

# Practical Implementation of Sequence Selection for Nonlinear Probabilistic Shaping

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**Abstract:** We propose two novel techniques to implement sequence selection (SS) for fiber nonlinearity mitigation, demonstrating a nonlinear shaping gain of 0.24 bits/s/Hz, just 0.1bits/s/Hz below the SS capacity lower bound. © 2022 The Author(s)

## 1. Introduction

Probabilistic amplitude shaping (PAS) is employed in current high capacity optical fiber communication systems to maximize the spectral efficiency and provide rate flexibility [1, 2]. Besides its well studied advantages in the linear regime, PAS has been investigated also for the opportunity to reduce nonlinear effects [3–6]. Recently, sequence selection (SS) has been proposed as a theoretical approach to lower bound the capacity of the nonlinear fiber channel [7, 8]. In the SS approach, the input distribution is optimized by selecting only “good sequences” (generating less nonlinear interference) for transmission. Unfortunately, SS does not provide a practical way to map information bits on good sequences. Therefore, a practical implementation of SS to be included in PAS is missing, except for a preliminary attempt using constant composition DM [9]. In this work, we propose two novel techniques to practically implement SS.

## 2. Implementation of Sequence Selection

Here we present two possible practical implementations of SS, namely bit scrambling SS (BSSS) and symbol interleaving SS (SISS). In both implementations, blocks of input bits are mapped to  $N_t$  different test sequences of  $n$  dual-polarization QAM (4D) symbols. The sequences are compared according to a proper metric and the best one is transmitted. Pilot bits or symbols and  $N_t - 1$  different invertible transformations are used to ensure that the test sequences behave as if they were independently drawn from a given distribution (uniform or shaped), while remaining univocally decodable. The two techniques differ for the employed invertible transformation, for the concatenation order of the latter with FEC, and for the use of pilot bits or symbols. The performance of both techniques depends on the acceptance rate  $1/N_t$ , on the rate loss due to pilot bits or symbols, and on the accuracy of the selection metric. The choice of the metric, which must be computed  $N_t$  times, also impacts on the computational complexity. While a thorough complexity analysis is deferred to a future study, here we consider two extreme cases: the more accurate and complex nonlinear interference (NLI) metric  $\|\mathbf{x} - \mathbf{y}\|$ , where  $\mathbf{x}$  is the transmitted sequence and  $\mathbf{y}$  the corresponding sequence obtained through a numerically emulated single-channel noiseless propagation of  $\mathbf{x}$  [8]; and the extremely simple windowed Kurtosis (WK) [6].

### 2.1. Bit Scrambling SS

The working principle of BSSS is sketched at the top of Fig. 1 for  $N_t = 2$  test sequences. Given a sequence  $\mathbf{b}$  of information bits, the two test sequences are obtained by prepending the pilot bit 0 and 1 to, respectively,  $\mathbf{b}$  and  $\mathbf{t} \oplus \mathbf{b}$ , where  $\mathbf{t}$  is a random-like bit sequence (fixed and known to the receiver), and  $\oplus$  denotes the XOR operation. Each test sequence, after going through the DM (if PAS is used) and the FEC encoder, is mapped to a sequence of  $n$  4D symbols. The corresponding metrics are then evaluated and the best sequence is transmitted. At the receiver side, after dispersion compensation (and possibly other processing blocks not shown here for simplicity), the received sequence is processed by the QAM demapper, FEC decoder, and inverse DM. Finally, the information bits are recovered by removing the pilot bit and, if the latter is equal to 1, by applying again the XOR operation to descramble the bit sequence. The technique can be extended to the case of  $N_t > 2$  test sequences by using  $\lceil \log_2 N_t \rceil$  pilot bits<sup>1</sup> and  $N_t - 1$  different scrambling sequences  $\mathbf{t}_1, \dots, \mathbf{t}_{N_t-1}$ .

BSSS is conceived to approximate the theoretical SS approach proposed in [7, 8] to lower-bound channel capacity. It provides practically independent test sequences with minimum rate loss, is compatible with the PAS

<sup>1</sup>  $\lceil x \rceil$  denotes the smallest integer greater than or equal to  $x$ .

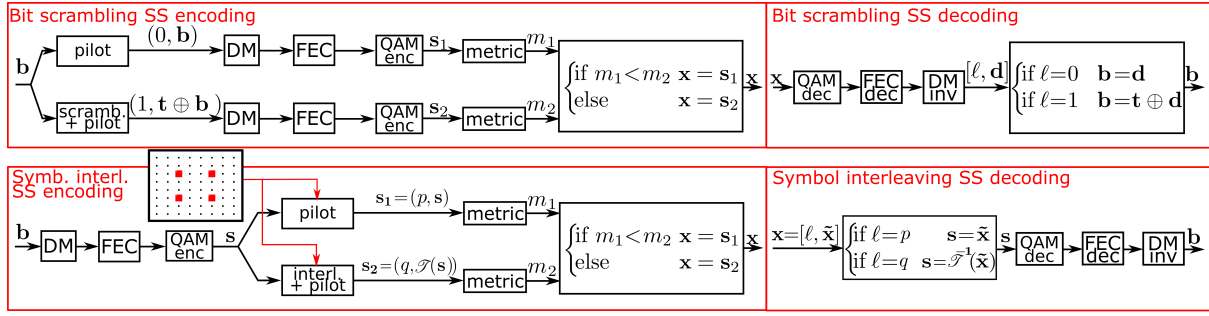


Figure 1. Bit scrambling and symbol interleaving SS , with  $N_t = 2$  test sequences.

approach with reverse FEC concatenation, and ensures that the pilot bits are protected by FEC. However, it poses a constrain on the length  $n$  of the test sequences (it must correspond to one or more FEC codewords) and requires applying the DM and FEC encoder to each test sequence, with a subsequent increase of complexity. To overcome these limitations, the alternative SISS technique proposed below may be employed.

## 2.2. Symbol Interleaving SS

The working principle of SISS is sketched at the bottom of Fig. 1 for  $N_t = 2$  test sequences. In this case, the DM, FEC encoder, and QAM mapper are directly applied to the information bit sequence  $\mathbf{b}$  to obtain the sequence of 4D symbols  $\mathbf{s}$ . Then, the two test sequences are obtained by prepending two different pilot symbols,  $p$  and  $q$ , respectively to  $\mathbf{s}$  and  $\mathcal{T}(\mathbf{s})$ , where  $\mathcal{T}(\cdot)$  denotes interleaving according to a randomly-selected permutation table (fixed and known to the receiver). The random permutation yields a sufficiently different new sequence (in terms of generated NLI), but with the same amplitude distribution induced by PAS within the block. The corresponding metrics are then evaluated and the best sequence is transmitted. At the receiver side, deinterleaving  $\mathcal{T}^{-1}(\cdot)$  is performed or not depending on the value of the pilot symbol, followed by QAM demapping, FEC decoding, and inverse DM. The technique can be extended to the case of  $N_t > 2$  test sequences by using one or more pilot symbols and  $N_t - 1$  different interleaving maps. Pilot symbols, which are not protected by FEC, are drawn from the dual-polarization QPSK constellation shown in the inset of Fig. 1, properly carved out of the full QAM constellation to ensure a sufficiently low error rate. With this choice, one pilot symbol allows the identification of up to 16 different interleaving maps, so that  $\lceil \log_2(N_t)/4 \rceil$  pilot symbols are used in total. In contrast with BSSS, the length of the test sequences is not constrained by the length of the FEC codewords. In fact, if desired, the symbol sequence  $\mathbf{s}$  can be divided into several shorter subsequences, which are then individually processed by the remaining blocks of the scheme of Fig. 1.

## 3. System Performance

The two implementations of SS are tested by means of numerical simulations on a  $30 \times 100$  km single mode fiber link with EDFA ( $\beta_2 = 21.7 \text{ ps}^2/\text{km}$ ,  $\gamma = 1.27 \text{ W}^{-1}\text{m}^{-1}$ ,  $\alpha_{\text{dB}} = 0.2 \text{ dB/km}$ ,  $N_F = 5 \text{ dB}$ ). Five 46.5 GBd WDM channels, spaced 50 GHz apart, are propagated across the link. Each WDM channel is obtained by modulating root-raised-cosine pulses with 0.05 roll-off by the dual polarization PAS-64QAM symbols obtained according to the schemes of Fig. 1. Enumerative sphere shaping (ESS) [10] with rate  $R = 1.3$  bits/amplitude (slightly increased to compensate for the rate loss due to pilot bits or symbols when BSSS or SISS are employed) and blocklength 256 (optimized for nonlinear performance) is used to implement PAS. BSSS and SISS are implemented on sequences of  $n = 256$  4D symbols, after PAS. FEC encoding and decoding are omitted from the simulations. At the receiver side, after channel demultiplexing, dispersion compensation, matched filtering, symbol-time sampling, and compensation of the mean phase rotation, the spectral efficiency (SE) corresponding to the achievable information rate with bit-wise decoding is evaluated [3].

Fig. 2(a) reports the SE obtained with the BSSS scheme ( $N_t = 256$ ) and two different metrics; with i.i.d. Maxwell-Boltzmann (MB) distributed symbols (optimal in the linear regime) [11]; with ESS alone; and the theoretical SE achievable by the SS approach (computed as explained in [8] with acceptance rate  $\eta = 10^{-3}$  and averaged cost function). BSSS with the NLI metric provides a gain of 0.24 and 0.08 bits/s/Hz w.r.t. to MB and ESS, respectively. On the other hand, BSSS with the WK metric (which basically measures the intensity fluctuations within the block) performs only slightly better than ESS. Eventually, the SS lower bound shows that an additional gain of at least 0.1 bits/s/Hz is theoretically achievable by improving the selection metric and increasing the number of tested sequences.

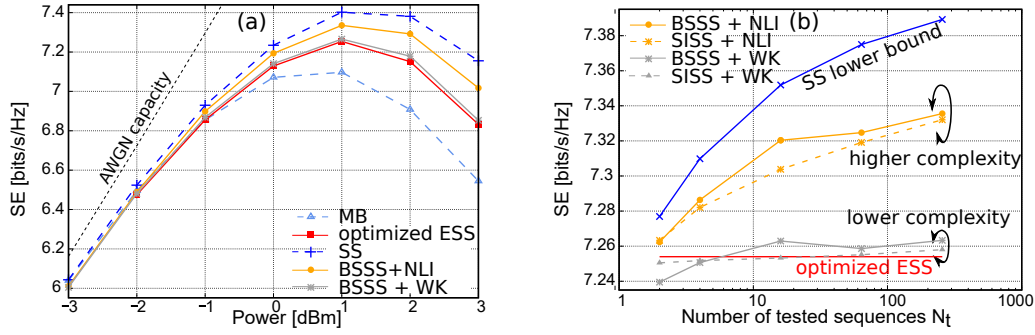


Figure 2. (a) SE with different shaping techniques; (b) SE vs number of tested sequences  $N_t$

Next, Fig. 2(b) shows the performance at the optimal power when changing the number of tested sequences  $N_t$ , for both BSSS and SISS and for both the NLI and WK metrics. The SS lower bound (with same acceptance rate  $1/N_t$  and averaged cost function [8]) is also reported as a reference. In general, the performance improves when increasing the number of tested sequences  $N_t$ . The gain obtained with SISS is slightly below the one obtained with BSSS, mainly because of the larger rate loss due to the use of pilot symbols compared to pilot bits. The complex NLI metric is always superior to the simple WK metric, while the theoretical bound shows that higher gains can be achieved by further improving the selection metric.

#### 4. Conclusion

Two practical implementations of SS have been proposed. The two techniques, which differ in the concatenation order with FEC and PAS, provide a similar nonlinear shaping gain, outperforming linearly optimized PAS by 0.24 bit/s/Hz, and optimal-blocklength (for the nonlinear regime) ESS by 0.08 bit/s/Hz. A variable acceptance rate and two different selection metrics have been considered, showing the possibility to reduce complexity by sacrificing performance, and highlighting the importance of finding an accurate metric with sufficiently low complexity.

#### Acknowledgment

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