# Characterization of Coupled Multi-core Fiber Nonlinearity using Modified CW-SPM Method

### Taiji Sakamoto, Ryota Imada, Kazuhide Nakajima

Access Network Service Systems Laboratories, NTT Corporation, 1-7-1, Hanabatake, Tsukuba Ibaraki, 305-0805 Japan taiji.sakamoto.un@hco.ntt.co.jp

**Abstract:** Narrow bandwidth continuous-wave-based SPM method is proposed for measuring the effective nonlinear coefficient  $\gamma_{eff}$  of coupled multi-core fibers. We confirmed that the measured  $\gamma_{eff}$  with 2- or 4-core fibers agreed well with the theoretical model. © 2022 The Author(s)

## 1. Introduction

A coupled multi-core fiber (MCF) is an attractive space division multiplexing (SDM) fiber for high capacity transmission since it has a much smaller core pitch and accommodates larger number of cores than those of uncoupled MCFs. The coupled MCF has unique properties owing to the random mode coupling in the MCF such as the narrow group delay spread specified by the spatial mode dispersion (SMD) coefficient in the ps/ $\sqrt{km}$  unit [1]. The lower nonlinearity of the coupled MCF has been experimentally confirmed in a 4-core transmission by comparing the single mode fiber (SMF) with the comparable fiber loss and effective area of the core mode [2]. Although the advantage of the nonlinear performance of the coupled MCF has not yet been investigated [3, 4], a measurement method for the nonlinear property of the coupled MCF has not yet been investigated, which is mandatory for fabricated fiber characterization and transmission system design.

In this paper, we investigate the applicability of the continuous-wave (CW)-based self-phase modulation (SPM) method [5] by modifying the wavelength difference between two CW lights to more than 10 times smaller value to suppress the spectral fluctuation and pulse distortion induced by the mode coupling and dispersion. We also reveal that the modified CW-SPM method well characterizes an effective nonlinear coefficient [3] in a coupled MCF with various core numbers and coupling coefficients.

## 2. Theoretical treatment and measurement principle

The mode propagation in a coupled MCF is expressed by [3]

$$\frac{\partial \overline{A}}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 \overline{A}}{\partial t^2} = ik\gamma(|\overline{A}|^2)\overline{A} \qquad (1),$$

where  $\overline{A}$  is the slowly varying field envelope of the propagation mode,  $\gamma$  is the nonlinear coefficient, and k is the nonlinear reduction factor (definition is shown in Sec. 3). Here, higher-order chromatic dispersion and loss are ignored for simplicity. Although there are multiple modes in the coupled MCF, they are expressed as one mode in Eq. (1) with the averaged group delay, chromatic dispersion  $\beta_2$ , etc., among all propagation modes owing to the random mode coupling. The  $|\overline{A}|^2$  is the total electric power of all modes. As the right term represents the well-known SPM effect,  $k\gamma$  can be treated as the effective nonlinear coefficient  $\gamma_{\text{eff}}$  that characterizes the nonlinear performance of the coupled MCF. This equation also indicates that a similar methodology for the  $\gamma$  measurement of SMF using SPM can be applied for the coupled MCF when each field (propagating through each mode) has the same envelope field. The conventional CW-SPM method utilizes two CWs with the appropriate wavelength interval  $\Delta\lambda$ . The two CWs are multiplexed with same linear polarization state, and input into the fiber under test (FUT). The  $\Delta\lambda$  is determined taking into account the total dispersion of the FUT since the propagating field envelope should not be distorted, and typically the  $\Delta\lambda$  and FUT lengths are set to about 0.3 nm and 500 m, respectively [5].



Fig. 1 Measurement setup for effective nonlinear coefficient of coupled MCF

There are two important considerations when applying the CW-SPM method to the coupled MCF. One is the spectral fluctuation owing to the unstable mode coupling in the MCF. The nonlinear phase shift in the propagating sinusoidal pulse can be derived from the spectral power ratio between the main and sub peak  $(I_1 / I_2)$ . However, the core-to-core insertion loss has a strong spectral dependency owing to the modal dispersion, and the spectrum can be asymmetric with the conventional condition. The other consideration is the pulse distortion by the modal dispersion. Typically, coupled MCF has tens ps/ $\sqrt{km}$  modal dispersion, which has more impact on the pulse distortion than the chromatic dispersion of tens of ps/nm/km. We here propose a modified CW-SPM method where the  $\Delta\lambda$  is reduced to a more than 10 times smaller value to suppress the spectral fluctuation and modal dispersion induced pulse distortion. The point of this technique is based on the fact that the peak ratio is derived from the nonlinear phase shift  $\phi_{max}$  as

 $I_1/I_2 = \{J_0^2(\phi_{max}/2) + J_1^2(\phi_{max}/2)\}/\{J_1^2(\phi_{max}/2) + J_2^2(\phi_{max}/2)\}$  (2), and is thus theoretically independent of  $\Delta\lambda$ . Figure 1 shows the proposed setup for measuring the  $\gamma_{\text{eff}}$  of the coupled MCF. The input configuration is the same for the conventional CW-SPM method except for the  $\Delta\lambda$  setup. The  $\Delta\lambda$  value in the modified CW-SPM was too small to spectrally distinguish the measured peaks with the conventional optical spectrum analyzer (OSA). Here, the coherent and signal processing-based ultra-high wavelength resolution OSA (AP2071A, APEX technologies) with a spectral resolution of 0.8 pm was used in the experiment.

#### 3. Results and discussion

Table 1 shows the structural parameters of the FUTs. The GeO<sub>2</sub> doped cores or Fluorine doped cladding coupled MCFs with 2 and 4 cores and 18~25 µm core pitch were used. Figures 2(a) and (b) show the example spectrum using a 500-m-long 2-core fiber (ID = 4) when  $\Delta\lambda$  is 0.28 nm and 0.01 nm, respectively. The bending radius of the FUT was 80 mm, and the OSA used for Figs. 2(a) and (b) was the conventional and coherent-based one, respectively. The input/output SMF was spliced with one of the cores of the MCF. The obtained spectrum was well symmetric and the peak ratio was stably measured by reducing the  $\Delta\lambda$  adequately. Figure 3 shows the  $\Delta\lambda$  dependence of  $I_1 / I_2$  with the SMF and 2-core fiber. The plots show the averaged value of the acquired 20 spectra and the error bar shows the max./min. values. The  $I_1 / I_2$  was independent of  $\Delta\lambda$  for the SMF measurement. However, the measured  $I_1 / I_2$  value for the 2-core fiber was  $\Delta\lambda$  dependent, which was expected owing to the modal dispersion. We experimentally confirmed that the peak ratio became stable and almost  $\Delta\lambda$  independent by reducing the  $\Delta\lambda$  to below 0.05 nm. Although the maximum  $\Delta\lambda$  for the measurement differs depending on the total modal dispersion of each MCF, the fiber length or  $\Delta\lambda$  was set with a margin to 500 m or 0.01 nm, respectively, for all FUTs taking into account that all FUTs have comparable SMD coefficients as shown in Table 1. Figure 4 shows the nonlinear phase shift  $\phi_{max}$  calculated from the measured  $I_1 / I_2$  as a function of the total input power  $P_{in}$ . The plots are the averaged value among the 10

Table. 1 Structural parameters of FUTs						
ID	# of cores	Core pitch Λ (μm)	Dopant	Core radius a (µm)	Relativ index difference $\Delta(\%)$	SMD coefficient (ps/√km)
1	1	NA	$GeO_2$ core	4.2	0.34	NA
2	2	20	${\rm GeO}_2$ core	4.5	0.44	31
3	2	25	$GeO_2 \ core$	4.5	0.40	43
4	2	20	F cladding	4.5	0.38	42
5	2	25	F cladding	4.4	0.35	41
6	2	21	F cladding	3.3	0.67	25
7	2	21	F cladding	4.8	0.30	30
8	4	18	F cladding	4.2	0.33	16
$I_1/I_2$ (dB)	50 45 40 35 30 25 0.01	SMF @	Fiber @ $P_{in} = P_{in} = 20.1 di$	22.7 dBn Bm	•	
Fig. 3 AA dependence of $I_1/I_2$						



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Fig. 4 Input power vs. nonlinear phase shift

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times spectrum data. The results for SMF and coupled MCFs with ID = 3, 8 9 are shown as examples. The solid lines correspond to the fitted results. We observed well linearity between  $P_{in}$  and  $\phi_{max}$  for each result. The nonlinear coefficient in W<sup>-1</sup>km<sup>-1</sup> unit can be derived by  $1/(2L_{eff}) d\phi/dP_{in}$ .

To analyze the validity of the measured values, the obtained values were compared with the calculated ones. The effective nonlinear coefficient  $\gamma_{\text{eff}}$  (= $k\gamma$  in Eq. (1)) is expressed by

$$\gamma_{eff} = k\gamma = \sum_{k \le l}^{M} \frac{32}{2^{\delta_{kl}}} \frac{1}{6M(2M+1)} \frac{n_2 \varpi_0}{cA_{eff}^{kl}}$$
(3)

where *M* is the number of cores, and  $A^{kl}_{eff}$  is the conventional (*k*=*l*) or inter-modal (*k*≠*l*) effective area defined as the effective area of the fundamental mode divided by  $f_{kkll}$  (see [3] for a detailed definition). The fiber bending and twisting significantly affects the mode field in the coupled MCF. We calculated the rotation angle dependence on  $A^{kl}_{eff}$  and  $\gamma_{eff}$  values for the 2-core fiber with a bending radius of 80 mm. The  $A^{11}_{eff}$  and  $A^{12}_{eff}$  corresponds to the  $A_{eff}$  of the even mode and between the even and odd modes, respectively. The  $\theta$  is defined as the rotation angle, where  $\theta = 0$  indicates that the two cores are both in the line of the bending direction. While the  $A^{kl}_{eff}$  greatly changes in the vicinity of  $\theta = 90^{\circ}$  [1], it is found that the  $\gamma_{eff}$  value is almost insensitive for  $\theta$ . This behavior can be roughly understood from the electrical fields. For  $\theta = 0^{\circ}$ , each mode propagates in different cores and  $A^{11}_{eff}$  and  $A^{22}_{eff}$  become almost  $A_{eff}$  of the core mode, and  $A^{12}_{eff}$  is large enough to ignore the inter-modal nonlinear effect. For  $\theta = 90^{\circ}$  in contrast,  $A^{11}_{eff}$  and  $A^{22}_{eff}$  has a comparable value. In either case, the  $\gamma_{eff}$  value becomes the same by balancing the nonlinear contribution between  $A^{kk}_{eff}$  and  $A^{kl}_{eff}$ . The rotation direction insensitivity on  $\gamma_{eff}$  was also confirmed for the other FUT including the 4-core fiber. Thus, the probability distribution of the bending direction in the 500-m FUT does not affect the  $\gamma_{eff}$  measurement. We then calculated the  $\gamma_{eff}$  of each FUT and compared the

measured and calculated  $\gamma_{eff}$  as shown in Fig. 5. The nonlinear index  $n_2$  of the FUTs were estimated on the basis of the data in [6]. The fiber IDs are noted in the figure. Note that  $\gamma_{eff}$ indicated here does not mean how much the nonlinear effect is reduced from that for the SMF in the mode division multiplexed (MDM) transmission with MIMO processing. This is because the  $\gamma_{eff}$  value was characterized when the total input power to the FUT was the same on the basis of Eq. (1). This indicates that the input power per core in the MCF is M times smaller than that for the SMF. The obtained  $\gamma_{eff}$  is, however, an inevitable parameter for estimating the nonlinear impairment of the MDM transmission system by numerically solving Eq. (1) by taking into account the input signal pattern difference and modal dispersion induced nonlinear mitigation. The relationship between the measured and calculated  $\gamma_{eff}$  is well correlated and almost on the line of y = x, and thus the applicability of the proposed measurement method was proved.



Fig. 5 Comparison of calculated and measured  $\gamma_{eff}$ 

#### 5. Conclusion

We proposed a modified CW-SPM method for measuring the effective nonlinear coefficient  $\gamma_{eff}$  of the coupled MCF, where the wavelength difference between two CWs was reduced to 0.01 nm to suppress the spectral and pulse distortion of the propagated light in the fiber. The measured  $\gamma_{eff}$  obtained with various coupled MCFs with 2~4 cores agreed well with the theoretical model.

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