Learned Perturbation-Aided Advanced Digital Backpropagation with Nonlinear Compensation Fusion for Subcarrier-Multiplexing Systems

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Abstract: A learned perturbation-aided advanced digital backpropagation (LP-ADBP) with nonlinear compensation fusion is proposed for subcarrier-multiplexing systems. 1-Step and 5-Step LP-ADBP provide similar performance as 5-Step and 10-Step ADBP but save \sim 73.3% and \sim 44.5% complexity, respectively. © 2022 The Author(s)

1. Introduction

Digital subcarrier-multiplexing (SCM) has been proven as a promising solution for 800G and beyond coherent optical fiber transmission systems [1]. However, the performance of a digital SCM system is still limited not only by the self-subcarrier nonlinearity (SSN), but also by the cross-subcarrier nonlinearity (CSN) which can hardly be mitigated via conventional digital backpropagation (DBP). An advanced digital backpropagation (ADBP) is especially proposed for SCM systems [2] based on a simple cross-phase modulation (XPM) model [3] to jointly compensate for SSN and CSN. This algorithm provides extraordinary nonlinear compensation (NLC) performance and becomes an NLC benchmark in SCM systems. Unfortunately, the high complexity burden hinders the practical application of ADBP. Although several modifications have been reported to further reduce the complexity of ADBP [4–7], promoting the practical implementations of NLC in SCM systems still needs further research.

In the past 5 years, learned DBP (LDBP) has attracted extensive attention from researchers [8]. This technique interprets the split step Fourier method (SSFM) of DBP as a deep neural network (NN) by mapping the chromatic dispersion compensation (CDC) and the NLC to a hidden layer and a nonlinear activation function, respectively. This approach fully exploits the potential of conventional DBP and becomes a new NLC benchmark. Furthermore, with the aid of the first-order perturbation analysis, the nonlinear steps of LDBP can be improved [9]. These LDBP-related algorithms may be promising effective and low-complexity NLC solutions in digital SCM systems.

In this paper, we propose a learned perturbation-aided ADBP (LP-ADBP) with nonlinear compensation fusion for SCM systems. By fusing the perturbation-based SSN (PSSN) and CSN compensation together with only one matrix and leveraging the NN to optimize the parameters, LP-ADBP shows a powerful NLC capability with low computational complexity. Verified in a 9-channel 120-GBaud 8-subcarrier dual-polarization 16QAM (DP-16QAM) 1600 km numerical system, 1-Step and 5-Step LP-ADBP provide similar performance compared with 5-Step and 10-Step ADBP but save about 73.3% and 44.5% complexity, respectively.

2. Principle of the proposed LP-ADBP

When considering both self-phase modulation (SPM) and intra-channel cross-phase modulation (IXPM) in the SSN compensation of ADBP for the k-th probe subcarrier based on the first-order perturbation analysis, the SSN compensation can be updated to a PSSN compensation according to [10] as follows:

$$E_{PSSN,k,x/y}(t) \approx E_{CDC,k,x/y}(t) e^{-j\sum_{n}^{\infty} \left((\gamma_{1} + \gamma_{2}) \left| E_{CDC,k,x/y}(t + nT_{S}) \right|^{2} + \gamma_{1} \left| E_{CDC,k,y/x}(t + nT_{S}) \right|^{2} \right) h(n)} - E_{CDC,k,y/x}(t) j \sum_{n}^{\infty} \gamma_{2} E_{CDC,k,x/y}(t + nT_{S}) E_{CDC,i,y/x}^{*}(t + nT_{S}) h(n)$$

$$= E_{CDC,k,x/y}(t) e^{-j\phi_{PSSN,k,x/y}(t)} - E_{CDC,k,y/x}(t) w_{P,k,yx}(t)$$
(1)

where $E_{CDC,k,x/y}(t)$ denotes the x or y polarization signal of the k-th probe subcarrier after CDC. h(n) represents the perturbation coefficients which can be equivalent to the tap weights of an FIR filter [10]. γ_1 and γ_2 are the



Fig. 1. Block diagram of one NLC step in the proposed LP-ADBP.

nonlinear coefficients to be optimized, and T_S denotes the sampling period.

The CSN compensation matrix $M'_k(t)$ in one NLC step of ADBP for the k-th probe subcarrier can be approximated as

$$M'_{k}(t) = \begin{bmatrix} e^{-j\xi\phi_{k,x}(t)} & -\xi w_{k,yx}(t)e^{-j\xi[\phi_{k,x}(t)+\phi_{k,y}(t)]/2} \\ \xi w^{*}_{k,yx}(t)e^{-j\xi[\phi_{k,x}(t)+\phi_{k,y}(t)]/2} & e^{-j\xi\phi_{k,y}(t)} \end{bmatrix}$$
(2)

where ξ is the compensation factor to be optimized. The CSN-induced nonlinear phase noise (NPN) of the x or y polarization $\phi_{k,x/y}(t)$ and the CSN-induced nonlinear polarization cross-talk (NPC) $w_{k,yx}(t)$ can be expressed as

$$\phi_{k,x/y}(t) = \sum_{i \neq k}^{N_{\text{SCM}}} \left(2 \left| E_{PSSN,i,x/y}(t) \right|^2 + \left| E_{PSSN,i,y/x}(t) \right|^2 \right) \otimes h_{i,N}(t)$$
(3)

$$w_{k,yx}(t) = \sum_{i \neq k}^{N_{\text{SCM}}} \left(E_{PSSN,i,x}(t) E_{PSSN,i,y}^*(t) \right) \otimes jh_{i,N}(t)$$

$$\tag{4}$$

where N_{SCM} denotes the number of subcarriers, $\mathscr{F}(h_{i,N}(t)) = H_{i,N}(\omega) = \sum_{m \in \mathcal{M}} \frac{8}{9} \gamma \frac{1 - e^{-\alpha L + j\Delta\beta'_i \omega L}}{\alpha - j\Delta\beta'_i \omega} e^{-j\Delta\beta'_i \omega m L}$ denotes

the CSN transfer function of the *i*-th interfering subcarrier related to the *k*-th probe subcarrier with N spans per NLC step, where $M = \{-\frac{N}{2} + 1, -\frac{N}{2} + 2, ..., \frac{N}{2}\}$. \mathscr{F} and γ represent Fourier transform and fiber nonlinear coefficient, respectively. $\Delta\beta_i' = \beta_2(\omega_i - \omega_k)$ denotes the group velocity difference between the *k*-th and *i*-th subcarrier with β_2 , ω_i , and ω_k denoting the second order dispersion coefficient, the central angular frequency of the *i*-th subcarrier and the *k*-th subcarrier, respectively.

Then, comparing equations (1), (3), and (4), we can find that $\phi_{k,x/y}(t)$ in equation (3) has an identical form as $\phi_{PSSN,k,x/y}(t)$ in equation (1), and $w_{k,yx}(t)$ in equation (4) is similar to $w_{P,k,yx}(t)$ in equation (1). This inspired us to think that the PSSN and CSN compensation may be fused together by sharing the $|\cdot|^2$ and $(\cdot)(\cdot)^*$ terms. Unfortunately, the compensation of CSN depends on the signal after PSSN compensation, which means the NLC compensation fusion reported in [7] is no longer applicable when perturbation analysis is considered. However, if we approximate that PSSN and CSN can be calculated separately, fusing the compensation of PSSN and CSN together is beneficial to save computational complexity. Therefore, we compute PSSN and CSN in parallel and fuse the compensation together with a fusion matrix $P'_k(t)$ written as

$$P'_{k}(t) = \begin{bmatrix} e^{-j(\xi\phi_{k,x}(t) + \phi_{PSSN,k,x}(t))} & -\xi w_{k,yx}(t)e^{-j[\phi_{k,x}(t) + \phi_{k,y}(t)]/2} - w_{P,k,yx}(t) \\ \xi w^{*}_{k,yx}(t)e^{-j[\phi_{k,x}(t) + \phi_{k,y}(t)]/2} + w^{*}_{P,k,yx}(t) & e^{-j(\xi\phi_{k,y}(t) + \phi_{PSSN,k,y}(t))} \end{bmatrix}$$
(5)

Then, one NLC step for the *k*-th probe subcarrier can be expressed as $[E_{NLC,k,x}(t) \quad E_{NLC,k,y}(t)]^T = P'_k(t)[E_{CDC,k,x}(t) \quad E_{CDC,k,y}(t)]^T$. Finally, since ξ , γ , γ_1 , γ_2 , and h(n) need to be optimized jointly, we interpret the above perturbation-aided ADBP into a DNN and leverage the optimization capability of supervised learning. We call this proposed algorithm learned perturbation-aided ADBP (LP-ADBP). The block diagram of one NLC step in LP-ADBP is shown in Fig.1. Mean squared error (MSE) is set as the loss function for backpropagation



Fig. 2. Block diagram of the 9-channel 8-subcarrier SCM simulation system.



Fig. 3. (a) The performance of different algorithms. (b) The complexity of LP-ADBP measured by real multiplications per sample (RMs/Sample) with respect to N_h and steps. (c) The complexity comparison between 5-Step ADBP, 10-Step ADBP, 1-Step LP-ADBP, and 5-Step LP-ADBP.

and we adopt Adam as the optimizer. To cope with the time-varying impairments like the phase noise during the training process, we adopt the static layer training scheme with 3 stages reported in [11]. Symmetry constraint is imposed on the h(n) and $N_h = 11$ taps are considered.

3. System setup and results

The system setup is shown in Fig. 2. A 9×120-GBaud wavelength-division multiplexing (WDM) coherent simulation system with 150-GHz channel spacing is considered. In each channel, 8 subcarriers are equally distributed with DP-16QAM modulation format and root-raised cosine (RRC) pulse shaping (0.1 roll-off). Both the transmitter laser and local oscillator linewidths are set to 100 kHz in order to consider the laser phase noise. For the transmission link, each span consists of an 80-km single-mode fiber ($\beta_2 = -21.684 \text{ } ps^2/km$, $\gamma = 1.3 / W/km$) and an Erbium-doped fiber amplifier (EDFA) with a 5-dB noise figure. After 20-span transmission, the central WDM channel is filtered out with an optical band-pass filter (OBPF) and then coherently received. The offline processing includes frequency offset compensation (FOC), subcarrier de-multiplexing with an ideal brick-wall filter, downsampling, CDC or NLC, matched filtering (MF), adaptive filtering (AF), carrier phase recovery (CPR), and hard decision. $Q^2 = 20\log_{10}[\sqrt{2} \text{ erfc}^{-1}(2 \text{ BER})]$ measures the performance where erfc^{-1} denotes the inverse complementary error function and BER denotes the bit error rate.

The results are shown in Fig. 3 (a) and the complexity of LP-ADBP measured by real multiplications per sample (RMs/Sample) is shown in Fig. 3 (b). Since the training process can be finished before implementation, the complexity of LP-ADBP only adds $2N_h$ RMs/Sample compared with the fusion version of ADBP in [7] due to the consideration of perturbation. For the 1 step per link complexity, the proposed LP-ADBP achieves 0.26-dB Q^2 factor improvement compared with CDC at the optimal launch power, providing about another 0.17-dB Q^2 factor improvement compared to 1-Step ADBP. This improvement is reasonable since 1-Step LP-ADBP is a little more computationally expensive (adding 14 RMs/Sample) than 1-Step ADBP. However,1-Step LP-ADBP provides similar NLC performance as 5-Step ADBP but saves 831 RMs/Sample, which is about a 73.3% complexity reduction as shown in Fig. 3 (c). Besides, 5-Step LP-ADBP performs similarly to 10-Step ADBP with only a 0.04-dB Q^2 factor penalty at the optimal launch power. Meanwhile, as shown in Fig. 3 (c), 5-Step LP-ADBP only requires 1204 RMs/Sample, saving about 44.5% complexity compared with 10-Step ADBP. Above all, the proposed LP-ADBP is a more cost-effective NLC algorithm for the SCM system.

4. Conclusion

To promote the practical application of NLC in SCM systems, we propose an LP-ADBP algorithm by fusing the perturbation-based SSN and CSN compensation together with only one matrix and leveraging the NN to optimize the parameters. 1-Step and 5-Step LP-ADBP provide similar performance as 5-Step and 10-Step ADBP but save \sim 73.3% and \sim 44.5% complexity, respectively. With the cost-effective advantage, LP-ADBP may be considered a promising NLC algorithm in SCM systems.

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