A Closed-form Expression for the ISRS GN Model Supporting Distributed Raman Amplification

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Abstract: A closed-form model for the nonlinear interference in distributed Raman amplified links is presented, the formula accounts for both forward and backward pumping. The model accurately estimates the received SNR over a 10 THz bandwidth. © 2022 The Author(s)

1. Introduction

Ultra-wideband (UWB) transmission has attracted considerable attention in recent years as a cost-effective solution to satisfy the ever-increasing volumes of data traffic. To achieve real-time prediction of the performance of UWB optical fibre transmission systems, approximations in closed form are used. Of particular interest there are closed-form expressions derived using the inter-channel stimulated Raman scattering (ISRS) Gaussian noise (GN) model [1], due to their simplicity and efficiency in estimating the NLI in UWB systems. This model includes the effect of ISRS and approximations in closed-form have been developed in [2–4]. However, among the weaknesses of these models are that the closed-form expressions do not support distributed Raman amplification (DRA), as they were developed for lumped amplifier solutions. A closed-form expression supporting forward (FW) Raman amplification in the presence of ISRS has recently been proposed in [5], however, it was validated only over C-band systems.

In this work, we have developed a closed-form expression of the ISRS GN model [1] supporting both separate and combined FW-DRA and backward (BW) DRA. This was enabled by deriving for the first time a semi-analytical solution to model the signal profile in the presence of DRA and ISRS. The proposed closed-form formulation is valid for arbitrary-order DRA, Gaussian constellations and supports an arbitrary number of Raman pumps. The derived expression was validated by simulations of signal transmission with 10 THz bandwidth using experimentally measured Raman gain spectrum and wavelength-dependent attenuation profile. This work also represents the first closed-form expression supporting both FW-DRA and BW-DRA in the presence of ISRS.

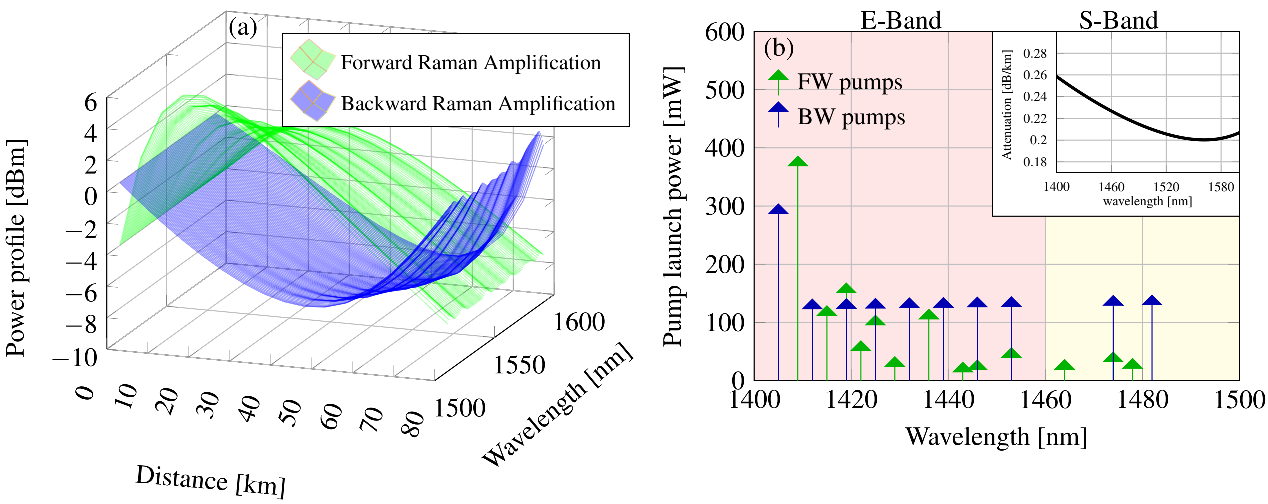

Fig. 1. (a) Per-channel launch power evolution along the fibre distance for FW-DRA (green) and BW-DRA (blue). (b) Attenuation profile and pumps' power and wavelength allocation which yields the power profiles shown in figure (a).

2. The closed-form expression

The signal-to-noise ratio for the i-th channel (SNR_i) at the end of the span after amplification can be estimated as $\text{SNR}_i^{-1} \approx \text{SNR}_{\text{NLI},i}^{-1} + \text{SNR}_{\text{ASE},i}^{-1} + \text{SNR}_{\text{TRX},i}^{-1}$, where $\text{SNR}_{\text{NLI},i}$, $\text{SNR}_{\text{ASE},i}$ $\text{SNR}_{\text{TRX},i}$ originate from fibre nonlinearity, amplifier noise and transceiver noise, respectively. This work is devoted to the calculation of $\text{SNR}_{\text{NLI},i}$. The first step in the derivation of the closed-form expression is to find a suitable function to represent the signal power evolution along the fibre distance in the presence of DRA. To that end, a formula similar to the one proposed in [6] is derived accounting for FW and BW Raman pumps; this formula is then approximated using a first order Taylor expansion, such that the normalised signal profile $\rho(f_i,z) = P(z,f_i)/P(L,f_i)$ is represented as $\rho(z,f_i) = e^{-\alpha z}[1-(C_f P_f L_{eff} + C_b P_b \tilde{L}_{eff})(f_i - \hat{f})]$ [7], with $L_{eff}(\zeta) = (1-e^{-\alpha_f z})/\alpha_f$ and $\tilde{L}_{eff}(\zeta) = (e^{-\alpha_b (L-z)} - e^{-\alpha_b L})/\alpha_b$, where f_i is the frequency of the channel of interest, L is the span length, α, α_f and α_b are the fibre attenuation at the signal, FW- and BW-DRA wavelengths, respectively, $\hat{f}$ is the average frequency of the FW and BW pumps, P_f, and P_b are the total launch power respectively from the WDM channels together with any FW pumps, and the BW pumps, C_f and C_b is the slope of a linear regression of the normalized Raman gain spectrum. The coefficients α, C_f, C_b, α_f, α_b are channel-dependent parameters and matched using nonlinear least-squares fitting to reproduce the solution of the Raman differential equations in the presence of DRA. These parameters model respectively the fibre loss, the gain/loss due to FW-DRA and BW-DRA together with ISRS and how fast the channel gain/loss due to the FW-DRA and BW-DRA together with ISRS extinguishes along the fibre.

Using the proposed semi-analytical solution of the Raman equations for $\rho(f_i,z)$, the $\text{SNR}_{\text{NLI},i}$ can be obtained as

$$\begin{aligned}
\text{SNR}_{\text{NLI},i}^{-1} \approx T^2 \sum_{\substack{0\le l_1+l_2\le 1\\ 0\le l_1'+l_2'\le 1}} \left(\frac{-\tilde{T}_f}{T}\right)^{l_1+l_1'} \left(\frac{\tilde{T}_b}{T}\right)^{l_2+l_2'} \Bigg(& \frac{16}{27}\frac{\pi\gamma^2 P_i^2 n^{1+\varepsilon}}{B_i^2 \phi_i(\alpha_l+\alpha_l')} \Bigg\{ 2(\kappa_f \kappa_f' + \kappa_b \kappa_b') \left[\text{asinh}\left(\frac{3\phi_i B_i^2}{8\pi\alpha_l}\right) + \text{asinh}\left(\frac{3\phi_i B_i^2}{8\pi\alpha_l'}\right)\right] \\
& + 4\ln\left(\sqrt{\frac{\phi_i L}{2\pi}} B_i\right) \Bigg[-(\kappa_f\kappa_b' + \kappa_b\kappa_f')\left(\text{sign}\left(\frac{\alpha_1}{\phi_i}\right)e^{-|\alpha_1 L|} + \text{sign}\left(\frac{\alpha_1'}{\phi_i}\right)e^{-|\alpha_1' L|}\right) + (\kappa_f\kappa_b' - \kappa_b\kappa_f')\Big(\text{sign}(-\phi_i)\, e^{-|\alpha_1 L|} \\
& + \text{sign}(\phi_i)\, e^{-|\alpha_1' L|}\Big)\Bigg]\Bigg\} + \frac{32}{27}\sum_{k=1,k\neq i}^{N_{\text{ch}}} \frac{n\gamma^2 P_k^2}{\phi_{i,k} B_k (\alpha_l + \alpha_l')} \Bigg\{ 2(\kappa_f\kappa_f' + \kappa_b\kappa_b')\left[\text{atan}\left(\frac{\phi_{i,k} B_i}{2\alpha_l}\right) + \text{atan}\left(\frac{\phi_{i,k} B_i}{2\alpha_l'}\right)\right] \\
& + \pi\Bigg[-(\kappa_f\kappa_b' + \kappa_b\kappa_f')\left(\text{sign}\left(\frac{\alpha_1}{\phi_{i,k}}\right)e^{-|\alpha_1 L|} + \text{sign}\left(\frac{\alpha_1'}{\phi_{i,k}}\right)e^{-|\alpha_1' L|}\right) + (\kappa_f\kappa_b' - \kappa_b\kappa_f')\left(\text{sign}(-\phi_{i,k})\, e^{-|\alpha_1 L|} + \text{sign}(\phi_{i,k})\, e^{-|\alpha_1' L|}\right)\Bigg]\Bigg\}\Bigg).
\end{aligned} \tag{1}$$

In this equation, $T_f = -[P_f C_f (f-\hat{f})]/\alpha_f$, $T_b = -[P_b C_b (f-\hat{f})]/\alpha_b$, $T = 1 + T_f - T_b e^{-\alpha_b L}$, $\alpha_1 = \alpha + l_1\alpha_f - l_2\alpha_b$, $\kappa_f = e^{-(\alpha + l_1\alpha_f)L}$, $\kappa_b = e^{-l_2\alpha_b L}$, $\phi_i = -4\pi^2(\beta_2 + 2\pi\beta_3 f_i)$, $\phi_{i,k} = -4\pi^2(f_k - f_i)[\beta_2 + \pi\beta_3(f_i + f_k)]$, n is the number of spans, P_i is the channel launch power with bandwidth B_i, γ is the nonlinear coefficient, N_{ch} is the number of channels, ε is the coherent factor. The parameters β_2 and β_3 are, respectively, the group velocity dispersion parameter and its linear slope. A detailed derivation of Eq. (1) can be found in [7].

3. Transmission setup and pumps optimisation

The transmission setup consists of a WDM signal with N_{ch}=101 channels spaced by 100 GHz and centred at 1550 nm. Each channel was modulated at the symbol rate of 96 GBd, with Gaussian symbols. This setup results in a total bandwidth of 10 THz, ranging from 1511 nm to 1592 nm. The transmission bandwidth was chosen such that all the Raman pumps were at least 4 THz away from the lowest-wavelength channel such that the impact of the signal-pump and pump-pump interaction, such as cross-phase modulation, in the NLI could be neglected [8]. The span length considered is 80 km and an ITU-T G652.D fibre is considered with Raman gain spectrum as shown in [9], nonlinear coefficient and dispersion parameters are $\gamma = 1.16$ W^{-1}km^{-1}, $D = 17$ ps nm^{-1}km^{-1}, $S = 0.0895$ ps nm^{-2}km^{-1}, respectively.

For the DRA, the number of pumps, and their wavelengths and powers are chosen based on an "find minimum of constrained nonlinear multivariable" optimisation algorithm. Two scenarios were considered: FW-DRA and BW-DRA. For FW-DRA, pumps are optimised such that half of the launch power is recovered on the receiver, while for BW-DRA pumps are optimised such that the launch power is fully recovered in the receiver. For both scenarios, the number of pumps is chosen such that the received power variation between the channels is less than 1 dB. For both scenarios, the per-channel power profile along the distance are shown in Fig. 1.a and the optimised pumps' wavelengths and powers that yield this profile are shown in Fig. 1.b, considering a spectrally uniform launch power profile, where each channel carries 0 dBm for BW-DRA and -4 dBm for FW-DBA. Note that for FW-DRA, lower per-channel launch power is chosen and half of the power is recovered in order to limit the per-channel-power peak along the distance to less than 6 dBm, as shown in Fig. 1.a.

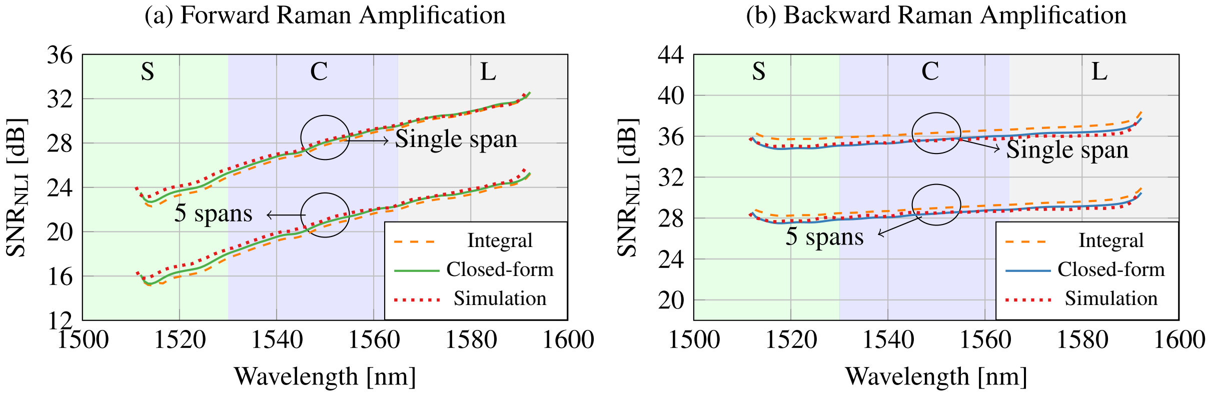

Fig. 2. Nonlinear performance after 1 x 80 km and 5 x 80 km transmission for (a) FW-DRA and (b) BW-DRA.

4. Results

The $\mathrm{SNR_{NLI}}$ as a function of wavelength is shown in Fig. 2 for both of the scenarios described in the last section. In order to verify the accuracy of the closed-form expression shown in Eq. (1), the $\mathrm{SNR_{NLI}}$ is also computed using the integral ISRS GN model [1] and split-step Fourier method simulations. For the latter, to ensure accurate results a local-error method [10] was used with goal local error of 10^{-10}. Each result is averaged over four simulations with 2^{17} Gaussian symbols per channel. Fig. 2.a shows the $\mathrm{SNR_{NLI}}$ for FW-DRA, using single span and 5 spans transmission. The high-power levels in short wavelengths (see Fig. 1.a), reduce the $\mathrm{SNR_{NLI}}$, degrading the performance of those channels; on the other hand, the performance of long-wavelength channels is higher, due to their reduced power levels, yielding to a tilt in the $\mathrm{SNR_{NLI}}$ profile. Fig. 2.b shows the same results for the case of BW-DRA; in that case the interaction between fibre attenuation, dispersion and power profile (see Fig. 1.a) yields a relatively flat $\mathrm{SNR_{NLI}}$ profile. Note that, the BW-DRA performs better in terms of $\mathrm{SNR_{NLI}}$ when compared with FW-DRA case, because of the reduced per-channel power evolution along the fibre.

In terms of accuracy, for a single span FW-DRA transmission, maximum per-channel errors of 0.4 dB and 0.5 dB were found between the closed-form expression and the integral ISRS GN model, and between the closed-form expression and the split-step simulation, respectively. Same transmission errors applies to 5 spans FW-DRA transmission. The same analyses for a single span BW-DRA transmission yields maximum per-channel error values of 0.9 dB and 0.5 dB, and for 5 spans BW-DRA transmission these values are 0.7 dB and 0.4 dB.

5. Conclusions

A closed-form expression that can accurately evaluate the NLI in UWB transmission systems using distributed Raman amplification was proposed. The approach was enabled by deriving a semi-analytical solution for the channel signal profile evolution along the fibre distance. The formula accounts for both forward and backward amplification, supporting arbitrary number of pumps and was applied in modelling a 10 THz Raman amplified link system. Its accuracy was validated using the integral ISRS GN model and split-step Fourier method simulations. Using this formula, the NLI is calculated in a few microseconds, making it suitable for intelligent UWB network planning tools, and rapid evaluations of system and network performance.

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References

1. D. Semrau, R. I. Killey, and P. Bayvel, “The Gaussian noise model in the presence of inter-channel stimulated Raman scattering,” J. Light. Technol. **36**, 3046–3055 (2018).
2. D. Semrau, R. I. Killey, and P. Bayvel, “A closed-form approximation of the Gaussian noise model in the presence of inter-channel stimulated raman scattering,” J. Light. Technol. **37**, 1924–1936 (2019).
3. D. Semrau, E. Sillekens, R. I. Killey, and P. Bayvel, “A modulation format correction formula for the Gaussian noise model in the presence of inter-channel stimulated Raman scattering,” J. Light. Technol. **37**, 5122–5131 (2019).
4. P. Poggiolini, M. R. Zefreh, G. Bosco, F. Forghieri, and S. Piciaccia, “Accurate non-linearity fully-closed-form formula based on the GN/EGN model and large-data-set fitting,” in *OFC,* (2019).
5. M. R. Zefreh, F. Forghieri, S. Piciaccia, and P. Poggiolini, “A closed-form nonlinearity model for forward-Raman-amplified WDM optical links,” in *OFC,* (2021).
6. M. Zirngibl, “Analytical model of Raman gain effects in massive wavelength division multiplexed transmission systems,” Electron. Lett. **34**, 789–790 (1998).
7. H. Buglia, M. Jarmolovičius, A. Vasylchenkova, E. Sillekens, R. I. Killey, P. Bayvel, and L. Galdino, “A closed-form expression for the ISRS GN model supporting distributed Raman amplification,” arXiv:2210.09067 [eess.SP] (2022).
8. M. A. Iqbal, G. D. Rosa, L. Krzczanowicz, I. Phillips, P. Harper, A. Richter, and W. Forysiak, “Impact of pump-signal overlap in S+C+L band discrete Raman amplifiers,” Opt. Express **28.**, 18440–18448 (2020).
9. D. Semrau, L. Galdino, E. Sillekens, D. Lavery, R. I. Killey, and P. Bayvel, “Modulation format dependent, closed-form formula for estimating nonlinear interference in S+C+L band systems,” in *ECOC,* (2019).
10. O. Sinkin, R. Holzlohner, J. Zweck, and C. Menyuk, “Optimization of the split-step Fourier method in modeling optical-fiber communications systems,” J. Light. Technol. **21**, 61–68 (2003).