# Adaptive Turbo Equalization of Probabilistically Shaped Constellations

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**Abstract:** Fiber nonlinearity compensation of probabilistically shaped constellations with adaptive turbo equalization is investigated for the first time. Potential for more than 100% transmission reach extension is demonstrated by combining probabilistic shaping, single-channel digital backpropagation, and adaptive turbo equalization. © 2022 The Author(s)

### 1. Introduction

Fiber nonlinearity compensation (NLC) is a key enabler for reach and data rate maximization of coherent wavelength division multiplexing (WDM) systems over single-mode fibers. Nevertheless, despite the progress made over the years, finding ways of applying NLC effectively to overcome fiber nonlinear interference (NLI) is still a work in progress, with most solutions targeting the intra-channel NLI [1]. Nowadays, state-of-the-art coherent optical transceivers combine probabilistically shaped (PS) constellations and powerful forward error correction (FEC) to maximize transmission throughput and reach [2]. In this scenario, the joint design of modulation, FEC and NLC is necessary to achieve higher transmission performance. Recently, turbo equalization (TEq) schemes have been proposed for this purpose [3, 4], achieving compensation of inter-channel NLI in WDM systems. In a TEq scheme, the receiver iterates between channel equalization and FEC decoding, aiming to improve the performance of both functionalities by exchanging soft-information from decoder to equalizer and vice-versa. It has been demonstrated that TEq has the potential to increase throughput [3] and transmission reach [4] in coherent WDM systems using uniform quadrature and amplitude (QAM) constellations. However, the use of TEq has not been yet investigated for PS QAM constellations.

In this paper, the TEq scheme from [4] is analyzed for PS signals, and its effectiveness demonstrated to be higher than for uniform signals. The combined performance of PS, digital backpropagation (DBP), and adaptive TEq is evaluated, highlighting the potential for transmission reach extension of WDM systems.

## 2. Turbo equalization method

The high-level description of the employed turbo equalizer is given in Fig. 1. After single shot DBP or electronic dispersion compensation (EDC), the assumed channel model takes the shape  $\mathbf{r}_i = \mathbf{H}_i \mathbf{s}_i + \mathbf{n}_i$ , where *i* represents time instant, and  $\mathbf{r}_i, \mathbf{s}_i$  and  $\mathbf{n}_i$  are vectorized, dual polarization received symbols, transmitted symbols and channel noise, respectively. The sizes of these vectors are 2N, where N is the length of the 1 sample per symbol equalizer. The matrix **H** is the assumed linear interference matrix of an appropriate size. The extrinsic log-likelihood ratios (LLRs) out of the FEC decoder are used to estimate soft-symbols, represented by mean  $\bar{s}$  and variance  $\sigma_s^2$ . This can be thought of as a soft mapping function. A recursive least squares (RLS) algorithm is used to estimate channel coefficients  $\mathbf{H}_i$  at time *i*, assuming a linear interference channel model  $\mathbf{r}_i = \mathbf{H}_i \mathbf{\bar{s}}_i$ . The estimated channel coefficients are then used to obtain a linear minimum mean squared error (LMMSE) equalizer coefficients  $\mathbf{w}_i$ , which are then used to obtain equalized symbols  $\hat{s}_i = \mathbf{w}_i^H [\mathbf{r}_i - \mathbf{H}_i \bar{\mathbf{s}}_i]$ . To obtain the posterior symbol distributions, an equivalent channel model is assumed of the form  $\hat{\mathbf{s}}_i = \mathbf{M}_i \mathbf{s}_i + \eta_i$ . The noise term  $\eta_i$  is assumed Gaussian, leading to a Gaussian likelihood function  $p(\hat{\mathbf{s}}_i | \mathbf{s}_i)$ . When probabilistic shaping is applied, the posterior distirbution of the equivalen channel is obtained as  $p(\mathbf{s}_i|\mathbf{\hat{s}}_i) \propto p(\mathbf{\hat{s}}_i|\mathbf{s}_i) p(\mathbf{s}_i)$ , where  $p(\mathbf{s}_i)$  is the prior distribution of the symbols. The posterior distributions are then used to obtain LLRs for the next FEC iteration using conventional bit-metric decoding. For space considerations, further details on the derivation of the LMMSE coefficients and the parameters of this equivalent model (i.e. coefficients of the equivalent matrix M and covariance matrix of the noise term  $\eta$ ) are omitted, and can be found here [4]



Fig. 1: Block diagram of the turbo equalizer used in this work.

#### 3. Simulation setup and results

The simulation setup assumes a transmission of eleven polarization-multiplexed (PM) WDM carriers modulated at 32 GBd in a 37.5 GHz frequency grid spacing. The sequence of shaped constellation symbols is generated by encoding pseudo random bit sequences with a constant composition distribution matcher (CCDM) [5] of length l = 4056 to a Maxwell-Boltzman probability mass function of a desired entropy. The soft-FEC is implemented with two ARJ4A low-density parity-check (LDPC) codes [6] of rates R = 0.7273, 0.4286 and lengths n = 22528, 28672, respectively. The range of FEC code rates used in this paper is obtained by using different puncturing rates of the basics codes. The coded symbols in each LDPC frame are interleaved before upsampling and pulse shaping. Pilot-symbols are uniformly inserted in between coded symbols at a rate of 5% for equalization, carrier phase recovery, and SNR estimation. An upsampling rate of 16 samples/symbol and pulse shaping with a root-raised cosine (RRC) filter with a roll-off factor of 0.01 are applied. Every Monte Carlo trial runs a sequence of symbols consisting of 18 LDPC code blocks per polarization. Performance metrics shown in the results are averaged over five Monte Carlo trials (> 2.2M information bits used in total). The nonlinear fiber channel is simulated using the symmetric split-step Fourier (SSFM) to solve the Manakov fiber model [7]. The model assumes transmission over multiple 50 km spans of standard, single mode fiber, with fiber attenuation compensated by Erbium-doped fiber amplifiers (EDFAs) with a noise figure of 4.5 dB. The SSFM is set to run with a fixed 100 m step-size. The fiber parameters of attenuation, nonlinear coefficient and chromatic dispersion are chosen to be  $\alpha = 0.2$  dB/km,  $\gamma = 1.3$  W<sup>-1</sup>km<sup>-1</sup>, and D = 17 ps/nm/km, respectively. Local oscillators are assumed to be ideal, i.e. the carrier phase recovery only compensates for the nonlinear phase noise. Dynamic polarization effects were not considered. At the receiver, the performance of the central WDM channel is evaluated. The same sequence of digital signal processing (DSP) blocks used in [4] is assumed at the receiver, adapted to process PS signals. Moreover, the bit interleaver assumed in [4] is replaced by a symbol interleaver instead, to make the modulation scheme compatible with CCDM. To compare performance of uniform and shaped constellations, the net information rate (IR) is fixed at 6.5 bits/QAMsymbol and different configurations of FEC rate and constellation entropy are set, as depicted in Table 1.

Modulation	Entropy [bits]	k	n	R = k/n	IR [bits/symb]
shaped 1024QAM	8.5	16384	20480	0.8000	6.50
uniform 256QAM	8	16384	20160	0.8127	6.50
uniform 1024QAM	10	16384	25200	0.6502	6.50

Table 1: Modulation and FEC parameters of the three tested configurations.

Shaped 256QAM is not included in the analysis because, to achieve the target net IR with an entropy much lower than 8 bits, the required severe puncturing significantly degrades the performance of the chosen FEC. Instead, PS 1024QAM was identified as the best choice in terms of flexibility and performance. Uniform 1024QAM is included as a basis for comparison of the effectiveness of the TEq for PS and non-PS schemes.

In Fig. 2a and Fig. 2b, performance curves of BER and effectively received signal-to-noise ratio (SNR<sub>eff</sub>) of the central channel as a function of fiber launch power per WDM channel are shown for the PS PM-1024QAM signals after  $29 \times 50$  km of transmission. Here, SNR<sub>eff</sub> is the average SNR estimated at the output of the LMMSE equalizer. After 10 TEq iterations the performance gain at the optimal launch power ( $\approx -2.0$  dBm per channel) saturates and the SNR<sub>eff</sub> is improved by extra  $\approx 0.80$  dB on top of 1.0 dB of the DBP gain. This allows for error-free transmission. In Fig. 3, the transmission reach extension of receivers employing DBP+TEq is compared



(a) post-rice ber versus noer input power. (b) Sirkeff versus noer input power.



Fig. 3: Post-FEC BER as a function of the number of fiber spans for all three investigated configurations.

to receivers that only implement EDC. For the uniform constellations, the transmission reach is extended by 8 spans (400 km), whereas 11 spans (550 km) are gained for PS 1024QAM. The extra gain for the PS signals is related to the fact that shaped constellations tend to enhance the NLI in the nonlinear fiber channel due to their higher kurtosis [8]. Overall, at the same net IR, the combined gains of PS, single-channel DBP, and TEq have allowed  $(29-13)/13 \approx 120\%$  of transmission reach extension, compared to EDC. Next, the benefits of compensating the inter-channel NLI with TEq are explicitly evaluated. In Table 2, the minimum SNR<sub>eff</sub> required for error-free performance is presented for all modulation schemes detailed in Table 1. From the results in Table 2, it can be noted that after the nonlinear fiber channel, if the receiver is set to perform EDC only, all modulations show a penalty from 0.2 to 0.5 dB w.r.t the AWGN channel performance due to the non-Gaussian noise, leading to a penalty of the Gaussian bit-metric decoder. By replacing EDC with DBP, the SNR<sub>eff</sub> increases, but the decoding threshold also goes up. This effect is most probably due to the fact that, after compensating for intra-channel NLI, the remaining inter-channel NLI would appear as time-varying intersymbol interference (ISI), further increasing the mismatch between the true channel and the memoryless Gaussian auxiliary channel model assumed by the softdecoder [9]. Thus, the increase in SNR<sub>eff</sub> after DBP does not translate linearly to post-FEC BER improvement. Finally, if DBP is followed by TEq, the decoding threshold is completely recovered for 256QAM and almost completely recovered for shaped 1024QAM (0.2 dB penalty remains). Hence, by reducing the impact of the timevarying ISI, the TEq also helps the mismatched soft-decoding process at the receiver.

Modulation	required SNR	required SNR <sub>eff</sub>	required SNR <sub>eff</sub>	required SNR <sub>eff</sub>
	(AWGN ch.)	(fiber ch. +EDC)	(fiber ch. +DBP)	(fiber ch. +DBP+ TEq)
shaped 1024QAM	$\approx 20.7 \text{ dB}$	$\approx 21.2 \text{ dB}$	$\approx 22.1 \text{ dB}$	$\approx 20.9 \text{ dB}$
uniform 256QAM	$\approx 21.9 \text{ dB}$	$\approx$ 22.4 dB	$\approx 22.8 \text{ dB}$	$\approx 21.9 \text{ dB}$
uniform 1024QAM	$\approx 22.8 \text{ dB}$	$\approx 23.0 \text{ dB}$	$\approx 23.6 \text{ dB}$	$\approx 22.6 \text{ dB}$

Table 2: Required SNR at the decoding threshold for different modulations schemes and receiver configurations.

# 4. Conclusions

The effectiveness of adaptive turbo equalization for nonlinearity compensation was assessed for probabilistically shaped constellations and shown to be higher than for uniform signals. A potential for more than 100% extension in transmission reach is verified in simulations by combining probabilistic shaping, digital backpropagation and adaptive turbo equalization.

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