Definition of Mode Field Diameter for Few-Mode Fibers based on Stationary Expression of Propagation Constant

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Abstract: We derive a novel definition of mode field diameters (MFDs) for few-mode fibers (FMFs) from the wave equation. The MFDs obtained with our definition are useful in evaluating splice losses of each mode in FMFs. © 2022 The Author(s)

1. Introduction

Few-mode fibers (FMFs) and few-mode multi-core fibers are attracting attention as transmission media that can realize large capacity space division multiplexing systems [1,2]. These fibers use several linearly polarized (LP) modes as the transmission channels. In transmission systems using these fibers, the mode dependent loss that occurs at splice points limits the transmission capacity [3]. Hence, predicting the splice losses of each transmission mode is essential for ensuring the interconnectivity, interoperability, and transmission performance of the systems.

The mode field diameter (MFD) is a key fundamental parameter of single-mode fibers (SMFs) [4,5], as it enables us to estimate the splice loss and other characteristics. Matsui et al. have experimentally clarified that the MFDs obtained from the far-field pattern (FFP) enable us to estimate splice losses of each mode in FMFs, whereas those obtained from the near-field pattern (NFP) do not [6]. This means that the MFDs based on the conventional FFP and NFP based definition are inconsistent with each other for some higher-order modes. A new NFP-based MFD definition is required since the MFD is often calculated from the NFP, for example, when designing the fibers.

In this paper, we propose a novel MFD definition based on the NFP that suits all LP modes in FMFs. We derive our definition from the wave equation. Numerical simulations clarify that the proposed MFD definition is consistent with the conventional FFP definition. We also demonstrate that the proposed definition is more useful than the conventional NFP definition in evaluating the splice losses of LP modes with non-zero azimuthal numbers.

2. MFD Definition based on Stationary Expression of Propagation Constant

This section derives a novel MFD definition from the wave equation. Propagation characteristics for any LP mode are determined by solving the following wave equation [7]

$$\frac{\partial^2\psi_{\nu\mu}}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_{\nu\mu}}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi_{\nu\mu}}{\partial\theta^2}+\left[k^2n^2(r)-\beta_{\nu\mu}^2\right]\psi_{\nu\mu}=0, \tag{1}$$

where $\psi_{\nu\mu}$ and $\beta_{\nu\mu}$ represent the near-field distribution and propagation constant, respectively, for an LP mode with azimuthal number ν and radial number μ. r and θ denote the radial distance and angular position, respectively. k stands for the wavenumber, and n indicates the refractive index. In the weakly guiding approximation, the near-field distribution can be resolved into radial and angular variables as follows:

$$\psi_{\nu\mu}=f_{\nu\mu}\cos(\nu\theta), \tag{2}$$

where $f_{\nu\mu}$ represents the radial near-field distribution. Then, we can obtain the following scalar wave equation as

$$d^2 f_{\nu\mu}/dr^2+(1/r)df_{\nu\mu}/dr+(k^2n^2-\beta_{\nu\mu}^2-\nu^2/r^2)f_{\nu\mu}=0. \tag{3}$$

Applying the variational principle to Eq. (3), we can obtain the following expression

$$\beta_{\nu\mu}^2=\frac{k^2\int_0^\infty n^2 f_{\nu\mu}^2 rdr}{\int_0^\infty f_{\nu\mu}^2 rdr}-\frac{\int_0^\infty (df_{\nu\mu}/dr)^2 rdr+\nu^2\int_0^\infty (1/r)f_{\nu\mu}^2 rdr}{\int_0^\infty f_{\nu\mu}^2 rdr}. \tag{4}$$

Equation (4) is a stationary expression for the propagation constant. Using the second term of Eq. (4), we propose a novel MFD definition for an arbitrary LP mode as

$$2w_{\nu\mu(s)}=2\sqrt{2}\left[\frac{\int_0^\infty f_{\nu\mu}^2 rdr}{\int_0^\infty (df_{\nu\mu}/dr)^2 rdr+\nu^2\int_0^\infty (1/r)f_{\nu\mu}^2 rdr}\right]^{1/2}, \tag{5}$$

where $2w_{\nu\mu(s)}$ represents the MFD based on the stationary expression. When the near-field distribution is exactly Laguerre Gaussian mode, the above formula yields a $(\nu+2\mu-1)^{1/2}$ times smaller value than its spot size. Equation (5) can be interpreted as a generalized formula of the well-known Petermann's NFP definition [8] mentioned later.

3. Numerical Simulations

This section confirms the usefulness of the proposed definition through numerical simulations. As an example, we consider a step-index fiber with the relative-index difference of 0.45% and the core diameter of 21.0 µm. This fiber supports four-LP modes at the wavelength of 1550 nm. We first compare the MFDs calculated by the proposed definition with those calculated by conventional definitions standardized for SMFs [4,5]. Then, we demonstrate that the proposed definition is useful in evaluating splice losses of each mode.

3.1 Comparison of Proposed Definition with Other Definitions

According to the international standards for SMFs [4,5], the MFD can be obtained from the far-field distribution as

$$2w_{\nu\mu(f)} = \frac{\lambda}{\pi}\left[2\frac{\int_0^{\pi/2} F_{\nu\mu}^2 \sin\phi\cos\phi d\phi}{\int_0^{\pi/2} F_{\nu\mu}^2 \sin^3\phi\cos\phi d\phi}\right]^{1/2}, \tag{6}$$

where $F^2{}_{\nu\mu}$ represents the far-field intensity distribution. λ stands for the wavelength of the test light. φ denotes the divergence angle. We refer to the above formula as the FFP definition. The MFD can be also determined from the near-field distribution as

$$2w_{\nu\mu(n)} = 2\sqrt{2}\left[\frac{\int_0^{\infty} f_{\nu\mu}^2 rdr}{\int_0^{\infty} (df_{\nu\mu}/dr)^2 rdr}\right]^{1/2}. \tag{7}$$

We refer to Eq. (7) as Petermann's NFP definition. We compare MFDs obtained by the proposed definition with those obtained by the above two definitions. For LP modes with the azimuthal number of zero, Eqs. (5) and (7) are completely identical with each other.

Figure 1 shows the MFDs with respect to the wavelength calculated by each definition, where (a)–(d) represent the results for LP_{01}, LP_{11}, LP_{21}, and LP_{02} modes, respectively. The red, blue, and green lines represent the results calculated by Eqs. (5), (6), and (7), respectively. Note that the green lines were omitted from Figs. 1(a) and 1(d) because Eq. (5) yields identical results to Eq. (7) for LP modes with ν=0. We found that the MFDs calculated by the proposed definition agreed with those calculated by the FFP definition. It must also be emphasized that the MFDs calculated by Eq. (7) differ from those calculated by Eq. (6) for LP modes with non-zero azimuthal numbers.

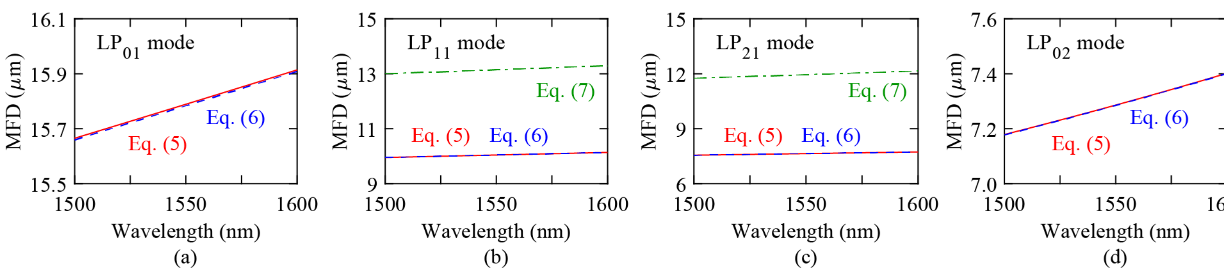

Fig. 1. MFDs versus wavelength for (a) LP_{01}, (b) LP_{11}, (c) LP_{21}, and (d) LP_{02} modes. Red, blue, and green lines represent the results calculated by Eqs. (5), (6), and (7), respectively.

3.2 Evaluation of Splice Losses

When two identical optical fibers are spliced with the axial misalignment of s, the relation between amplitude a_1 and a_2 of the same mode in the input and output fibers is given by

$$a_2 / a_1 = \iint \psi_{\nu\mu}(r-s)\psi_{\nu\mu}(r) rdrd\theta \Big/ \iint \psi_{\nu\mu}^2(r) rdrd\theta, \tag{8}$$

Using Eq. (2) and Fourier transformation, we can rewrite Eq. (8) as [9]

$$\frac{a_2}{a_1} = \frac{\iint F_{\nu\mu}^2(\rho)\cos^2(\nu\Theta)\exp[j\rho s\cos(\theta_s - \Theta)]\rho d\rho d\Theta}{\iint F_{\nu\mu}^2(\rho)\cos^2(\nu\Theta)\rho d\rho d\Theta}, \tag{9}$$

where ρ and Θ symbolize the radial distance and angular position in the far-field, respectively. θ_s denotes the direction of the misalignment. When misalignment s is small, the exponential term in Eq. (9) can be approximated as

$$\exp[j\rho s\cos(\theta_s - \Theta)] \cong 1 + j\rho s\cos(\theta_s - \Theta) - [\rho s\cos(\theta_s - \Theta)]^2 / 2. \tag{10}$$

Substituting Eq. (10) into Eq. (9) yields the following relation

$$\frac{a_2}{a_1}=1-\frac{s^2}{4}\frac{\int F_{\nu\mu}^2(\rho)\rho^3 d\rho}{\int F_{\nu\mu}^2(\rho)\rho d\rho}\quad(\nu\neq 1),\quad\text{and}\quad \frac{a_2}{a_1}=1-\frac{s^2}{8}(1+2\cos^2\theta_s)\frac{\int F_{\nu\mu}^2(\rho)\rho^3 d\rho}{\int F_{\nu\mu}^2(\rho)\rho d\rho}\quad(\nu=1). \tag{11}$$

Considering the average over the direction of misalignment θ_s, Eq. (11) can be expressed for all ν as

$$\frac{a_2}{a_1}=1-\frac{s^2}{4}\frac{\int F_{\nu\mu}^2(\rho)\rho^3 d\rho}{\int F_{\nu\mu}^2(\rho)\rho d\rho}=1-\frac{k^2 s^2}{4}\frac{\int_0^{\pi/2} F_{\nu\mu}^2 \sin^3\phi\cos\phi d\phi}{\int_0^{\pi/2} F_{\nu\mu}^2 \sin\phi\cos\phi d\phi}=1-\frac{s^2}{2w_{\nu\mu(s)}^2}, \tag{12}$$

where we used the relation $\rho=k\sin\varphi$. If the misalignment s is small, the power coupling efficiency $\eta_{\nu\mu}$ between the $LP_{\nu\mu}$ modes at a splice point is

$$\eta_{\nu\mu}=\left(a_2/a_1\right)^2=\left[1-s^2/(2w_{\nu\mu(s)}^2)\right]^2\cong\exp\left[-s^2/w_{\nu\mu(f)}^2\right]. \tag{13}$$

Equation (13) demonstrates that the power coupling efficiency, which corresponds to the splice loss, can be given by the same expression as the conventional Gaussian model for any LP modes. Equation (13) also demonstrates that the splice losses for any LP mode can be evaluated using the conventional FFP definition written as Eq. (6). This means that the splice losses can also be evaluated using the proposed MFD definition because the MFDs obtained by the proposed and FFP definitions are the same as demonstrated in Fig. 1. Moreover, we see in Eq. (13) that the MFDs of each mode should preferably have similar values to suppress the mode dependent loss at a splice point. Generally, there is a trade-off between meeting the above requirements and suppressing macro- and micro-bending losses of higher-order modes. We have to consider this point when designing FMFs.

Figure 2 shows the splice losses with respect to the axial misalignment, where (a)–(d) represent the results for LP_{01}, LP_{11}, LP_{21}, and LP_{02} modes, respectively. The red and green lines represent the results estimated using Eqs. (5) and (7), respectively. The black circles indicate the theoretical splice losses calculated from the overlap integral of the near fields. Note that we did not plot the splice losses estimated using Eq. (6) since they are the same as the red line. The green lines were also omitted from Figs. 2(a) and 2(d) for the same reason of Fig. 1. We confirmed from Fig. 2 that the splice losses estimated using the proposed definition agreed well with the theoretical ones, whereas those estimated using Eq. (7) did not. This finding leads to the conclusion that the splice losses of each mode can be evaluated using the proposed definition and the FFP definition.

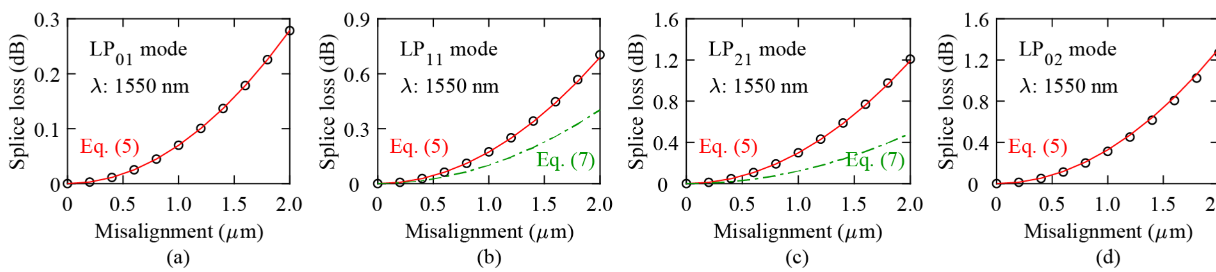

Fig. 2. Splice losses with respect to the axial misalignment for (a) LP_{01}, (b) LP_{11}, (c) LP_{21}, and (d) LP_{02} modes. Black circles represent the theoretical splice losses calculated by the overlap integral between the electric field. Red and green lines indicate the results estimated using Eqs. (5) and (7), respectively.

4. Conclusions

We proposed a novel MFD definition for FMFs based on a stationary expression of the propagation constant. The proposed definition is derived from the wave equation. We clarified through numerical simulations that the proposed definition is consistent with the FFP definition, whereas the conventional NFP definition is inconsistent with the other two definitions for LP modes with non-zero azimuthal numbers. We also clarified that the splice losses estimated using the proposed definition agreed well with the theoretical ones, whereas those estimated by the conventional NFP definition did not for LP modes with non-zero azimuthal numbers. From these results, we conclude that the proposed definition is more useful than the conventional NFP definition in evaluating the splice losses of each mode in FMFs. We believe that the findings of this work will be useful in standardizing the MFD for FMFs.

5. References

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