Distortion Analysis and Equivalent Multiplicative and Additive Noise Model for High-Speed DAC and ADC

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Abstract: By analyzing the distortions of time-interleaving convertors, equivalent model with both multiplicative and additive noises is proposed and constructed by tone-based measurements. Experiments show that the proposed model estimates system performance with 0.2 dB accuracy.

1. Introduction

The rapidly development of optical communication system makes strict requirements on the conversion speed of digital-to-analog convertors (DACs) and analog-to-digital convertors (ADCs). Convertors with interleaving structure, e.g., in time domain [1-3] or in frequency domain [4, 5] are mandatory for modern optical communications employing symbol rate exceeding 10 Gbaud. For such high-speed multiplexed convertors, the distortion comes from the imperfections of sub-DACs/ADCs, as well as the imbalance among them. Those distortion limit the system performance and they are considered as complex nonlinear effect [2]. Thus, investigating the distortions of multiplexed convertors and quantitatively evaluating their actual impact on communication system performance is necessary.

Effective number of bits (ENOB) and signal-to-noise-and-distortion ratio (SINAD) are widely used to describe the distortion of DACs and ADCs [6] because those two metrics are easy to be measured. However, the actual distortion caused by high-speed DAC and ADC is verified to be more complex than the ENOB-equivalent additive noise experimentally [7]. In [7], the equivalent noise constructed by orthogonal term accurately estimates the distortion of DAC and ADC and the system performance, just as its performances in other optical and wireless communication systems [8, 9]. However, the orthogonal term is the difference between the actual nonlinear system output and the best linear approximation so that it changes with the input signal [9]. As a result, it should be measured again if the input signal changes. In addition, accurate measurement for orthogonal term is prohibitively hard [8]. Thus, more accurate and practical method to estimate the distortion of high-speed DAC and ADC is necessary.

In this paper, the characteristics of different impairment sources in a typical time-interleaving DAC is analyzed. Unlike the conventional ENOB-equivalent additive noise model, we proposed an equivalent model with both multiplicative and additive noises to estimate the distortion of high-speed multiplexed DAC/ADC and the communication system performance. More importantly, both noises could be measured practically by conventional tone-based measurement. Experiment shows that the Q value of the 31.5 Gbaud PAM8 transmission system which is distorted by DAC and ADC only, could be estimated with 0.2 dB accuracy.

2. Distortion analysis for time-interleaving convertors

The quantization noise of DAC/ADC comes from its finite amplitude resolution, i.e., the number of bits b. It is assumed that the full range of the analog signal is divided into 2^b quantization intervals, and each amplitude of the analog signal is mapped to a discrete amplitude level, which results in a quantization error. Such quantization error is modeled as an additive noise, and the theoretical signal to noise ratio (SNR) is approximate to $6.02b+1.76$ *(dB)* [2]. In addition, the amount of quantization noise does not change with the input signal [10]. Based on the concept of ideal quantization noise, the ENOB of a practical DAC/ADC can be written as $(SINAD_{dB}-1.76)/6.02$. Here, the SINAD and ENOB are measured by single frequency tone stimuli, whose definitions are shown in Eq. (1). The SINAD is the ratio between the power of fundamental tone and the power of total harmonics distortions (THD) and noise, excluding DC. The amplitude of the stimuli tone usually covers the full range of the convertors. If not, the ENOB calculation includes the normalization factor, i.e., "Scale" in Eq. (1).

$$SINAD_{dB} = 10\,log_{10}\left(\frac{P_{FUND}}{P_{THD}+P_{Noise}}\right),\ ENOB_{bit} = \frac{SINAD_{dB}-1.76-20log_{10}(Scale)}{6.02},\ Scale = \frac{Input\ amplitude}{Fullscale\ amplitude} \quad (1)$$

To focus on the distortion of DAC and ADC, we consider the simplest communication system which only includes DAC and ADC. Fig. 1 shows an example with a commercially available 84 GSa/s 8-bit DAC and an 80 GSa/s 8-bit

ADC embedded in a commercially available digital storage oscilloscope. We measured the SINAD for different input amplitudes and calculate ENOB according to Eq. (1). To cover the wide-frequency band of actual communication signal, SINAD is the ratio between the averaged power of the fundamental tones with different frequencies and the averaged power of harmonics and noise. Experimental results in Fig. 2 shows that both ENOB and SINAD change along with the scale of input signal. In particular, the measured ENOBs is not constant for different amplitudes even the normalization is applied. This phenomenon suggests that the ENOB-equivalent additive noise model based on the concept of ideal quantization error is not sufficient to describe the distortion of a practical high-speed DAC/ADC.

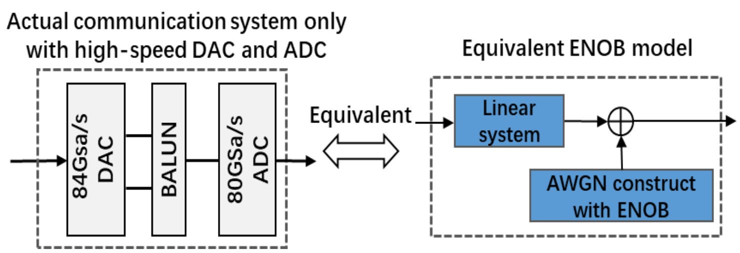

Fig. 1 Experiment setup for measuring SINAD and ENOB, and equivalent additive noise model construct with ENOB

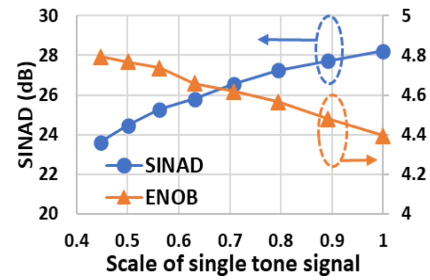

Fig. 2 Measured SINAD and ENOB for different input signal amplitudes

To precisely model the noise of high-speed convertors, it is necessary to make their distortion characteristics clear. Taking a typical time-interleaving DAC as an example, its schematic with M sub-DACs is shown in Fig. 3. If the sampling rate of high-speed DAC is f_s, M sub-DACs are provided with the same relatively-low-speed clock frequency f_s/M. The distortions of such time-interleaving DAC are not only the distortion of sub-DACs but also the mismatches among them. Mismatches among sub-DACs contain DC offset imbalance DC_m, gain imbalance $1+g_m$, and timing error mismatch δ_m. Based on these understanding, the DC offset imbalance generates an M-periodic additive distortion, and gain imbalance causes an M-periodic multiplicative distortion. Like clock jitter, the distortion of timing mismatch is also multiplicative. The combined impact of additive and multiplicative distortion results in the non-constant ENOB and SINAD. As echoed in Fig. 2, the variation of ~4 dB SINAD and ~0.5 bits ENOB demonstrates the complicated distortion mechanism.

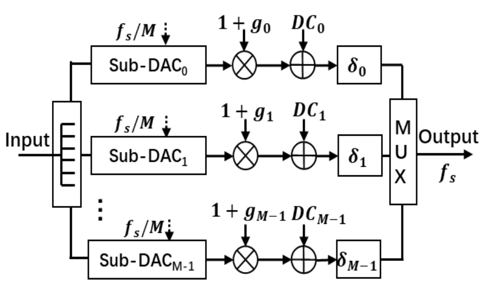

Fig. 3 Schematic diagram of time-interleaving DAC with M sub-DACs

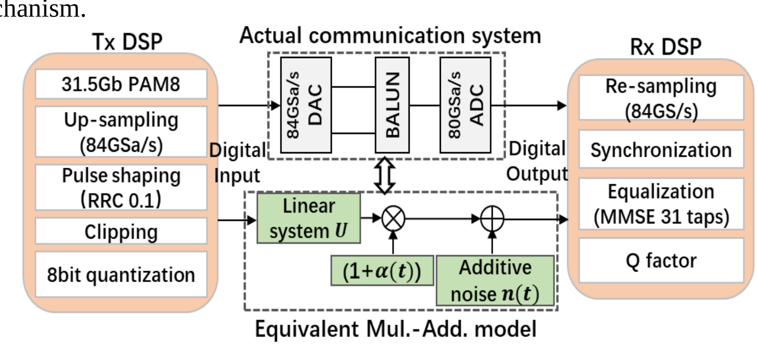

Fig. 4 Experimental setup and DSP flow of actual communication system, and the equivalent Mul.-Add. noise model. MMSE: minimum mean square error

3. DAC and ADC performance estimation by using the equivalent multiplicative and additive noise model

3.1 Equivalent multiplicative and additive noise model

Based on the distortion analysis for time-interleaving convertors, the equivalent multiplicative and additive (Mul-Add.) noise model is proposed in Fig. 4. The proposed model includes the linear system U, multiplicative coefficient 1+$\alpha(t)$, and additive noise $n(t)$. Here, the linear system is modeled by finite impulse response (FIR) filter. The $\alpha(t)$ is a white Gaussian random sequence with constant variance, where the variance represents the multiplicative noise power. For additive noise $n(t)$, it is an AWGN with constant power.

In our model, the additive and multiplicative distortions are approximated by the AWGN rather than the actual deterministic periodical signals so that the model could be measured practically. The reason of such assumption is that the high-speed DACs and ADCs includes hundreds of sub-DAC/ADC and the mismatch of each sub-DAC/ADC is random. From the perspective of system performance estimation, it just needs to obtain the influence of the distortion rather the actual distortion waveform. The accuracy of such approximation will be verified by experiment in following sections.

3.2 Linear fitting method for constructing the equivalent multiplicative and additive noise model

To obtain the multiplicative noise and additive noise power, a linear fitting method based on the tone measurements is proposed. Like the measurements of SINAD and ENOB, single frequency tone is injected into the system. Then, fundamental tone and all other spectral components excluding DC at output are counted. A scatter plot of the total power of harmonics and noise versus power of the fundamental tones is shown in Fig. 5. To cover the wide-frequency band, the power is averaged over different tone frequencies. The linear fitting ranges the amplitude of input sine signal from zero to full-scale. The intercept of fitting line that respects the total distortion power with zero input, can be considered as the power of additive noise, which is independent with input signal. The fitting slop represents the variance of multiplicative noise $\alpha(t)$.

The spectrum of all-zero input shows equally spaced spurs caused by the DC offset imbalance. The relationship between fundamental tone power and the total harmonic distortion (THD) and noise power is close to linear. This is quite different from the nonlinear behavior of non-multiplexed DAC/ADC, trans-impedance amplifier, and driver where the harmonic distortion increases much faster than fundamental tone does [11]. This also indicates that the imbalance among sub-convertors is the dominate distortion and the nonlinearity of sub-convertors could be ignored.

3.3 Experiment verification

The experiment setup and DSP flow of communication system with only high-speed DAC and ADC are shown in Fig. 4. The transmitted signal is 31.5 Gbaud PAM8 signal with root-raised-cosine pulse shaping. Different RMSs of input signal are applied before 8-bit quantification. For the Rx DSP of digital output, 31 taps equalizer (MMSE) is operated before decision and Q calculation. Both the digital output of actual communication system and that of equivalent models are processed by a same DSP flow. For the conventional equivalent ENOB model in Fig. 1, measured ENOB with full-scale sine signal is used to construct the additive noise.

Fig. 6 gives the Q results of actual communication system and two equivalent models. The error of ENOB model is up to 2.6 dB, proving that the distortion of high-speed DAC and ADC cannot be regarded as the additive noise. The equivalent Mul-Add. noise model matches well with the experiment within the error of 0.2 dB.

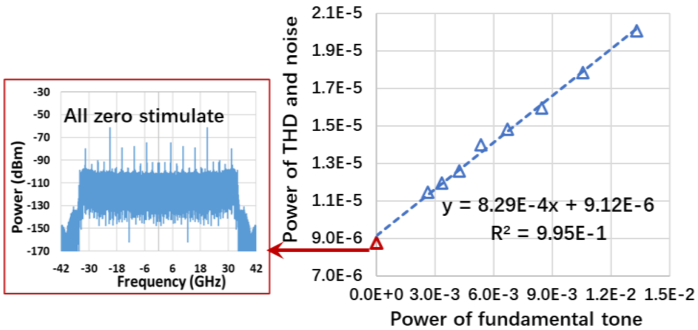

Fig. 5 Linear fitting for fundamental power and power of THD and noise in linear unit

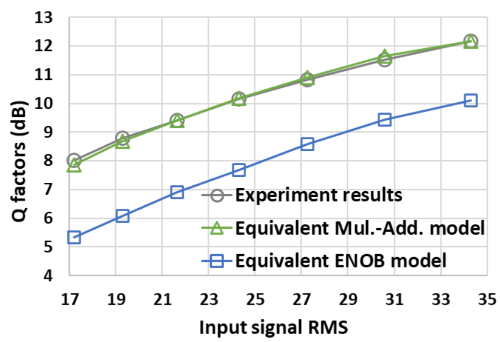

Fig. 6 Q factors vs. DAC input signal RMS. DAC full swing: -127~127

4. Conclusions

In this paper, the distortion mechanisms of typical time-interleaving convertors were analyzed. In multiplexed convertors, the distortions caused by sub-DAC mismatches, such as DC offset imbalance, gain imbalance, and timing mismatch, contribute to the additive and multiplicative distortions. Based on this analysis, the equivalent multiplicative and additive noise model, which could be constructed by the simple tone-based measurements, was proposed. Experimental results showed that the proposed equivalent model can estimate the performance of high-speed DAC and ADC communication system with 0.2 dB accurately whereas conventional ENOB model has 2.6 dB error.

5. References

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