Probabilistic Shaping Methods for Linear and Nonlinear Channels

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Abstract: The main methods and achievements regarding probabilistic shaping are reviewed, highlighting the primary difficulties and opportunities.

1. Introduction

Probabilistic shaping is a popular technique employed in modern coherent optical communication systems to improve performance and flexibility [1–4]. It is implemented by replacing the conventional quadrature amplitude modulation (QAM) encoder (and decoder) with a distribution matcher (DM), which maps (and demap) bits to (from) constellation symbols with a desired probability distribution, without any other modification in the transceiver processing chain. The development of low-complexity DMs is the key to the success of probabilistic shaping. In the linear regime, probabilistic shaping allows approaching the Shannon capacity, closing the 1.53 dB SNR gap of uniform constellations [1, 5], and finely tuning the information rate. On top of that, current research aims at updating probabilistic shaping, trying to tailor it to the nonlinear regime, in the hope of obtaining additional gains and boosting system performance. The main differences between linear and nonlinear shaping are highlighted in Fig. 1(a) and discussed in the rest of the paper.

2. Probabilistic Shaping for Linear Channels

In the linear regime with a given constellation, the Maxwell–Boltzmann (MB) distribution with i.i.d. symbols is optimal, as it minimizes the mean energy per symbol for a given rate, closing the gap to Shannon capacity [5].

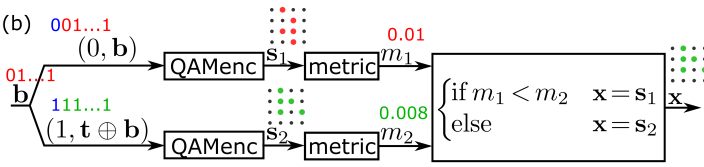
The most popular approach for the practical implementation of probabilistic shaping is the probabilistic amplitude shaping (PAS) scheme, which allows to generate symbols from a QAM constellation with desired rate and distribution [1, 2]. In a nutshell, a DM maps part of the information bits to shaped amplitudes that are sent to the forward error correction (FEC) encoder, while the remaining information bits, together with the FEC parity bits, are mapped to the sign bits to produce QAM symbols. In the linear regime, the main challenge is designing a DM that closely approaches the MB performance with reasonable computational complexity. Some of the most promising approaches for (linear) DM include enumerative sphere shaping (ESS) [6, 7], constant composition DM (CCDM) [8], and hierarchical DM (HiDM) [9–11]. While ESS minimizes the rate loss for a given block length, the HiDM reduces the computational complexity and/or hardware requirements with respect to the equivalent-length (single-layer) DM. For a comparison between the different approaches in the linear regime refer to [7, 12] and references therein.

3. Probabilistic Shaping for Nonlinear Channels

Preliminary approaches to probabilistic shaping for nonlinearity mitigation have focused on the adaptation and optimization of existing shaping schemes that were originally developed for linear channels. For instance, it has been shown that only a negligible gain can be obtained by optimizing the target marginal distribution of the QAM symbols [13]. A higher *nonlinear shaping gain* is obtained by optimizing the blocklength [14, 15] and the bandwidth [16] over which PAS is applied, since the constraint induced by the DM on the signal amplitudes within each blocklength may reduce the intensity fluctuations of the signal that are responsible for the generation of nonlinear interference. In particular, short-blocklength ESS was shown to outperform CCDM also in the nonlinear regime, both in simulations [12] and experiments [17]. Unfortunately, the nonlinear shaping gain mentioned above vanishes or strongly diminishes when adding a carrier phase recovery (CPR) at the receiver [18]. This is explained by the ability of CPR to mitigate the nonlinear phase noise induced by the intensity fluctuations of the signal, making PAS blocklength optimization unnecessary [12, 19]. Kurtosis-limited ESS [20] and band-trellis ESS [21] have been also developed to limit the intensity fluctuations of the signal and improve the nonlinear tolerance of ESS, but they provide a significant gain only for single span links.

(a)

	Optimal distribution and performance	Sub-optimal implementation
Linear regime	i.i.d. Maxwell-Boltzmann	ESS, CCDM, HiDM,...
Nonlinear regime	lower bound by seq sel	Bit scrambling SS, List-encoding CCDM



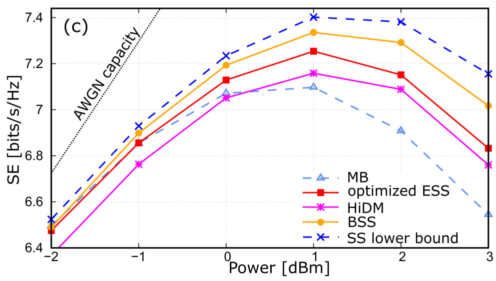


Figure 1. (a) Linear and nonlinear shaping (b) BSSS, (c) SE vs power.

Unlike the linear case, the optimal distribution and performance in the nonlinear case are unknown. Consequently, nonlinear shaping has been approached also from a more theoretical point of view. For example, the energy dispersion index (EDI) and other similar metrics can predict the PAS performance in the nonlinear regime [22, 23]. The same can be done with the nonlinear phase noise metric, with the additional benefits that it accounts for the impact of CPR and has no free parameters to be tuned by system simulations [12]. Hopefully, these metrics will help to design successful nonlinear constellation shaping, as in [24, 25].

Sequence selection (SS) has been recently proposed to investigate the ultimate potential and limitations of nonlinear constellation shaping [4,26,27]. In a nutshell, SS optimizes the input distribution by using only the input sequences that minimize the nonlinear interference, improving the spectral efficiency (SE). The SS approach has been used to derive an improved capacity lower bound for the dual-polarizion WDM channel and to demonstrate an unbounded growth of the AIR with power for a simplified nonlinear channel. While the good sequences can be obtained by a simple rejection sampling algorithm, a technique to map information bits on these good sequences has not yet been devised.

Bit-scrambling SS (BSSS) is a novel practical implementation of SS. BSSS scrambles the input bit sequence to generate alternative sequences, using pilot bits to identify the selected sequence and allow the decoding. The working principle of BSSS with 2 test sequences is sketched in Fig. 1(b). Given a stream of information bits $\mathbf{b}$, two test sequences $(0,\mathbf{b})$ and $(1,\mathbf{t}\oplus\mathbf{b})$ are obtained by prepending the pilot bit “0” and “1” to $\mathbf{b}$ and $\mathbf{t}\oplus\mathbf{b}$, respectively, where $\mathbf{t}$ is a random-like bit sequence (fixed and known to the receiver), and $\oplus$ indicates the XOR operation. After the QAM encoder, which can optionally include a DM, a metric is evaluated, and the best sequence is transmitted. At the receiver, after conventional detection and decoding, the pilot bit is used to determine if the XOR operation must be performed again to recover $\mathbf{b}$. The technique can be easily extended to $N_t = 2^{n_p}$ test sequences by using n_p pilot bits. The method approaches SS with acceptance rate $1/N_t$. The cost and performance of the technique mainly depend on N_t and on the used metric. In this work, the metric is the nonlinear interference $m = ||\mathbf{x}-\mathbf{y}||$, where $\mathbf{x}$ is the transmitted sequence and $\mathbf{y}$ the corresponding sequence obtained by a numerically emulated single-channel noiseless propagation of $\mathbf{x}$ followed by dispersion compensation, matched filtering [4].

4. Linear and Nonlinear PAS Performance

Dual polarization PAS-64 QAM symbols with rate $R = 4.6$ bits/2D-symbol are modulated with a root-raised-cosine shape with roll-off 0.05. The signal, corresponding to five 46.5 GBd channels with 50 GHz spacing, is sent into a 30×100 km SMF link with EDFAs ($\beta_2 = 21.7\,\mathrm{ps}^2/\mathrm{km}$, $\gamma = 1.27\,\mathrm{W}^{-1}\mathrm{m}^{-1}$, $\alpha_{\mathrm{dB}} = 0.2\,\mathrm{dB/km}$, $N_F = 5\,\mathrm{dB}$). At the receiver, the SE of the central channel is evaluated after dispersion compensation, matched filtering and sampling. SS and BSSS are implemented for sequences of length $n = 256$, starting from PAS sequences.

Fig. 1(c) shows the performance obtained with different shaping techniques. All curves approach the Shannon capacity in the linear regime, the best approximation being provided by the MB one. ESS with optimized block length 256 closely approaches the MB curve in the linear regime, and provides a nonlinear shaping gain of 0.15 bits/s/Hz with respect to (w.r.t.) the MB distribution. BSSS with $N_t = 256$ provides a gain of 0.08 bits/s/Hz and 0.24 bits/s/Hz w.r.t. optimized ESS and MB, respectively. The considered HiDM (5 layers with $N = [4,3,3,3,3]$, $k = [1,9,7,8,10]$, $M = [4,64,64,64,64]$) has a large rate loss that affects the performance in the linear regime; however, the HiDM outperforms the MB in the nonlinear regime and has limited computational cost and hardware requirements. All the curves are below the SS lower bound [4] (computed with acceptance rate $\eta = 10^{-3}$ and averaged cost function), showing that additional gains are achievable.

5. Conclusion

The implementation of linear and nonlinear probabilistic shaping has been analyzed, highlighting the main challenges and approaches. The nonlinear shaping gain of different approaches was determined, showing that further improvements can be obtained.

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