Modeling of Nonlinear Distortion in Space-Division Multiplexing

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Abstract

We review the main models of space-division multiplexing transmissions in fiber optics working in the nonlinear regime, including perturbative modeling, the ergodic Gaussian noise model, and their implications on system analysis.

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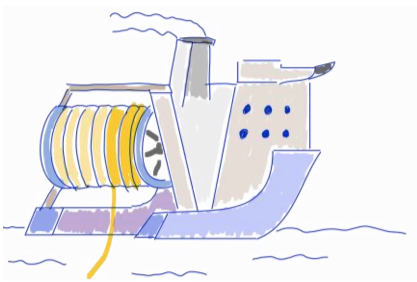
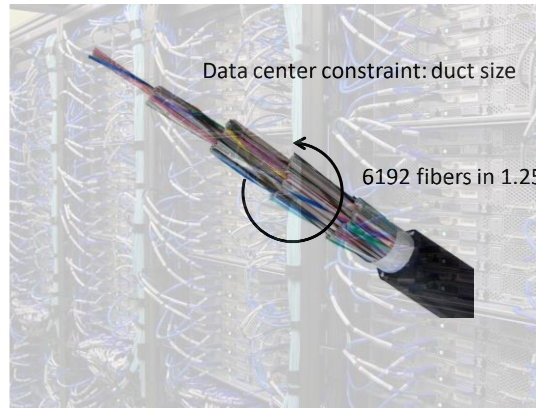
Motivation

Subsea constraint: tank size of the vessel





10,000 parallel fibers linking datacenters [Win]

- The challenge for the next future: increase the capacity with a spatial constraint
- The paradigm: capacity per cable rather than per fiber

[Win] P. Winzer, “Trasmission system capacity scaling through space-division multiplexing: a techno-economic perspective,” Academic Press, 2020
[Mat] T. Matsui, “Weakly Coupled Multicore Fiber Technology, Deployment, and Systems ,” Proc IEEE 2022.

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Motivation

 # spatial paths *bandwidth*

$$C = 2MB\log_2\left(1 + \frac{P_{\mathrm{WDM}}}{2N_0MB}\right)$$

bandwidth

Hollow-core fibers — *Freq. dependent parameters*

Multi-band transmission — *Raman effect*

Nyquist WDM — *Fiber slope*

PMD

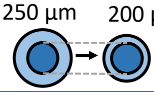
High symbol rates — *Kerr effects*

capacity

Linear crosstalk *Mode dispersion* *Extra Kerr effects*

Spatial dimension

250 µm 200 µm



Multi-core FMF Multi-mode

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Goal of this tutorial

- Describe the physical aspects underpinning nonlinearity in SDM
- Describe approximated models to simplify their analysis

Perturbative analysis

Review of nonlinear effects in SMF

Impact of coupling

interplay mode dispersion - Kerr

Gaussian noise models for SDM

Critical comparison

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Scalar → SDM

Scalar	SDM
Electric field A	$\vert A\rangle = \begin{bmatrix} A_{1x} \\ A_{1y} \\ \vdots \\ A_{2Nx} \\ A_{2Ny} \end{bmatrix}$
A^*	$\langle A\vert = [A_{1x}^* \quad A_{1y}^* \quad \dots \quad A_{2Nx}^* \quad A_{2Ny}^*]$
$\vert A\vert^2 = A^*A = AA^*$	$\langle A\vert A\rangle = \vert A_{1x}\vert^2 + \vert A_{2y}\vert^2 + \cdots \vert A_{2Ny}\vert^2 \neq \vert A\rangle\langle A\vert$
Phase shift $e^{j\varphi}$, such that $e^{j\varphi}e^{-j\varphi}$=1	Unitary matrix $\boldsymbol{U}$, such that $\boldsymbol{U}\boldsymbol{U}^\dagger = \boldsymbol{I}$
Real parameter, e.g., β_2, such that β_2=β_2^*	Hermitian matrix $\boldsymbol{B_2}$, such that $\boldsymbol{B_2} = \boldsymbol{B_2^\dagger}$
Deterministic propagation	Random coupling

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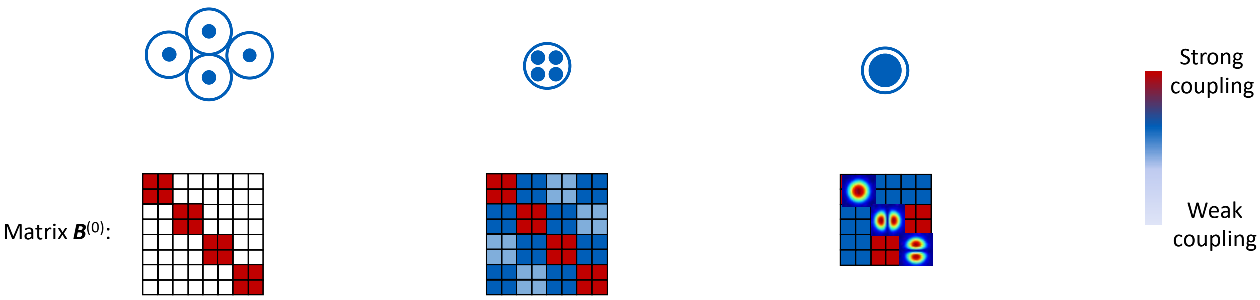
Linear Propagation in SDM

$$\frac{\partial |A\rangle}{\partial z} = -j\boldsymbol{B}^{(0)}|A\rangle - \boldsymbol{B}^{(1)}\frac{\partial |A\rangle}{\partial t}$$



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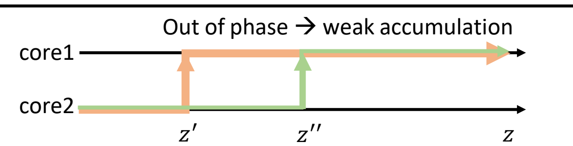
Coupling in Multi-core fibers



$$e^{j\beta_1(z-z')}e^{j\beta_2 z'} \neq e^{j\beta_1(z-z'')}e^{j\beta_2 z''}$$

$\frac{d\|A\rangle}{dz} = -j\begin{bmatrix}\beta_1 & \kappa \\ \kappa & \beta_2\end{bmatrix}\|A\rangle$	Weak-coupling	Random-coupling	Systematic coupling
	a, $\Lambda \gg a$	$\Lambda > a$	$\Lambda \sim a$
	$\beta_1 \sim \beta_2$ $\kappa \ll \beta_{1,2}$	$\beta_1 \sim \beta_2$ $\kappa < \beta_{1,2}$	$\beta_1 \neq \beta_2$ $\kappa \sim \beta_{1,2}$
End-to-end coupling:	Cores: weak Modes: weak	Cores: strong Modes: strong	Cores: strong Modes: **weak**
Mode dispersion scaling	Linear in z	Square root in z	Linear in z

K. Saitoh, "Multi-Core Fiber Technology for SDM: Coupling Mechanisms and Design," JLT 2022 Ref: T. Hayashi, "Randomly-coupled Multi-core Fiber Technology," Proc IEEE 2022

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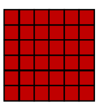
Kerr nonlinearity: SDM

- In a reference system tracking $\boldsymbol{B}^{(0)}$:

$$\frac{\partial|A_n\rangle}{\partial z} = -j\sum_{\ell\neq n}\boldsymbol{K}_{n\ell}|A_\ell\rangle - \boldsymbol{B}_1^{(1)}\frac{\partial|A_n\rangle}{\partial t} - j\gamma\left(\sum_{\ell}\kappa_{n\ell}\langle A_\ell|A_\ell\rangle\right)|A_n\rangle$$

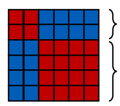
- Manakov averaging: *linear coupling* *nonlinear coupling*

Strong-coupling regime



$-j\gamma\kappa\langle A|A\rangle|A\rangle$

Group-coupling regime



$-j\gamma[\kappa_{11}\langle A_1|A_1\rangle + \kappa_{12}\langle A_2|A_2\rangle]|A_1\rangle$

Example: dual-polarization (Strong-coupling regime)

$-j\gamma\frac{8}{9}\left(|A_x|^2 + |A_y|^2\right)\begin{bmatrix}A_x\\A_y\end{bmatrix}$

C. Antonelli, M. Shtaif, and A. Mecozzi, "Modeling of Nonlinear Propagation in Space-Division Multiplexed Fiber-Optic Transmission," JLT 2016
S. Mumtaz, R. J. Essiambre, and G. P. Agrawal, "Nonlinear propagation in multimode and multicore fibers: Generalization of the Manakov equations," JLT 2013

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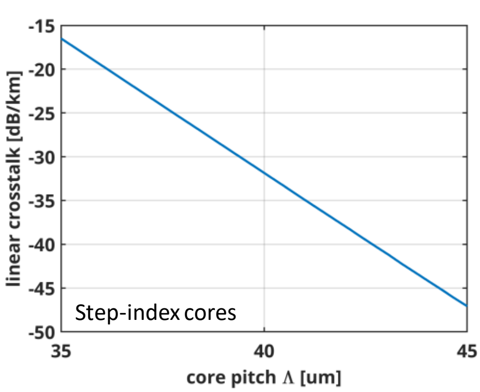
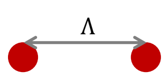
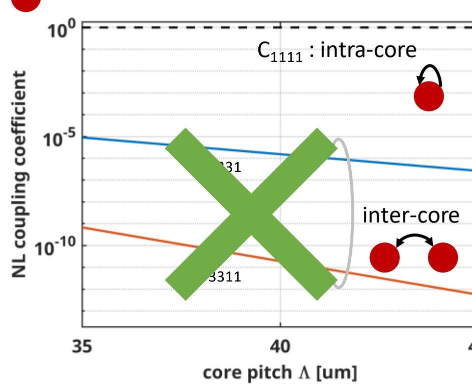
Multi-core fibers: linear vs nonlinear crosstalk

Linear coupling: $-j\boldsymbol{K}_{12}|A_2\rangle$

Kerr effect $= -j\gamma\sum_{n}\hat{e}_n\sum_{h,k,m}C_{nhkm}A_h^*A_kA_m$







S. Mumtaz , R. J. Essiambre, and G. P. Agrawal, "Reduction of Nonlinear Penalties Due to Linear Coupling in Multicore Optical Fibers," PTL 2012

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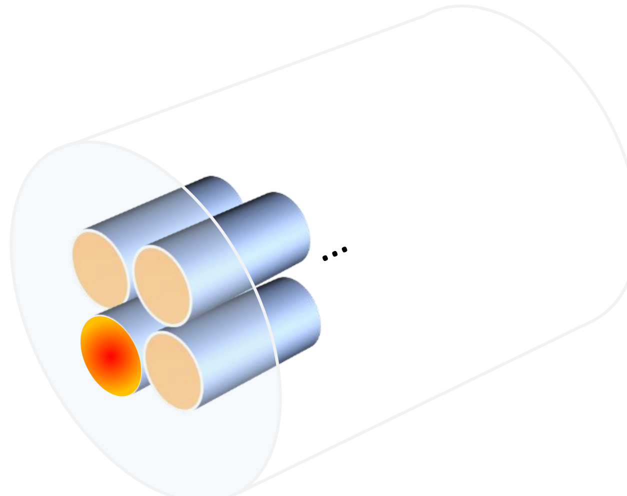
Multi-core fibers: Manakov coefficient

- Nonlinear coefficient scales almost inversely with the number of cores

$$\frac{\partial|A_n\rangle}{\partial z} = -\boldsymbol{B_1}\frac{\partial|A_n\rangle}{\partial t} - j\sum_{\ell\neq n}\boldsymbol{K}_{n\ell}|A_\ell\rangle$$
$$-j\gamma\left(\kappa_{nn}\langle A_n|A_n\rangle + \cancel{\sum_{\ell\neq n}\kappa_{n\ell}\langle A_\ell|A_\ell\rangle}\right)|A_n\rangle$$

$$\kappa_{nn} = \frac{8}{3}\frac{1}{2N+1}$$

N= # cores



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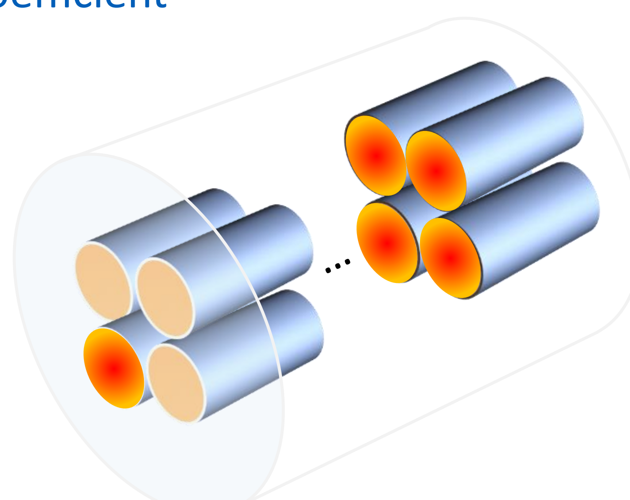

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Comparison

	Fiber bundle	MCF weakly coupling	MCF strong (randomly) coupling
End-to-end XTalk	0	$\kappa_{XT}P$	~ 0 (after MIMO)
Manakov coeff κ	$\frac{8}{9}$	$\frac{8}{9}$	$\frac{8}{3}\frac{1}{2N+1} < \frac{8}{9}$
Optimal power (@ equal link parameters)	$P_{NLT} = \left(\frac{P_{ASE}}{2a_{NL}}\right)^{1/3}$	Not impacted by linear XT, $\sim P_{NLT}$	Not impacted by linear XT, > P_{NLT} due to smaller κ
Capacity scaling @ optimal power	$AIR = N\log_2(1+SNR)$	$\sim AIR$	$N\log_2(1+(2N+1)^{\frac{2}{3}}SNR)$

M. Shtaif, C. Antonelli, A. Mecozzi, and X. Chen, "Challenges in Estimating the Information Capacity of the Fiber-Optic Channel," Proc. IEEE 2022
J. Downie et al. "Modeling the Techno-Economics of Multicore Optical Fibers in Subsea Transmission Systems," JLT 2022

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Experiments

Hayashi *et al.*: Randomly-Coupled Multi-Core Fiber Technology

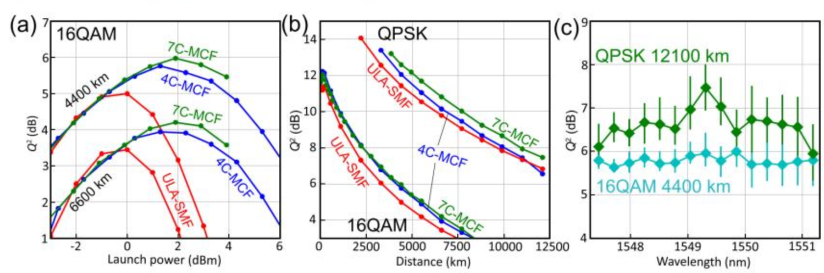

Fig. 21. *Q-factors for ultralow-loss large-effective-area SMF (ULA-SMF), 4C-MCF, and 7C-MCF as a function of (a) launch power (16 QAM) and (b) distance (QPSK and 16QAM). For 7C-MCF, (c) Q-factor as function of wavelength channel for a transmission distance of 4400 (16QAM) and 12 100 km (QPSK) [24].*

Ref: T. Hayashi, "Randomly-coupled Multi-core Fiber Technology," Proc IEEE 2022 (licensed CC BY 4.0)
[24] R. Ryf et al., "Coupled-core transmission over 7-core fiber," OFC 2019

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Perturbative models in single-mode fibers: review

- Four wave mixing (FWM) with unmodulated waves

$$A(z,\omega_i) \simeq A(0,\omega_i) - j\gamma \underbrace{\sum_{k,m,n} \eta_{kmni}(z) A^*(0,\omega_k) A(0,\omega_m) A(0,\omega_n)}_{\text{Nonlinear interference (NLI)}}$$

If all signals with the same power P, the FWM power scales as

$$P_{\mathrm{FWM}} = a_{\mathrm{NL}} P^3$$

$$\eta_{kmni} = \int_0^z e^{-\alpha\xi} e^{-j\Delta\beta_{kmni}\xi}\, \mathrm{d}\xi$$ FWM kernel or efficiency

$$\Delta\beta_{kmni} = \beta(\omega_m) + \beta(\omega_n) - \beta(\omega_k) - \beta(\omega_i)$$ FWM phase matching coefficient

$$\omega_i = \omega_m + \omega_n - \omega_k$$ FWM law of conservation of energy

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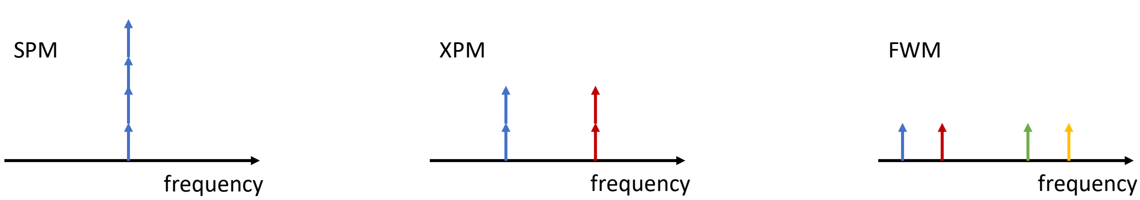
Four-wave mixing (FWM)

- FWM among unmodulated waves

$$\omega_n = \omega_k + \omega_m - \omega_\ell$$



- Is that all in SDM?

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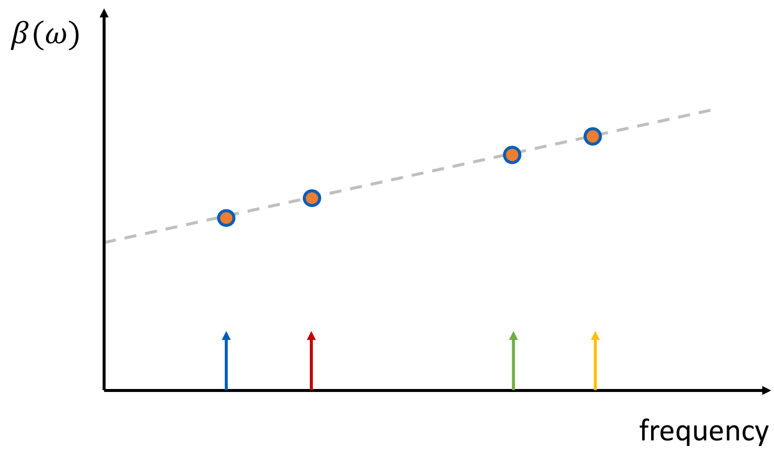
FWM efficiency



$\Delta\beta_{klmn} = 0$

On a line? Phase matching!

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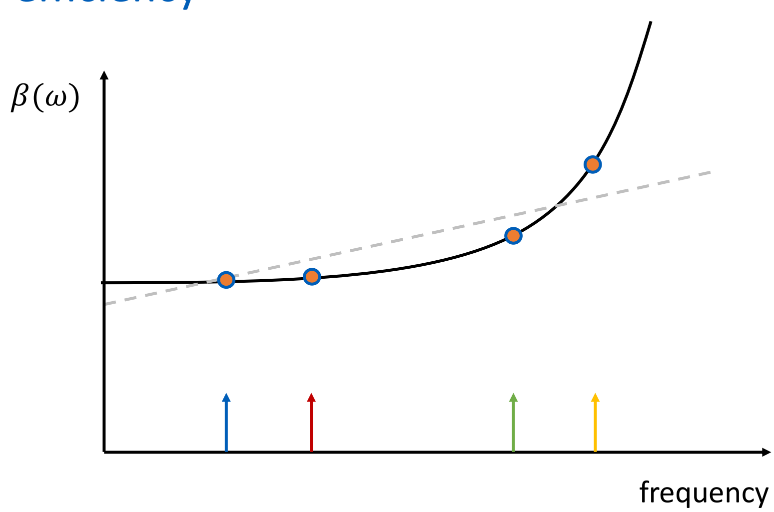
FWM efficiency



$\Delta\beta_{klmn} \neq 0$

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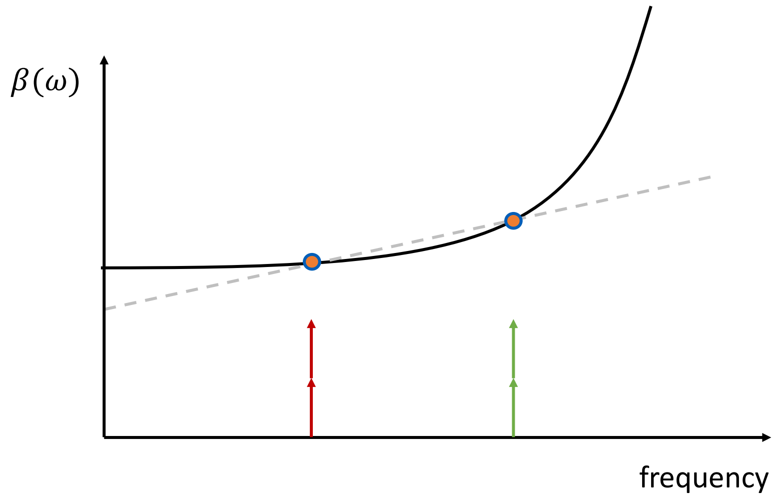
FWM efficiency



- What about in SDM?

Include signal modulation

$$A(z,\omega) \simeq A(0,\omega) - j\gamma \iint_{-\infty}^{\infty} \eta(\omega,\omega_1,\omega_2) A^*(0,\omega_1+\omega_2-\omega) A(0,\omega_1) A(0,\omega_2) \frac{\mathrm{d}\omega_1}{2\pi}\frac{\mathrm{d}\omega_2}{2\pi}$$

$$\eta = \int_0^z e^{-\alpha\xi} e^{-j\Delta\beta(\omega,\omega_1,\omega_2)\xi}\, \mathrm{d}\xi$$

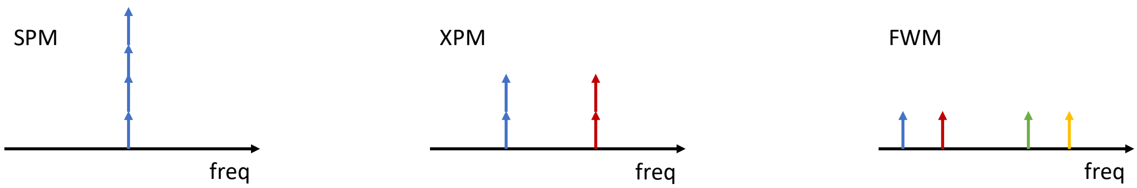
$$\Delta\beta = \beta(\omega_1) + \beta(\omega_2) - \beta(\omega_1+\omega_2-\omega) - \beta(\omega)$$

- What about in SDM? Just extra combinations among different polarizations?

FWM with modulation

- FWM among unmodulated waves

$$\omega_n = \omega_k + \omega_m - \omega_\ell$$



- FWM among modulated signals

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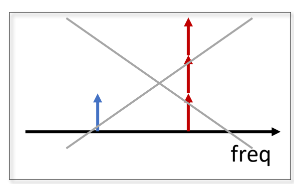
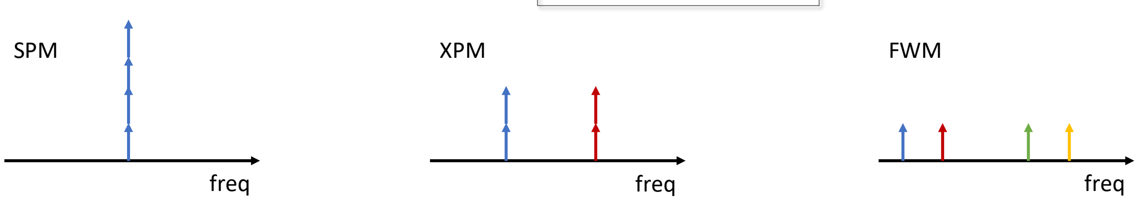
FWM with modulation

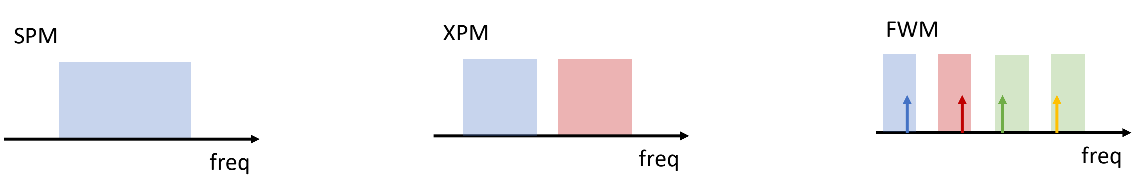


- FWM among unmodulated waves

$$\omega_n = \omega_k + \omega_m - \omega_\ell$$



- FWM among modulated signals



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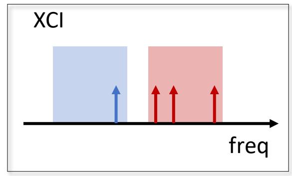
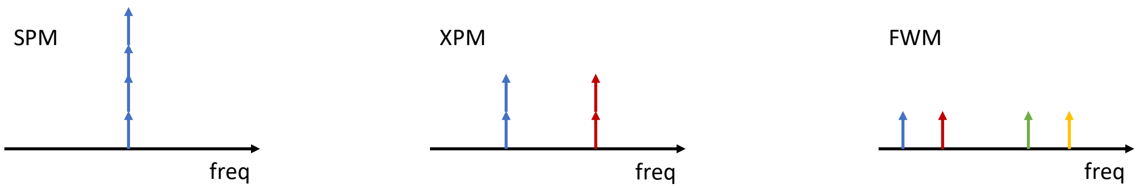
FWM with modulation

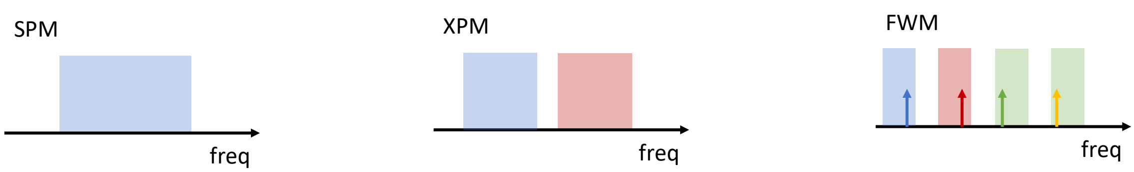


$$\omega_n = \omega_k + \omega_m - \omega_\ell$$

- FWM among unmodulated waves



- FWM among modulated signals



P. Poggiolini, "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems," JLT2012

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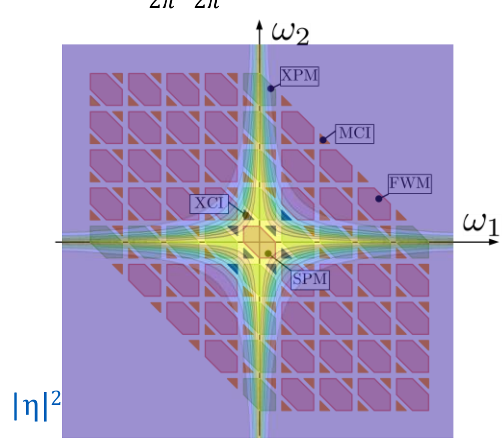
Impact of FWM kernel

$$A(z,\omega) \simeq A(0,\omega) - j\gamma \iint_{-\infty}^{\infty} \eta(\omega,\omega_1,\omega_2) A^*(0,\omega_1+\omega_2-\omega) A(0,\omega_1) A(0,\omega_2) \frac{\mathrm{d}\omega_1}{2\pi}\frac{\mathrm{d}\omega_2}{2\pi}$$

$$\eta = \int_0^z e^{-\alpha\xi} e^{-j\Delta\beta(\omega,\omega_1,\omega_2)\xi}\,\mathrm{d}\xi$$

- Note: XPM degenerates along the two axes.



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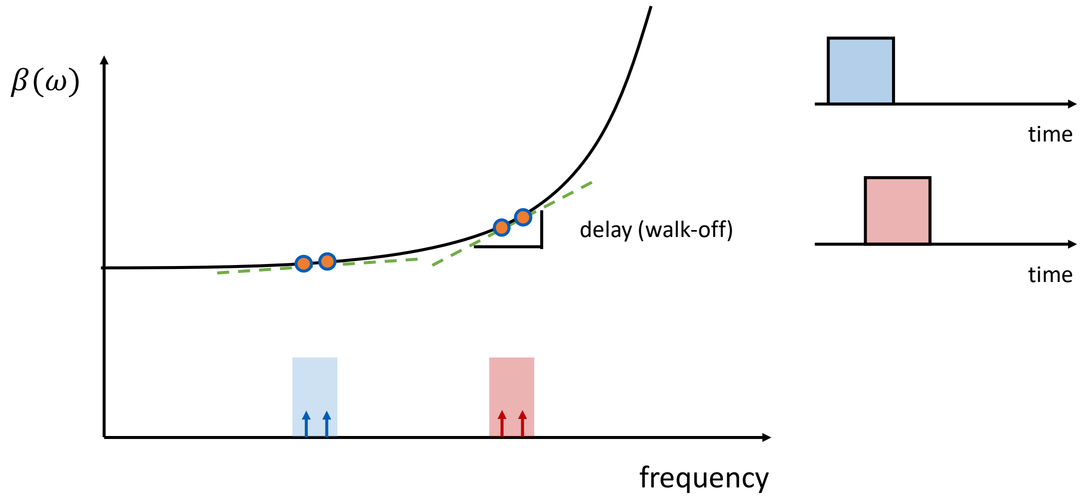
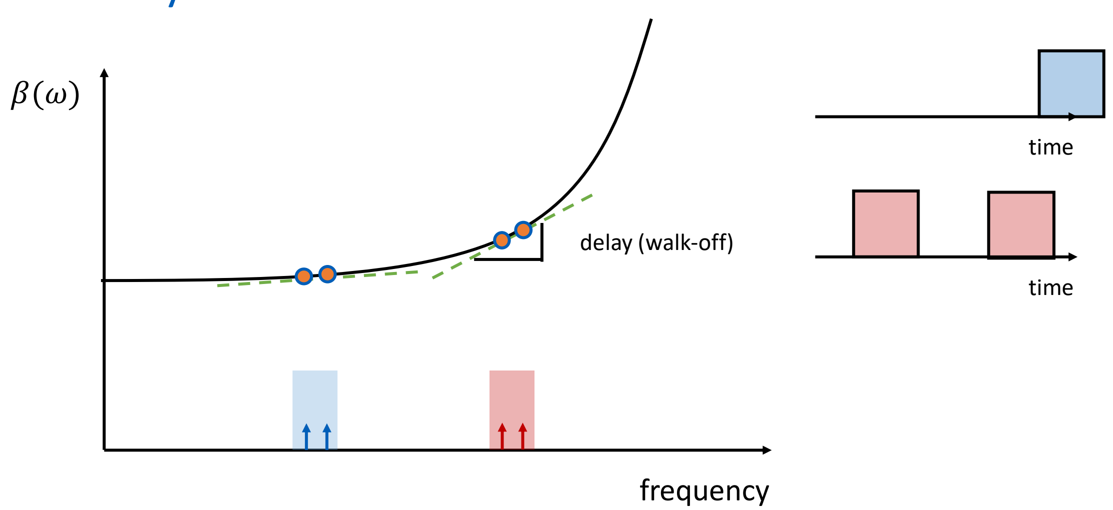
XPM efficiency



XPM efficiency

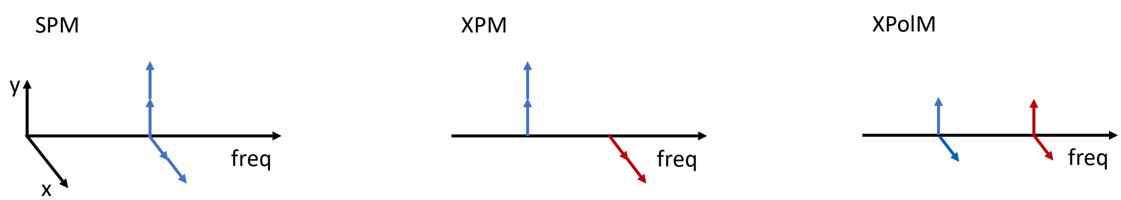


- XPM is mainly set by channel walk-off: what about in SDM?

FWM in the spatial dimension

- FWM among unmodulated waves

$\omega_n = \omega_k + \omega_m - \omega_\ell$



SPM	XPM	XPolM
$\lvert A_y \rvert^2 A_x$	$\lvert B_y \rvert^2 A_x$	$B_y B_x^* A_x$
Phase modulation	Phase modulation	Polarization crosstalk
$\langle A\vert A\rangle\vert A\rangle$	$\langle B\vert B\rangle\vert A\rangle$	$\vert B\rangle\langle B\vert A\rangle$

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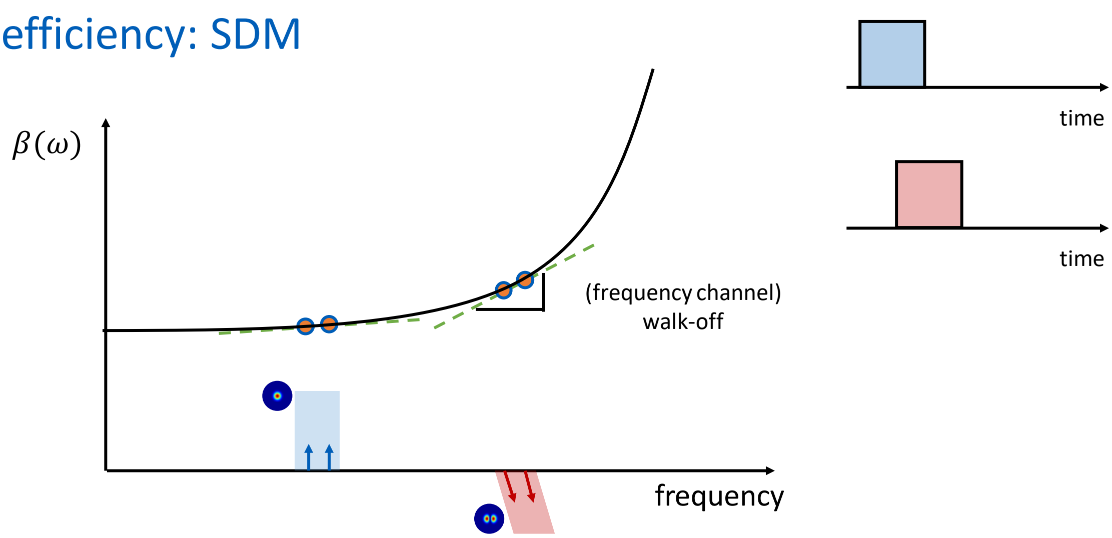
XPM efficiency: SDM



- DMGD may induce phase matching to channels well spaced away in the frequency

R.-J. Essiambre et al., "Experimental investigation of inter-modal four-wave mixing in few-mode fibers," PTL 2013.
G. Rademacher et al. "Intermodal Nonlinear Signal Distortions in Multi-Span Transmission With Few-Mode Fibers," PTL 2020

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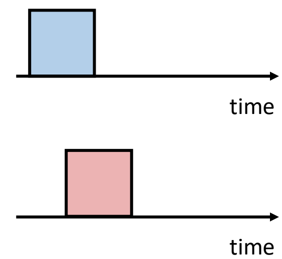
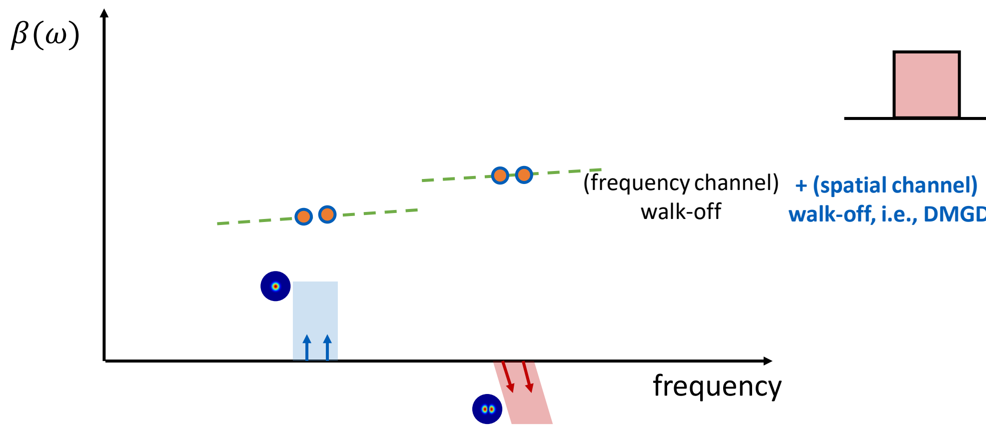
XPM efficiency: SDM





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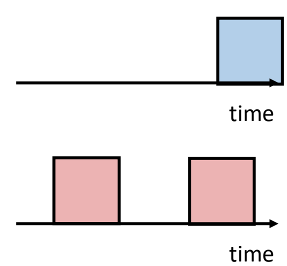
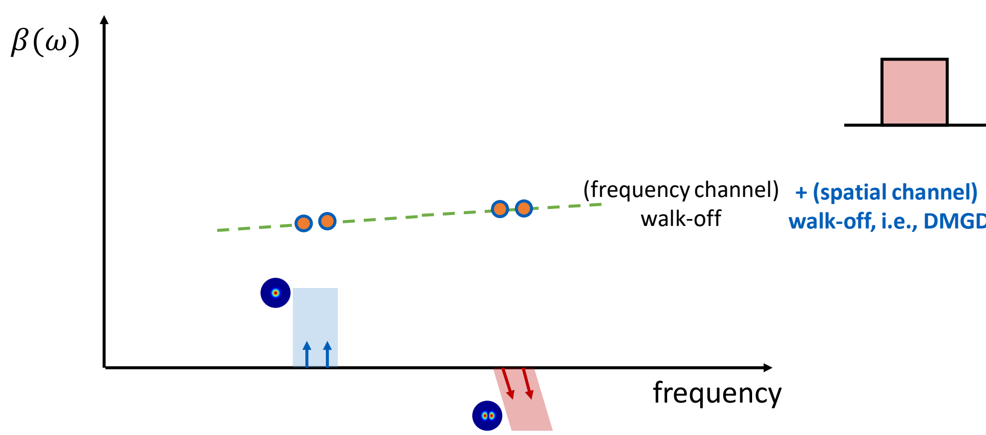
XPM efficiency: SDM





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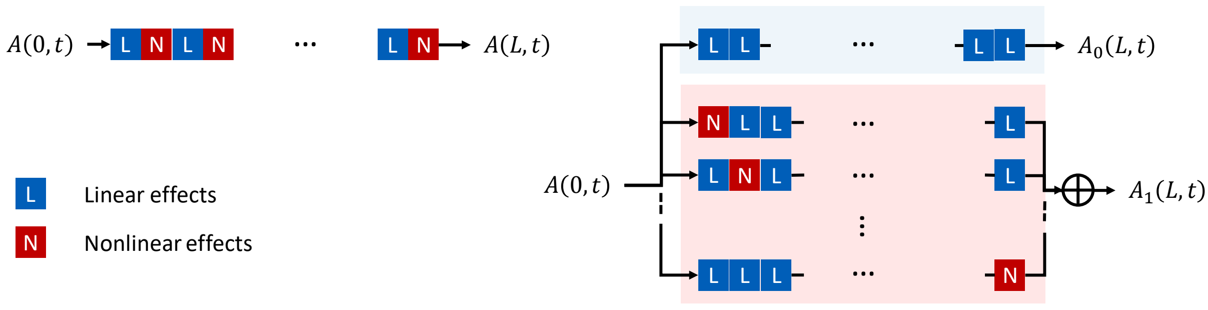
Perturbative nonlinearity: spatially resolved interpretation



$$A(L,t) \simeq A_0(L,t) + \gamma A_1(L,t)$$

- Perturbation: 1[st] order Taylor series in nonlinearity→ cross only one nonlinear block from input to output

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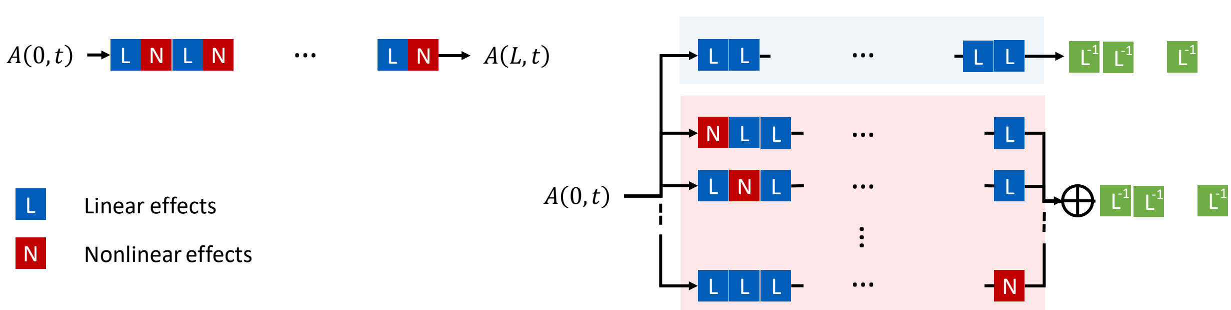
Perturbative nonlinearity: spatially resolved interpretation



$$A(L,t) \simeq A_0(L,t) + \gamma A_1(L,t)$$

- Perturbation: 1[st] order Taylor series in nonlinearity→ cross only one nonlinear block from input to output

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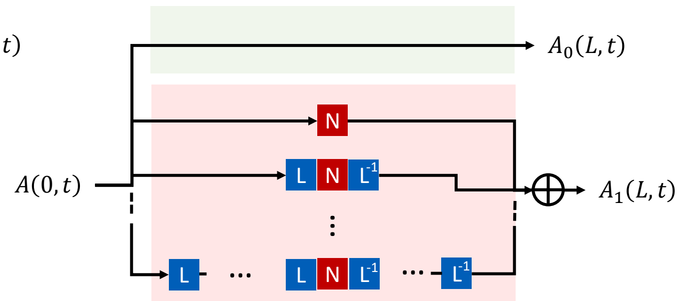
Perturbative approach



L Linear effects

N Nonlinear effects

$$A(L,t) \simeq A(0,t) + \gamma A_1(L,t)$$

P. Serena and A. Bononi, "A Time-Domain Extended Gaussian Noise Model," JLT 2015

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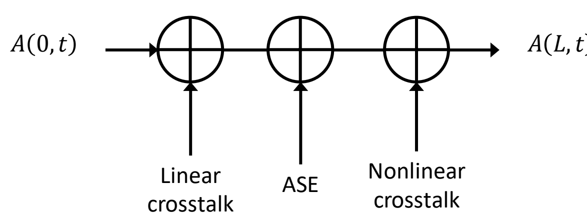
Perturbative approach



L Linear effects

N Nonlinear effects

- AWGN perturbative equivalent channel model

P. Serena and A. Bononi, "A Time-Domain Extended Gaussian Noise Model," JLT 2015

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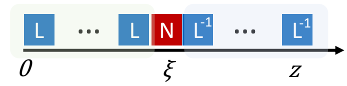
Discrete-time channel model



- Nonlinear interference

$$n_i = -j \int_0^z e^{-\mathcal{L}\xi} \mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right) \mathrm{d}\xi$$

A. Mecozzi and R.-J. Essiambre, "Nonlinear Shannon Limit in Pseudolinear Coherent Systems," JLT 2012

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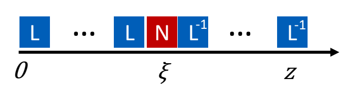
Discrete-time channel model



- Nonlinear interference

$$n_i = -j \int_0^z e^{-\mathcal{L}\xi} \mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right) \mathrm{d}\xi$$

$$\sum_k a_k p(0, t-kT)$$

A. Mecozzi and R.-J. Essiambre, "Nonlinear Shannon Limit in Pseudolinear Coherent Systems," JLT 2012

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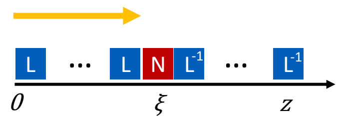
Discrete-time channel model



- Nonlinear interference

$$n_i = -j\int_0^z e^{-\mathcal{L}\xi}\mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right)\mathrm{d}\xi$$

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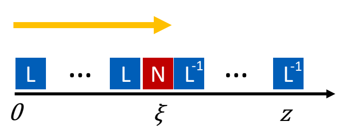
Discrete-time channel model



- Nonlinear interference

$$n_i = -j\int_0^z e^{-\mathcal{L}\xi}\mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right)\mathrm{d}\xi$$

$$\sum_{k,m,n} a_k^* a_m a_n p^*(\xi, t-kT)p(\xi, t-mT)p(\xi, t-nT)$$

A. Mecozzi and R.-J. Essiambre, “Nonlinear Shannon Limit in Pseudolinear Coherent Systems,” JLT 2012

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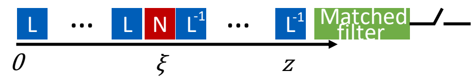
Discrete-time channel model



- Nonlinear interference

$$n_i = -j\int_0^z e^{-\mathcal{L}\xi}\mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right)\mathrm{d}\xi$$

A. Mecozzi and R.-J. Essiambre, "Nonlinear Shannon Limit in Pseudolinear Coherent Systems," JLT 2012

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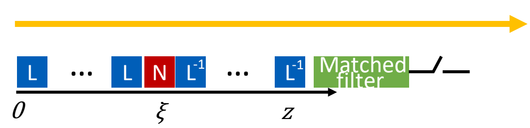
Discrete-time channel model



- Nonlinear interference

$$n_i = -j\int_0^z e^{-\mathcal{L}\xi}\mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right)\mathrm{d}\xi$$

$$\int_{-\infty}^{\infty} p^*(\xi, t-iT)\sum_{k,m,n} a_k^* a_m a_n p^*(\xi, t-kT)p(\xi, t-mT)p(\xi, t-nT)\,\mathrm{d}t$$

A. Mecozzi and R.-J. Essiambre, "Nonlinear Shannon Limit in Pseudolinear Coherent Systems," JLT 2012

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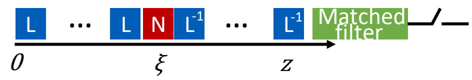
Discrete-time channel model



- Nonlinear interference

$$n_i = -j \int_0^z e^{-\mathcal{L}\xi} \mathcal{N}\left(e^{\mathcal{L}\xi}(A(0,t))\right) \mathrm{d}\xi$$

$$n_i = \sum_{k,m,n} a_k^* a_m a_n \mathcal{X}_{kmni}$$

$$\mathcal{X}_{kmni} = \int_0^z f(z) \int_{-\infty}^{\infty} p^*(\xi, t - iT) p^*(\xi, t - kT) p(\xi, t - mT) p(\xi, t - nT)\, \mathrm{d}t\, \mathrm{d}\xi$$

A. Mecozzi and R.-J. Essiambre, "Nonlinear Shannon Limit in Pseudolinear Coherent Systems," JLT 2012

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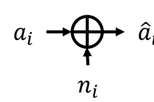
NLI power



- Discrete-time channel model:

$$\hat{a}_i = a_i + n_i = a_i - j \underbrace{\sum_{k,m,n} a_k^* a_m a_n \mathcal{X}_{kmni}}_{\text{NLI}}$$

- NLI power: $\mathbb{E}[|n_i|^2] = \sum_{\substack{k,m,n \\ h,p,r}} \mathbb{E}\left[a_k^* a_m a_n a_h a_p^* a_r^*\right] \mathcal{X}_{kmni} \mathcal{X}_{hpri}^*$

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Second-order noise

e.g., $\mathbb{E}[a_k^* a_m]\,\mathbb{E}[a_r^* a_n]\mathbb{E}[a_p^* a_h]$ →

if Gaussian

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}_2}\prod_{(i,j)\in\pi}\mathbb{E}[a_i^* a_j]$$

k*	m	n	h	p*	r*	
•	•	○	Δ	Δ	○	–
•	•	○	Δ	○	Δ	–
•	○	•	Δ	Δ	○	–
•	○	•	Δ	○	Δ	–
•	Δ	○	•	Δ	○	GN
•	Δ	○	•	○	Δ	GN

removed by carrier phase recovery

Discrete-time channel model $\hat{a}_i = a_i + n_i = a_i - j\sum_{k,m,n} a_k^* a_m a_n \mathcal{X}_{kmni}$

GN Model: NLI power $= P^3 \sum_{k,m,n} \mathcal{X}_{kmni}(\mathcal{X}_{kmni}^* + \mathcal{X}_{knmi}^*)$

A. Mecozzi and R. Essiambre, "Nonlinear Shannon limit in pseudolinear coherent systems," JLT 2012
P. Poggiolini, "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems," JLT 2012

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Second-order noise

e.g., $\mathbb{E}[a_k^* a_m]\,\mathbb{E}[a_r^* a_n]\mathbb{E}[a_p^* a_h]$ →

if Gaussian

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}_2}\prod_{(i,j)\in\pi}\mathbb{E}[a_i^* a_j]$$

k*	m	n	h	p*	r*	
•	•	○	Δ	Δ	○	–
•	•	○	Δ	○	Δ	–
•	○	•	Δ	Δ	○	–
•	○	•	Δ	○	Δ	–
•	Δ	○	•	Δ	○	GN
•	Δ	○	•	○	Δ	GN

removed by carrier phase recovery

Discrete-time channel model $\hat{a}_i = a_i + n_i = a_i - j\sum_{k,m,n} a_k^* a_m a_n \mathcal{X}_{kmni}$

GN Model: NLI power = $\iiint_{-\infty}^{\infty} |\eta(\omega,\omega_1,\omega_2)|^2 |P(\omega)|^2 |P(\omega+\omega_1+\omega_2)|^2 |P(\omega+\omega_1)|^2 |P(\omega+\omega_2)|^2 \frac{d\omega}{2\pi}\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}$

A. Mecozzi and R. Essiambre, "Nonlinear Shannon limit in pseudolinear coherent systems," JLT 2012
P. Poggiolini, "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems," JLT 2012

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Cumulant

if Gaussian

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}_2}\prod_{(i,j)\in\pi}\mathbb{E}[a_i^* a_j]$$

For any random variable

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}}\prod_{B\in\pi}\kappa[a_i; i\in B]$$

cumulant

one-to-one relation

Moments	Cumulants
$\mathbb{E}[x]$	$\mathbb{E}[x]$
$\mathbb{E}[x^2]$	σ^2
$\mathbb{E}[x^3]$	skewness
$\mathbb{E}[x^4]$	kurtosis
⋮	⋮

- Wikipedia page on cumulant
- C. L. Nikias and J. Mendel, "Signal Processing with Higher-Order Spectra," IEEE sig. Proc. Magazine, 1993

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Cumulant

if Gaussian

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}_2}\prod_{(i,j)\in\pi}\mathbb{E}[a_i^* a_j]$$

For any random variable

$$\mathbb{E}\left[\sum_{\substack{k,m,n\\h,p,r}} a_k^* a_m a_n a_h a_p^* a_r^*\right] = \sum_{\pi\in\mathcal{P}}\prod_{B\in\pi}\kappa[a_i; i\in B]$$

cumulant

k*	m	n	h	p*	r*	
●	●	○	Δ	Δ	○	–
●	●	○	Δ	○	Δ	–
●	○	●	Δ	Δ	○	–
●	○	●	Δ	○	Δ	–
●	Δ	○	●	Δ	○	GN
●	Δ	○	●	○	Δ	GN
●	●	○	●	●	○	FON
●	●	○	●	○	●	FON
●	○	●	●	●	○	FON
●	○	●	●	○	●	FON
●	●	●	○	●	○	–
●	●	●	○	○	●	–
○	●	○	●	●	●	–
○	○	●	●	●	●	–
○	●	●	○	●	●	FON
●	●	●	●	●	●	HON

- Wikipedia page on cumulant
- C. L. Nikias and J. Mendel, "Signal Processing with Higher-Order Spectra," IEEE sig. Proc. Magazine, 1993
- P. Serena and A. Bononi, "A time-domain extended Gaussian noise model," JLT 2015
- P. Serena et al., "The Enhanced Gaussian Noise Model Extended to Polarization-Dependent Loss," JLT 2020

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All-order NLI powers

- Second-order noise (GN)
- Fourth-order noise (FON)
- Higher-order noise (HON)

$$\sigma_{\mathrm{GN}}^2 = P^3 \sum_{k,m,n} \mathcal{X}_{kmni}(\mathcal{X}_{kmni}^* + \mathcal{X}_{knmi}^*)$$

$$\sigma_{\mathrm{FON}}^2 = \kappa_2 P \sum_{k,n} (|\mathcal{X}_{kkni} + \mathcal{X}_{knki}|^2 + |\mathcal{X}_{nkki}|^2)$$

$$\sigma_{\mathrm{HON}}^2 = \kappa_3 \sum_{n} |\mathcal{X}_{nnni}|^2$$

A. Mecozzi and R. Essiambre, "Nonlinear Shannon limit in pseudolinear coherent systems," JLT 2012
A. Carena, G. Bosco, V. Curri, Y. Jiang, P. Poggiolini, and F. Forghieri, "The EGN model of non-linear fiber propagation," OE 2014
P. Serena and A. Bononi, "A time-domain extended Gaussian noise model," JLT 2015
R. Dar, M. Feder, A. Mecozzi, andM. Shtaif, "Accumulation of nonlinear interference noise in multi-span fiber-optic systems," OE 2014
P. Poggiolini, G. Bosco, A. Carena, V. Curri, Y. Jiang,F. Forghieri, "A Simple and Effective Closed-Form GN Model Correction Formula Accounting for Signal Non-Gaussian Distribution," JLT 2015
A. Ghazisaeidi, "A Theory of Nonlinear Interactions between Signal and Amplified Spontaneous Emission Noise in Coherent Wavelength Division Multiplexed Systems." JLT 2017

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Include frequency channels

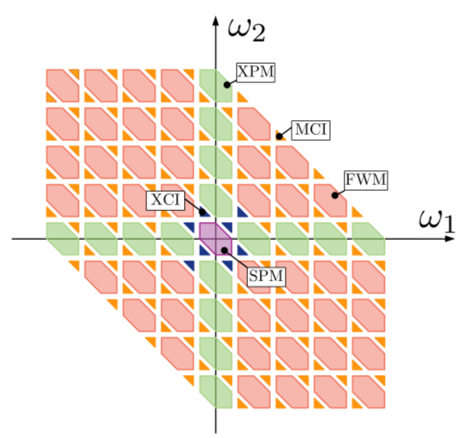
- Make the substitution for the supporting pulse:

$$p(z, t-nT) \rightarrow p(z, t-nT)e^{j\omega_k t}$$

Example: XPM

walk-off

$$\mathcal{X}_{kmni} = \int_0^z f(z) \int_{-\infty}^{\infty} p^*(\xi, t-kT-\boldsymbol{\tau}(\boldsymbol{z}))p(\xi, t-mT-\boldsymbol{\tau}(\boldsymbol{z})) p^*(\xi, t-iT)p(\xi, t-nT)\mathrm{d}t\,\mathrm{d}\xi$$



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Include spatial channels

time → $\mathbf{n} = [n_1, n_2 . n_3]$ ← space; ↑ frequency

- Make the substitution for the supporting pulse:

$$p(z, t - nT) \rightarrow p(z, t - n_1 T) e^{j\omega_{n_2} t} |n_3\rangle \triangleq |p_{\mathbf{n}}(z,t)\rangle$$

- Matched filter based detection

$$\sum_{\mathbf{n}} a_{\mathbf{n}} p_{\mathbf{n}}(z,t) \longrightarrow \boxed{\text{Matched filter}} \longrightarrow \sum_{\mathbf{n}} a_{\mathbf{n}} \int_{-\infty}^{\infty} \langle \tilde{p}_{\mathbf{i}}(z,\omega) | \tilde{p}_{\mathbf{n}}(z,\omega) \rangle \frac{d\omega}{2\pi} = a_{\mathbf{i}}$$

$$\begin{cases} n_i = \sum_{k,m,n} a_k^* a_m a_n \mathcal{X}_{kmni} \\ \mathcal{X}_{kmni} = \int_0^z f(z) \int_{-\infty}^{\infty} p^*(\xi, t - iT) p^*(\xi, t - kT) p(\xi, t - mT) p(\xi, t - nT)\, dt\, d\xi \\ \sigma_{GN}^2 = P^3 \sum_{k,m,n} \mathcal{X}_{kmni} (\mathcal{X}_{kmni}^* + \mathcal{X}_{knmi}^*) \end{cases}$$

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Include spatial channels

time → $\mathbf{n} = [n_1, n_2 . n_3]$ ← space; ↑ frequency

- Make the substitution for the supporting pulse:

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$$\begin{cases} n_{\mathbf{i}} = \sum_{\mathbf{k,m,n}} a_{\mathbf{k}}^* a_{\mathbf{m}} a_{\mathbf{n}} \mathcal{X}_{\mathbf{kmni}} \\ \qquad p^* \rightarrow \langle p| \qquad p \rightarrow |p\rangle \\ \mathcal{X}_{kmni} = \int_0^z f(z) \int_{-\infty}^{\infty} p^*(\xi, t - iT) p^*(\xi, t - kT) p(\xi, t - mT) p(\xi, t - nT)\, dt\, d\xi \\ \sigma_{GN}^2 = P^3 \sum_{k,m,n} \mathcal{X}_{kmni} (\mathcal{X}_{kmni}^* + \mathcal{X}_{knmi}^*) \end{cases}$$

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Include spatial channels

- Make the substitution for the supporting pulse:

$$p(z,t-nT) \rightarrow p(z,t-n_1T)e^{j\omega_{n_2}t}|n_3\rangle \triangleq |p_{\mathbf{n}}(z,t)\rangle$$

time → $\mathbf{n} = [n_1, n_2. n_3]$ ← *space*; ↑ *frequency*

- Matched filter based detection

$$\sum_{\mathbf{n}} a_{\mathbf{n}} p_{\mathbf{n}}(z,t) \longrightarrow \boxed{\text{Matched filter}} \longrightarrow \sum_{\mathbf{n}} a_{\mathbf{n}} \int_{-\infty}^{\infty} \langle \tilde{p}_{\mathbf{i}}(z,\omega)|\tilde{p}_{\mathbf{n}}(z,\omega)\rangle \frac{d\omega}{2\pi} = a_{\mathbf{i}}$$

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Include spatial channels

- Make the substitution for the supporting pulse:

$$p(z,t-nT) \rightarrow p(z,t-n_1T)e^{j\omega_{n_2}t}|n_3\rangle \triangleq |p_{\mathbf{n}}(z,t)\rangle$$

time → $\mathbf{n} = [n_1, n_2. n_3]$ ← *space*; ↑ *frequency*

- Matched filter based detection

$$\sum_{\mathbf{n}} a_{\mathbf{n}} p_{\mathbf{n}}(z,t) \longrightarrow \boxed{\text{Matched filter}} \longrightarrow \sum_{\mathbf{n}} a_{\mathbf{n}} \int_{-\infty}^{\infty} \langle \tilde{p}_{\mathbf{i}}(z,\omega)|\tilde{p}_{\mathbf{n}}(z,\omega)\rangle \frac{d\omega}{2\pi} = a_{\mathbf{i}}$$

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Include spatial channels

- Make the substitution for the supporting pulse:

$$p(z,t-nT) \rightarrow p(z,t-n_1T)e^{j\omega_{n_2}t}|n_3\rangle \triangleq |p_{\mathbf{n}}(z,t)\rangle$$

$$\mathbf{n} = [n_1, n_2.n_3]$$

time (→ n_1), space (→ n_3), frequency (→ n_2)

- Matched filter based detection

$$\sum_{\mathbf{n}} a_{\mathbf{n}} p_{\mathbf{n}}(z,t) \longrightarrow \boxed{\text{Matched filter}} \longrightarrow \sum_{\mathbf{n}} a_{\mathbf{n}} \int_{-\infty}^{\infty} \langle \tilde{p}_{\mathbf{i}}(z,\omega)|\tilde{p}_{\mathbf{n}}(z,\omega)\rangle \frac{d\omega}{2\pi} = a_{\mathbf{i}}$$

$$\begin{cases} n_{\mathbf{i}} = \sum_{\mathbf{k,m,n}} a_{\mathbf{k}}^* a_{\mathbf{m}} a_{\mathbf{n}} \mathcal{X}_{\mathbf{kmni}} \\ \mathcal{X}_{\mathbf{kmni}} = \int_0^z f(z) \int_{-\infty}^{\infty} \langle p_{\mathbf{k}}(\xi,t)|p_{\mathbf{m}}(\xi,t)\rangle\langle p_{\mathbf{i}}(\xi,t)|p_{\mathbf{n}}(\xi,t)\rangle \, dt \, d\xi \\ \sigma_{\mathrm{GN}}^2 = P^3 \sum_{\mathbf{k,m,n}} \mathcal{X}_{\mathbf{kmni}} (\mathcal{X}_{\mathbf{kmni}}^* + \mathcal{X}_{\mathbf{knmi}}^*) \end{cases}$$

No more degenerate

G. Rademacher et al. "Nonlinear Gaussian Noise Model for Multimode Fibers with Space-Division Multiplexing," JLT2016
P. Serena et al., "The Enhanced Gaussian Noise Model Extended to Polarization-Dependent Loss," JLT 2020

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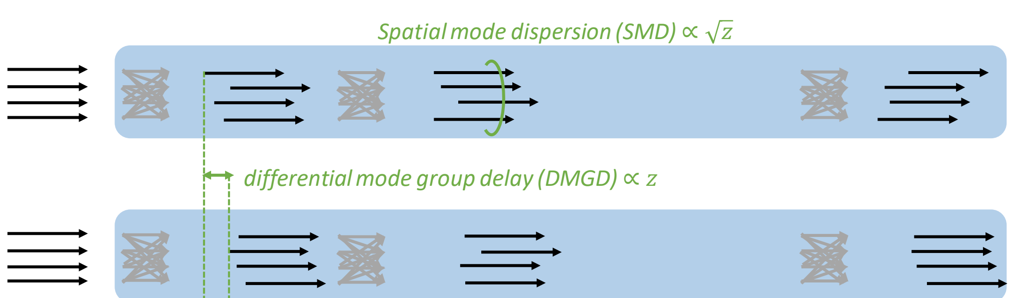
Random mode dispersion

- The main novelty brought by the spatial dimension is its randomness

$$|\tilde{p}(z,\omega)\rangle = \boldsymbol{U}(z,\omega)e^{-j\beta(\omega)z}|\tilde{p}(0,\omega)\rangle$$

($\boldsymbol{U}(z,\omega)$: random; $e^{-j\beta(\omega)z}$: deterministic)



SMD coefficient >> PMD coefficient of SMF

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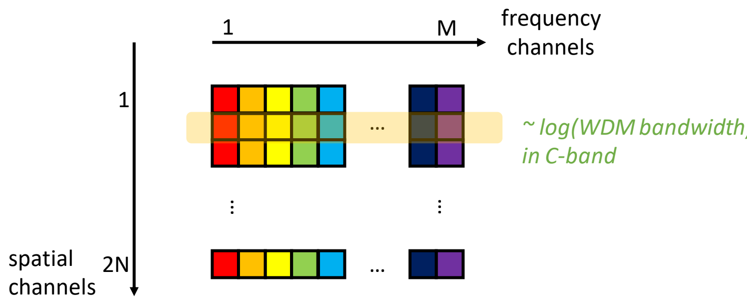
XPM scaling



- What is the scaling with the number of modes in the presence of mode dispersion?

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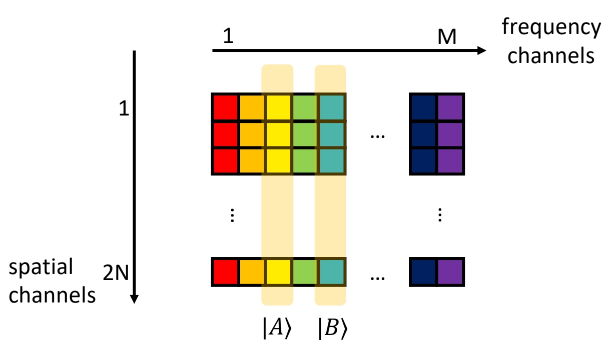
XPM scaling

Assumption:
- One group of strongly coupled modes for a total of 2N polarizations (spatial channels)



- What is the scaling with the number of modes in the presence of mode dispersion?

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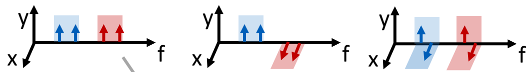
Strong-mode coupling: limiting cases



	Scalar	SMF (e.g., x-pol)	SDM (made of 2N pol)
XPM	$(B^*B + BB^*)A$ $= 2\|B\|^2 A$	$(2\|B_x\|^2 + \|B_y\|^2)A_x + B_x B_y^* A_y$	$(\langle B\|B\rangle \boldsymbol{I} + \|B\rangle\langle B\|)\|A\rangle$
XPM power	σ^2	$\sigma^2 + \frac{\sigma^2}{4} + \frac{\sigma^2}{4}$	$\sigma^2 + \frac{2N-1}{4}\sigma^2 + \frac{2N-1}{4}\sigma^2$ $= \frac{(2N+1)}{2}\sigma^2$
XPM power with high mode dispersion		$(2\|B_x\|^2 + \|B_y\|^2)A_x +$ $B_x B_y^* A_y$	

C. Antonelli, O. Golani, M. Shtaif, and A. Mecozzi, "Nonlinear interference noise in space-division multiplexed transmission through optical fibers," OE 2017

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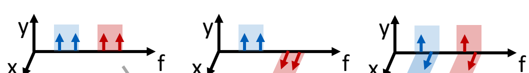
Strong-mode coupling: limiting cases



	Scalar	SMF (e.g., x-pol)	SDM (made of 2N pol)
XPM	$(B^*B + BB^*)A$ $= 2\|B\|^2 A$	$(2\|B_x\|^2 + \|B_y\|^2)A_x + B_x B_y^* A_y$	$(\langle B\|B\rangle \boldsymbol{I} + \|B\rangle\langle B\|)\|A\rangle$
XPM power	σ^2	$\sigma^2 + \frac{\sigma^2}{4} + \frac{\sigma^2}{4}$	$\sigma^2 + \frac{2N-1}{4}\sigma^2 + \frac{2N-1}{4}\sigma^2$ $= \frac{(2N+1)}{2}\sigma^2$
XPM power with high mode dispersion		$(\frac{3}{2}\|B_x\|^2 + \frac{3}{2}\|B_y\|^2)\ A_x +$ $B_x B_y^* A_y$ $\frac{9}{4}\sigma^2 + \frac{9}{4}\sigma^2 + 0$	$\frac{N}{2}\left(1 + \frac{1}{2N}\right)^2 \sigma^2$

C. Antonelli, O. Golani, M. Shtaif, and A. Mecozzi, "Nonlinear interference noise in space-division multiplexed transmission through optical fibers," OE 2017

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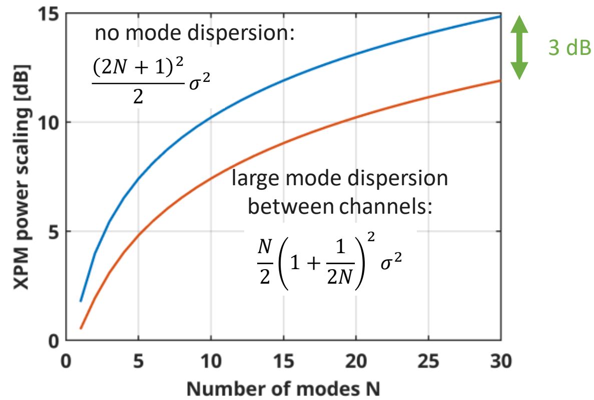
Asymptotics



- *What about with a finite mode dispersion?*

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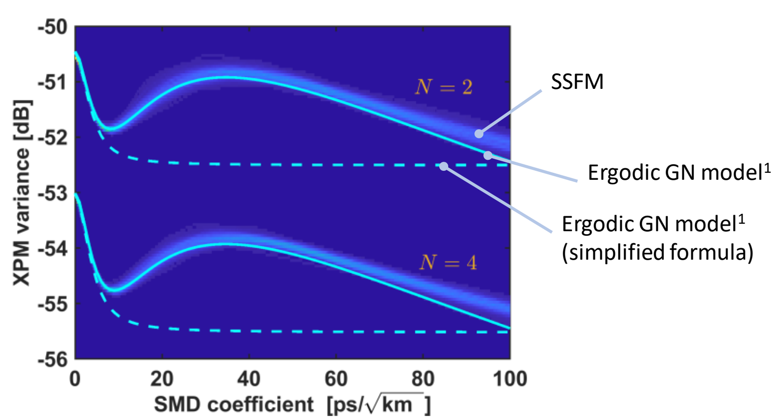
Arbitrary mode dispersion

- 100km
- 17 ps/nm/km
- Gaussian distributed symbols
- 49 Gbd, 100 GHz
- N: # number of modes
- strongly-coupled regime



1. P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, “The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling,” JLT 2022.

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GN Model with arbitrary mode dispersion

$$\eta_{\mathbf{kmni}} = \int_0^L e^{-\alpha z} e^{-j\beta(\omega+\omega_1)z} e^{-j\beta(\omega+\omega_2)z} e^{+j\beta(\omega)z} e^{+j\beta(\omega+\omega_1+\omega_2)z}\, \mathrm{d}z = \frac{1-e^{-\alpha L - j\Delta\beta L}}{\alpha + j\Delta\beta}$$

$$\boldsymbol{U}(z,\omega+\omega_1)\, \boldsymbol{U}(z,\omega+\omega_2) \boldsymbol{U}^\dagger(z,\omega)\, \boldsymbol{U}^\dagger(z,\omega+\omega_1+\omega_2)$$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, "The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling," JLT 2022.

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GN Model with arbitrary mode dispersion

$$\eta_{\mathbf{kmni}} = \int_0^L e^{-\alpha z}\, \boldsymbol{U}(z,\omega+\omega_1)\, \boldsymbol{U}(z,\omega+\omega_2) \boldsymbol{U}^\dagger(z,\omega)\, \boldsymbol{U}^\dagger(z,\omega+\omega_1+\omega_2)\; \mathrm{d}z = \quad ?$$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, "The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling," JLT 2022.

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GN Model with arbitrary mode dispersion

??

$$\eta_{\mathbf{kmni}} = \int_0^L \mathbb{E}\left[\boldsymbol{U}(z,\omega+\omega_1)\, \boldsymbol{U}(z,\omega+\omega_2)\boldsymbol{U}^\dagger(z,\omega)\, \boldsymbol{U}^\dagger(z,\omega+\omega_1+\omega_2) \right] dz = \ ?$$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, “The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling,” JLT 2022.

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GN Model with arbitrary mode dispersion

$$\eta_{\mathbf{kmni}} = \int_0^L \mathbb{E}\left[\boldsymbol{U}(z,\omega+\omega_1)\, \boldsymbol{U}(z,\omega+\omega_2)\boldsymbol{U}^\dagger(z,\omega)\, \boldsymbol{U}^\dagger(z,\omega+\omega_1+\omega_2) \right] dz = \ ?$$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, “The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling,” JLT 2022.

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GN Model with arbitrary mode dispersion

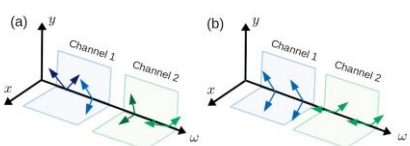
$$\eta_{\mathbf{kmni}} = \int_0^L \mathbb{E}\left[\boldsymbol{U}(z,\omega+\omega_1)\,\boldsymbol{U}(z,\omega+\omega_2)\boldsymbol{U}^{\dagger}(z,\omega)\,\boldsymbol{U}^{\dagger}(z,\omega+\omega_1+\omega_2) \right] dz = \ ?$$

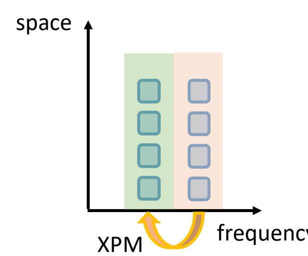
$$\sigma_{\mathrm{GN}}^2 = P^3 \sum_{\mathbf{k,m,n}} \mathcal{X}_{\mathbf{kmni}}(\mathcal{X}_{\mathbf{kmni}}^* + \mathcal{X}_{\mathbf{knmi}}^*)$$

$$\mathbb{E}\left[\sigma_{\mathrm{XPM}}^2\right]$$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, "The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling," JLT 2022.

Ergodic GN Model: simplified formula





Channel spacing

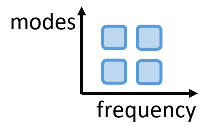
$$\sigma_{\mathrm{XPM}}^2 = \frac{2N+1}{2N}\left((2N+1)\sigma^2(\alpha) + \frac{(2N-1)\left(\alpha + \frac{\Delta\omega^2\mu^2}{N}\right)}{\alpha}\sigma^2\left(\alpha + \frac{\Delta\omega^2\mu^2}{N}\right) \right)$$

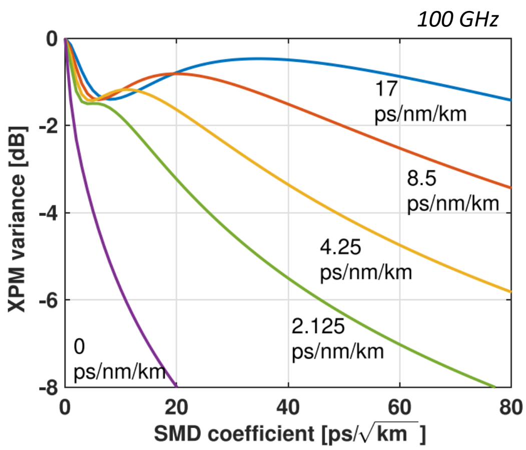
 # modes *one-dimensional case* *Spatial mode dispersion coefficient (SMD)*

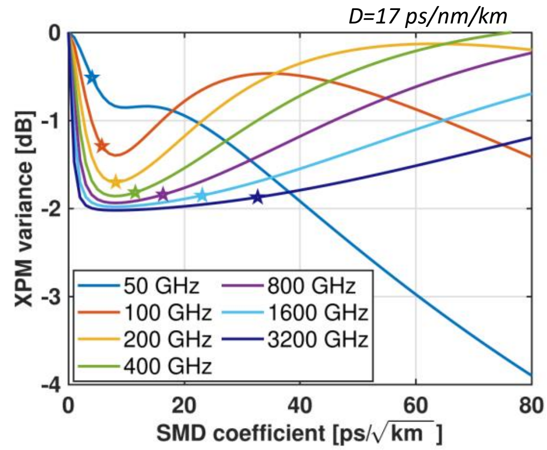
- Reliable up to SMD ~5 ps/$\sqrt{\text{km}}$

P. Serena, C. Lasagni, A. Bononi, C. Antonelli, and A. Mecozzi, "The Ergodic GN Model for Space Division Multiplexing With Strong Mode Coupling," JLT 2022.

Scalings







- Local XPM minimum at SMD = 8 ps/√km !

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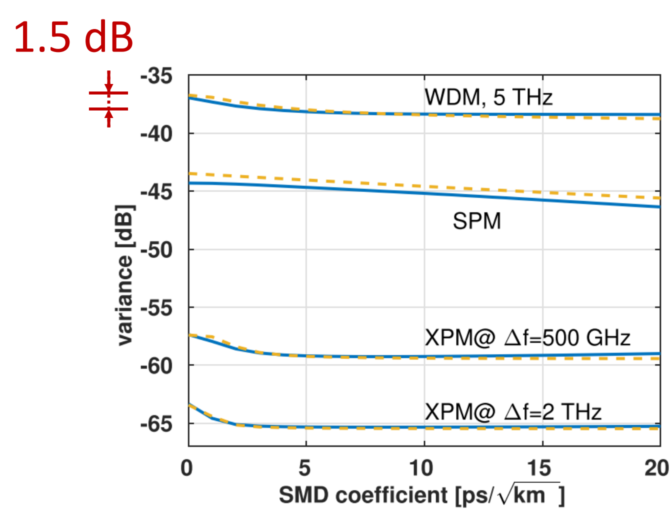
WDM case

- Solid: numerical ergodic GN model
- Dashed: closed-form formulas



SPM curve: C. Antonelli, O. Golani, M. Shtaif, and A. Mecozzi, "Propagation effects in few-mode fibers," ECOC17, W1B1

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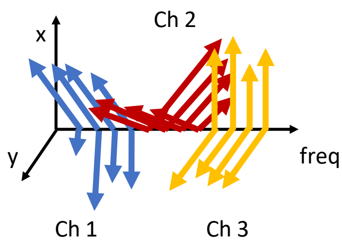
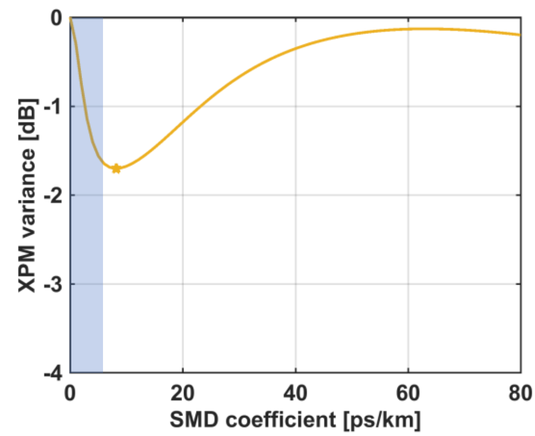
Scaling properties

- Inter-channel SMD dominates intra-channel SMD: XPM mitigation





Walk-off length = mode dispersion correlation length

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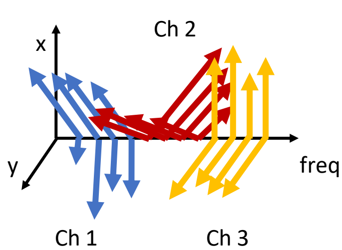
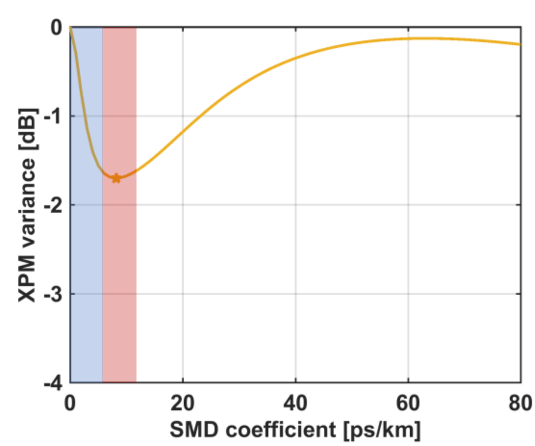
Scaling properties

- Inter-channel SMD dominates intra-channel SMD: XPM mitigation
- Plateau regime, complete channel decorrelation: max inter-channel SMD effect





Walk-off length = mode dispersion correlation length

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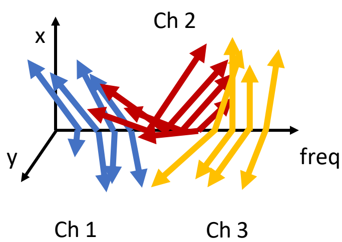
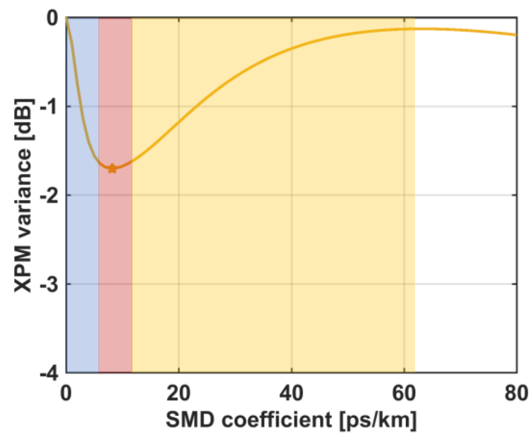
Scaling properties

- Inter-channel SMD dominates intra-channel SMD: XPM mitigation
- Plateau regime, complete channel decorrelation: max inter-channel SMD effect
- I
  ntra-channel SMD partially compensated walk-off: XPM inflation





Walk-off length = mode dispersion correlation length

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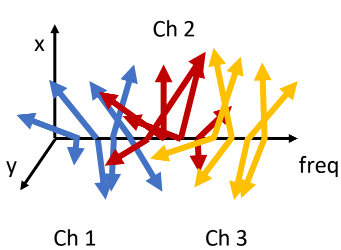
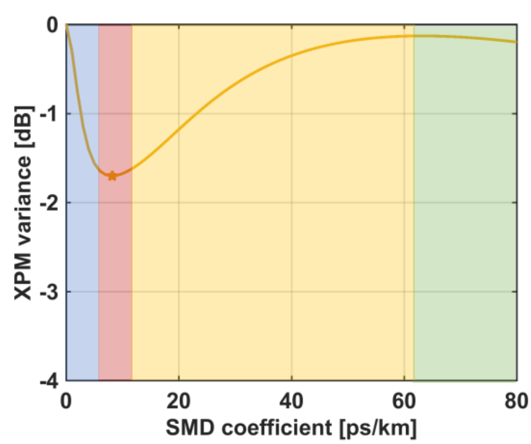
Scaling properties

- Inter-channel SMD dominates intra-channel SMD: XPM mitigation
- Plateau regime, complete channel decorrelation: max inter-channel SMD effect
- I
- Strong dispersive regime: full XPM mitigation
- f: XPM inflation





Walk-off length = mode dispersion correlation length

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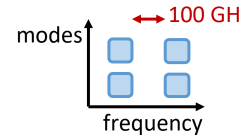
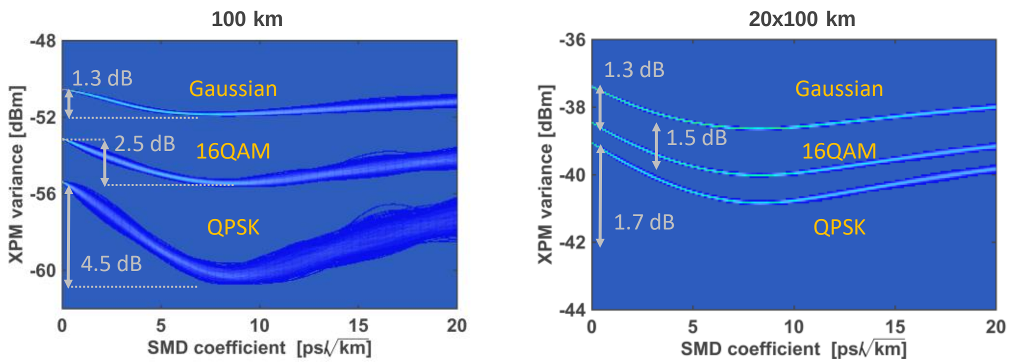
Dependence on the modulation format





- Mode dispersion is more effective with non-Gaussian formats

C. Lasagni, P. Serena, A. Bononi, A. Mecozzi, and C. Antonelli, "Dependence of nonlinear interference on mode dispersion and modulation format in strongly-coupled SDM transmissions," submitted to OE 2023

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Fourth-order noise

- Fourth-order noise is the dominant modulation-format dependent contribution

$$\frac{1}{\mathrm{SNR}} = \frac{1}{\mathrm{SNR_{ASE}}} + \frac{1}{\mathrm{SNR_{GN}}} - \frac{1}{\mathrm{SNR_{FON}}} + \frac{1}{\mathrm{SNR_{HON}}}$$

$$\sigma^2_{\mathrm{XPM,FON}} \simeq \frac{(2N+1)^2}{2N}\sigma^2_{\mathrm{XPM,FON,1}}(\alpha) + \frac{(2N-1)\left(\alpha + \frac{\Delta\omega^2\mu^2}{N}\right)}{2N\alpha}\sigma^2_{\mathrm{XPM,FON,1}}\left(\alpha + \frac{\Delta\omega^2\mu^2}{N}\right)$$

one-dimensional case

C. Lasagni, P. Serena, A. Bononi, A. Mecozzi, and C. Antonelli, "Dependence of nonlinear interference on mode dispersion and modulation format in strongly-coupled SDM transmissions," submitted to OE 2023

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Conclusions

- SDM is more complex than SMF, but perturbative analysis helps a lot
- Do not forget the interplay mode dispersion vs Kerr effect!
- Are you searching for a fast model? Try the ergodic GN model
- A local minimum of XPM appears after a mode dispersion of 8 ps/$\sqrt{\mathrm{km}}$: a best value for fiber design in strong coupling regime?
- Are you searching for extensions? Focus on MDL

Thank you

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