{Intra-modal nonlinear noise}} \\ & \underbrace{+2\sum_{q\neq p} \frac{\tilde{\gamma}_{ppqq}^2}{4} \iint_{-\infty}^{\infty} G_{\mathrm{TX}}^{(p)}(f_0+\Delta f_1) G_{\mathrm{TX}}^{(q)}(f_0+\Delta f_2) G_{\mathrm{TX}}^{(q)}(f_0+\Delta f_1+\Delta f_2) \left|\chi\left(\Delta\beta^{(ppqq)}\right)\right|^2 d\Delta f_1 d\Delta f_2}_{\text{Inter-modal nonlinear noise}}, \end{aligned} \tag{1}$$

where $G_{\mathrm{TX}}{}^{(p)}(f)$ is the PSD of the transmitted signal in mode p at frequency component f, including potentially several WDM channels and the two polarizations. $\chi(\Delta\beta)$ represents the normalized four-wave mixing efficiency. The nonlinear parameters $\tilde{\gamma}_{ppqq}$ between nondegenerate modes for FMFs are defined as $\bar{\gamma}_{ppqq} = (4/3)\big(2N_p/(2N_q+1)\big)(\omega_0 n_2/c\bar{A}_{pq})$ [10]. Further, we improve the above model by introducing MDL term to characterize the real loss difference of each mode channel in FMF links. The inevitable MDL changes the FWM efficiency (especially the inter-mode term), thereby affecting the normalized nonlinear noise power. In the following, the MDL is assumed to be independent of frequency. The intra-mode and inter-mode normalized four-wave mixing efficiency is expressed as

$$\chi\left(\Delta\beta^{(ppqq)}\right) = \int_0^{L_s} e^{-2\alpha_q z} e^{j\Delta\beta^{(ppqq)}(\Delta f_1,\Delta f_2)z} dz, \tag{2}$$

where L_S is the span length, α_q is the transmission loss of the mode q, z is transmission distance and $\Delta\beta$ $(\Delta f_1, \Delta f_2)$ is the phase matching condition parameter [6].

In order to theoretically investigate the effect of MDL and mode permutation technology on the nonlinear impairments of different spatial modes, we use the incoherent Gaussian noise (IGN) model by considering the non-coherent accumulation of NLI noise with spans to simulate and calculate the total NLI noise power spectral density. Link of FMF transmission systems with mode permutation is shown in Fig. 1(a), and the schematic diagram of typical 6-mode permutation strategies is presented in Fig. 1(b). The key of mode permutation is to perform span-by-span spatial channel conversion in a cyclic manner. And different mode permutation strategies have different permutation periods, which greatly affecting the nonlinear noise power of each mode at different transmission distances.

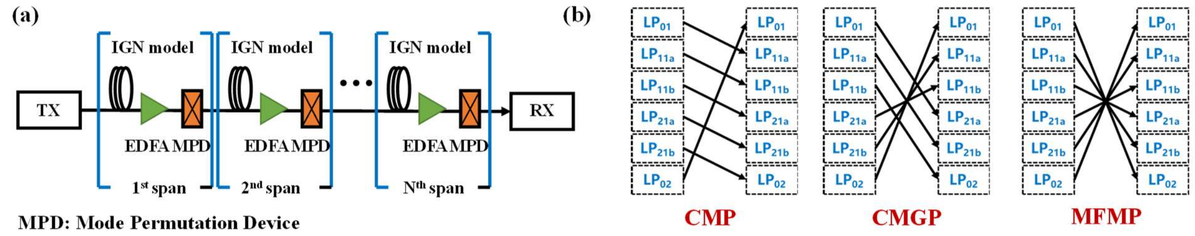

Fig. 1. (a) FMF transmission systems with mode permutation, evaluated using the IGN model, (b) mode permutation strategies (CMP: cyclic mode permutation [7], CMGP: cyclic mode-group permutation [8], MFMP: mirror-flipped mode permutation [9]). The CMP and CMGP schemes [7,8] take 6 and 4 spans as one period of permutation to realize the cyclic shift of all spatial modes, respectively, and the MFMP scheme [9] takes 2 spans as one period of permutation to realize more frequently modes exchanging.

3. Influence of MDL in 6-mode SDM transmission based on mode permutation

3.1 Influence of mode-dependent loss on nonlinear impairment scaling

We present the nonlinear impairment scaling in the 6-mode fiber [11], without loss of generality. The chromatic dispersion is 22.2 ps/(km·nm) for all modes, and DMGD values are 0, 42, 42, -41 ps/km for $LP_{01}/LP_{11}/LP_{21}/LP_{02}$ modes, respectively. Here, the symbol rate is 28 GBaud, the number of WDM channels is 5, the channel spacing is 28 GHz, the input power of each channel is 3 dBm, the span length is 60 km, and the noise figure of amplifiers is 5 dB.

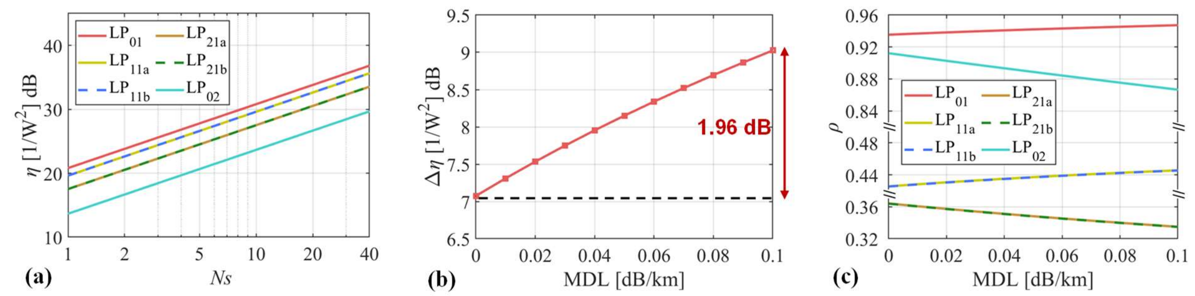

Fig. 2. (a) Normalized NLI noise variance η in dB versus the number of fiber spans N_s, (b) the difference of normalized NLI noise variance $\Delta\eta$ among all spatial modes almost linearly changes by 2 dB with MDL, (c) the nonlinear ratio ρ, defined as the ratio of the nonlinear signal distortion (representing intramodal interactions) to the strength of the overall (intra- + inter-modal), as a function of MDL, which significantly varies from mode to mode and becomes much lower inside a mode group.

To evaluate the influence of MDL on NLI noise, we first focus on the normalized NLI noise variance η, which is defined as the ratio of P_{NLI} to P_{TX}^3 [12]. Figure 2(a) shows η versus the number of spans N_s when all mode losses are 0.2 dB/km, i.e., MDL equals 0 dB/km, without mode permutation. The LP_{01} mode has the largest η due to its largest effective mode field area. The η curves of LP_{11a} and LP_{11b} modes coincide because they are degenerate modes and have equal DMGD and effective mode field areas, the same goes for the LP_{21a} and LP_{21b} modes. Further, we scan the difference of normalized NLI noise variance $\Delta\eta$ among all modes under different MDLs. Note that we set the MDL to be uniformly distributed among the four LP modes, and obtain the results of the MDL on $\Delta\eta$, shown in Fig. 2(b). It can be found that $\Delta\eta$ has an approximate linear relationship with MDL. Surprisingly, $\Delta\eta$ can be greatly increased by 1.96 dB when the MDL reaches 0.1 dB, which often occurs although with the best FMF drawing fabrication technique.

In order to study the influence of MDL on intra-mode and inter-mode NLI noise, we define a ratio of the nonlinear signal distortion, ρ, that represents the ratio of intra-modal interactions to the strength of the overall (intra- + inter-mode) NLI noise [6]. Figure 2(c) shows it for different modes as a function of MDL. For LP_{01} mode, the intra-mode NLI noise is almost unchanged, while the inter-mode NLI noise decreases with the mode loss, which leads to an increase in ρ. For other modes, both intra- and inter-mode terms reduce with the increase of mode loss.

3.2 Nonlinear impairment scaling in mode permutation systems

We further explore the nonlinear impairment scaling of mode-permutation-based FMF transmission systems using the same parameters of the 6-mode fiber mentioned in section 3.1. When all mode losses are 0.2 dB/km, i.e., MDL equals 0 dB/km, the normalized NLI noise variance η versus the number of spans N_s for different mode permutation strategies are shown in Fig. 3(a)&(b), and the shaded area represents the η variation range of the 6 modes. We can see that mode permutation technology can dramatically equalize mode NLI noise compared to non-permutation structure. We could also find that in short distance, MFMP has smaller difference of η due to more frequent interactions between modes. While in long distance, CMP has the smallest difference of η due to the most sufficient mixing between modes.

The GN model treats the effect of NLI on WDM and MDM signals as additive white Gaussian noise, which is statistically independent of amplifier noise and transmitted signal. Under the assumptions of a matched baseband transfer function and no inter-symbol interference, the nonlinear signal-to-noise ratio ($\mathrm{SNR_{NL}}$) that can be directly measured on the electrical signal constellation can be calculated as follows [13]

$$\mathrm{SNR}_{\mathrm{NL}} = \frac{P_{\mathrm{TX}}}{R_S\left(G_{\mathrm{LIN}} + G_{\mathrm{NLI}}\left(f_0\right)\right)}, \quad (3)$$

where R_S is the symbol rate, G_{LIN} is the power spectral density of amplifier spontaneous emission (ASE) linear noise. We calculate $\mathrm{SNR_{NL}}$ as a function of launched power P_{TX} at N_s=2&32 without MDL in Fig. 3(c)&(d). Shaded areas represent the $\mathrm{SNR_{NL}}$ variation ranges of the 6 modes. At a low launch power, the ASE noise of linear effect dominates, while at a high launch power, the NLI noise of Kerr effects dominates. At N_s=2, the mode difference of $\mathrm{SNR_{NL}}$ for the MFMP strategy is relatively small regardless of P_{TX}, because its permutation period equals the span number. At N_s=32, the CMP strategy performs best, reducing the mode difference on NLI noise. Then we consider the actual 6-mode fiber with an MDL of 0.1 dB/km, $\mathrm{SNR_{NL}}$ changes with launched power P_{TX} at N_s=2&32 in Fig. 3(e)&(f). Compared to the case without MDL, the existence of MDL results in relatively big difference of ASE noise among modes, as shown at a low P_{TX}. The CMP strategy outperforms by 0.3 dB in terms of the difference of $\mathrm{SNR_{NL}}$ at N_s=32.

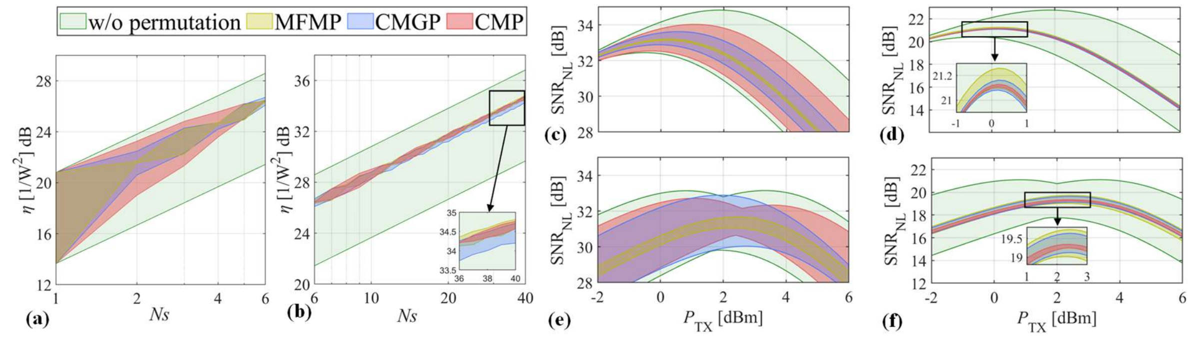

Fig. 3. (a)&(b) Normalized NLI noise variance η vs. the number of spans N_s for different mode permutation strategies; (c)&(d) $\mathrm{SNR_{NL}}$ vs. launched power P_{TX} at (c) N_s=2 and (d) N_s=32 without MDL; (e)&(f) $\mathrm{SNR_{NL}}$ vs. P_{TX} at (e) N_s=2 and (f) N_s=32 with an MDL of 0.1 dB/km. The shaded areas show that the ranges of $\mathrm{SNR_{NL}}$ are significantly broadened due to MDL, providing a more realistic and accurate analysis on the link performance in FMF transmission based on mode permutation.

4. Conclusion

We have studied the influence of MDL and mode permutation technique on nonlinear impairments in FMFs based on the IGN model for the first time. The difference of NLI noise variance has an approximate linear relationship with MDL increases. Moreover, the mode permutation can greatly average the NLI noise of each mode. CMP permutation strategy can achieve a lowest $\mathrm{SNR_{NL}}$ difference among modes of <0.3 dB with an MDL of 0.1 dB/km.

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