# Shaped Four-Dimensional Modulation Formats for Optical Fiber Communication Systems

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**Abstract:** We review the design of multidimensional modulations by maximizing generalized mutual information and compare the maximum transmission reach of recently introduced 4D formats. A model-based optimization for nonlinear-tolerant 4D modulations is also discussed. © 2021 The Author(s)

#### 1. Introduction

The design of multidimensional (MD) modulation formats has been considered as an effective approach to harvest performance gain in optical communications. For an additive white Gaussian noise (AWGN) channel, higher achievable information rates are to be expected from MD shaping when increasing the constellation dimensionality [1]. On the other hand, nonlinear effects in the optical channel could be mitigated by MD geometrical shaping [2]. This insight motivates the search for a linear noise and /or nonlinear interference (NLI)-tolerant modulation formats in a higher dimensional space.

Conventional MD formats are not *true* MD formats in the sense that they are only optimized in each dimension independently. This is the case of polarization-multiplexed 2D (PM-2D) formats. MD modulation formats with dependency between dimensions can be obtained by set-partitioning the regular QAM (SP-QAM) or via MD geometrical shaping. MD coded modulation encodes binary bits and then maps them onto consecutive 2D symbols or onto one MD symbol. For example in 4D space, polarization switched-QPSK [3] maps 3 bits onto two consecutive QPSK symbols while 4D2A8PSK [4] maps 5-7 bits onto two 8PSK symbols in both X and Y polarizations. More recently, modulation formats in 4D, 8D and 12D have been proposed by adding constraints in the optimization to enable larger gains in nonlinearity tolerance and to further extend the transmission reach [5–7].

In this paper, we focus on designing 4D modulation formats for soft-decision forward error correction (SD-FEC) with 20%-25% overhead by maximizing the generalized mutual information (GMI) and ,thus, increase transmission reach. Simulation comparisons for a set of 4D-optimized modulation formats, which outperform previously known 4D formats, are presented. Finally, to highlight future directions for the design of nonlinear-tolerant modulation in optical fiber systems, an optimization of dual-polarization (DP) modulation based on 4D NLI model [8] is performed.

### 2. GMI Computation and Design Methodology for Multidimensional Modulation

Due to its simplicity and flexibility, bit-interleaved coded modulation with SD-FEC is usually considered an attractive option for optical fiber communication systems [10], and hence, the use of information-theoretical performance metric GMI is preferred for coded modulation design [11].

For a discrete uniformly-distributed *N*-dimensional modulation with spectral efficiency (SE)  $m = \log_2 M$  bit/4D, the GMI under Gaussian noise assumption can be estimated via Gauss-Hermite quadrature as [11, Eq. (45)],



Fig. 1: Example of  $d_{ij}$  calculation and the constellation set  $I_k^b$  for 4D-OS128 [9] in first orthant of 4D space (2×2D).

$$GMI \approx m - \frac{1}{M\pi^{N/2}} \sum_{k=1}^{m} \sum_{b \in \{0,1\}} \sum_{i \in I_k^b} \sum_{l_1=1}^{J} \alpha_{l_1} \sum_{l_2=1}^{J} \alpha_{l_2} \cdots \sum_{l_N=1}^{J} \alpha_{l_N} \cdot \log_2 \frac{\sum_{p=1}^{M} \exp\left(-\frac{\|d_{ip}\|^2 + 2\sigma_z \Re(\xi_{l_1} + J\xi_{l_2})d_{ip}\right)}{\sigma_z^2}\right)}{\sum_{j \in I_k^b} \exp\left(-\frac{\|d_{ij}\|^2 + 2\sigma_z \Re(\xi_{l_1} + J\xi_{l_2})d_{ij}\right)}{\sigma_z^2}\right)},$$
(1)

where the quadrature nodes  $\xi_l$  and the weights  $\alpha_l$  can be easily found (numerically) for different values of J. In this paper, we use the quadrature nodes and weights for J = 10 in [11, Table III].  $d_{ij} \triangleq X_i - X_j$  denote the difference between two MD symbols,  $X_i = [x_i^{1,2}, x_i^{3,4}, \dots, x_i^{N-1,N}]$  denotes a MD symbol consisting of N/2 complex symbols,

 $\sigma_z^2$  is the noise variance per complex dimension and  $I_k^b \subset \{1, 2, ..., M\}$  with  $|I_k^b| = M/2$  is the set of indices of constellation points whose binary label is *b* at bit position *k*. Fig. 1 shows an example of computing  $d_{ij}$  of two 4D symbols as  $d(\bullet, \bullet)$  for 4D format 4D-OS128. In order to clearly show the dependency of 4D symbols, we use a similar color coding as in [9]: valid 4D symbols are the 2D projected symbols in the first/second 2D with the same color.

As shown in Eq. (1), GMI computation requires a joint consideration of the 4D coordinates and its binary labeling. A GMI-based optimization can find a constellation  $\mathcal{X}^*$  and a labeling  $\mathcal{L}^*$  for a given channel conditional PDF  $p_{Y|X}$  with a constraint on transmitted power  $\sigma_x^2$ , i.e.,  $\{\mathcal{X}^*, \mathcal{L}^*\} = \operatorname{argmax}_{\mathcal{X}, \mathcal{L}: E[||\mathcal{X}||^2] \le \sigma_x^2} G(\mathcal{X}, \mathcal{L}, p_{Y|X})$ , where *G* as an expression of GMI emphasizes the dependency of the GMI on the constellation, binary labeling, and channel law.

It is known that GMI-based optimization of large constellations and/or constellations with high dimensionality is computationally demanding. Therefore, an unconstrained optimization with at least hundreds of GMI evaluations is very challenging. Potentially irregular formats obtained from the optimization also impose strict requirements on the generation and detection of the signals, due to the need of high-resolution digital-to-analog/analog-to-digital converters. To solve the multi-parameter optimization challenges of MD geometric shaping and also to achieve a good performance-complexity tradeoff, constraints of constant modulus [4, 5] and orthant-symmetry (OS) [9] have been proposed to design *N*-dimensional formats. These solutions have shown a small performance loss with respect to the unconstrained optimizations in AWGN channel and achieve an even better performance in the optical fiber channel.

## 3. Simulation Results of 4D Geometric Shaping for Multi-Span Systems

To target a practical SD-FEC with 20%-25% overhead, the optimizations were performed for AWGN channel at an SNR in which GMI  $\approx 0.85m$  for six different SEs with  $m \in \{5, 6, 7, 8, 9, 10\}$ . In order to make the modulation more structured and reduce the optimization complexity, constraint of OS is used for m = 7, 8, 9, 10. Split-step Fourier method of the nonlinear Manakov equation with a step size of 100 m was performed to compare the modulation formats and predict system performance. The simulation parameters are given in Table 1 for the optical fibre link under consideration.

TX Parameters	
Symbol rate	45 Gbaud
No. of WDM channels	11
Channel spacing	50 GHz
Root-raised-cosine roll-off	10%
Fiber and Link Parameters	
Attenuation coeff. ( $\alpha$ )	0.21 dB/km
Disp. parameter $(D)$	16.9 ps/nm/km
Nonlinear coeff. $(\gamma)$	1.31 dB/km
Span length	80 km
EDFA noise figure	5 dB

Table 1: Simulation parameters.

In Fig. 2, the maximum transmission distance and the relative reach increase

in percentage at GMI = 0.85m of twelve modulation formats are evaluated. We observe that 4D-optimized formats achieve approximately 320-2160 km (9%-25%) reach increase w.r.t PM-QAM/4D-SP-QAM at the same information rates, which are highlighted by the orange shaded region. We note from Fig. 2 that larger reach increase in percentage can be achieved w.r.t the QAM modulations without gray labeling. Especially comparing to 4D-SP32 and 4D-SP512, the gains of 4D-optimized formats are more than 20%, which is mainly due to the superior performance of labeling.



Fig. 2: The maximum reach of various modulation formats for multi-span optical fiber transmission. The  $2 \times 2D$  projection of the modulations at normalized GMI of 0.95 are depicted as (a) - (f).

## 4. 4D NLI Model-aided 4D Geometric Shaping for Single-Span Transmission

As noted in the previous section, most of the modulation formats in Fig. 2 are designed for AWGN channel, only 4D-64PRS uses heuristic idea of constant-modulus constraint to improve the nonlinearity tolerance. For nonlinear fiber channel, NLI power models with considering modulation-dependent interference could provide a quick computation of the NLI power as a function of the input constellation, e.g., the enhanced Gaussian noise model [12] for PM-2D format and 4D NLI model [8] for a general DP-4D format. Accordingly, to design a nonlinear-tolerant 4D modulation, the optimization problem for a given optical fiber channel parameters  $\mathcal{P}$  can be reformulated as,

$$\{\mathcal{X}^*, \mathcal{L}^*\} = \operatorname*{argmax}_{\mathcal{X}, \mathcal{L}} G\left(\mathcal{X}, \mathcal{L}, \mathrm{SNR}_{\mathrm{opt}}(\mathcal{X}, \mathcal{P})\right), \tag{2}$$

where  $SNR_{opt}(X, \mathcal{P})$  denotes the optimum effective SNR at a given distance and depends on the modulation format.

In Fig. 3, the 4D modulation formats with a SE of 7 bit/4D are optimized with OS constraint via end-to-end learning following [13] by maximizing GMI. The simulations are implemented by solving two optimization problems: one is for AWGN channel with SNR=10 dB (AWGN-learned) and the other is for the 4D-model [8] with a single-channel, 234km single-span transmission system (4D model-learned). PM-QPSK as a format with a good nonlinearity tolerance and 4D-128SP-QAM [14] with 7 bit/4D are shown as references.

Fig. 3 (left) shows that the 4D model-learned modulation can tolerate higher nonlinearity that achieves up to 0.25 dB gain with respect to 4D-SP128-QAM in terms of SNR<sub>opt</sub> at 234 km. Fig. 3 (right) shows that in an AWGN channel with an SNR of 10 dB, AWGN-learned modulation format provide the gain around 0.22 bit/4D (in term of GMI) with respect to 4D-SP128-QAM, while the gain is around 0.18 bit/4D for 4D-learned modulation format. However, the gain of the 4D model-learned modulation in an optical channel with fiber length of 234 km is increased to 0.29 bit/4D, which is higher than that of the AWGN-learned format. This benefits from the improvement of SNR<sub>opt</sub> shown in Fig. 3 (left). It well indicates that 4D model-learned modulation leads to a good trade-off between linear and nonlinear shaping gain by increasing the linear shaping gain and maintaining a fair level of nonlinearity tolerance.



Fig. 3: The SNR<sub>opt</sub> at 234 km (left) and GMI (right) of geometrically shaped 4D modulation format optimized based on AWGN channel and 4D NLI model. In this example, M = 128.

## 5. Conclusions

We numerically assessed a series of multidimensional modulation formats for multi-span transmission systems. We showed that the 4D-optimized modulation formats can be a solution for multi-rate applications between 5 and 10 bit/dual-pol. In addition, up to 0.25 dB NLI gains in terms of SNR<sub>opt</sub> are demonstrated for 4D model-based modulation optimization over a regular 4D format for a single-span transmission system. The results in this work confirm that the multidimensional modulations could be a good alternative for high capacity transmission systems and offer substantial potential gains in the nonlinear optical fiber channel.

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- 1. G. Forney et al., "Efficient modulation for band-limited channels," IEEE Journal on Selected Areas in Communications 2, 632-647 (1984).
- 2. R. Dar et al., "On shaping gain in the nonlinear fiber-optic channel," in IEEE Int. Symp. on Inf. Theory, (2014), pp. 2794–2798.
- 3. M. Karlsson and E. Agrell, "Which is the most power-efficient modulation format in optical links?" Opt. Express 17 (2009).
- 4. K. Kojima et al., "Nonlinearity-tolerant four-dimensional 2A8PSK family for 5-7 bits/symbol spectral efficiency," JLT 35 (2017).
- B. Chen et al., "Polarization-ring-switching for nonlinearity-tolerant geometrically-shaped four-dimensional formats maximizing generalized mutual information," JLT 37 (2019).
- 6. B. Chen et al., "Eight-dimensional polarization-ring-switching modulation formats," PTL 31 (2019).
- 7. R.-J. Essiambre *et al.*, "Increased reach of long-haul transmission using a constant-power 4D format designed using neural networks," in *ECOC*, (2020).
- G. Liga *et al.*, "Extending fibre nonlinear interference power modelling to account for general dual-polarisation 4D modulation formats," Entropy 22 (2020).
- B. Chen *et al.*, "Analysis and experimental demonstration of orthant-symmetric four-dimensional 7 bit/4D-sym modulation for optical fiber communication," JLT 39 (2021).
- 10. B. P. Smith et al., "A pragmatic coded modulation scheme for high-spectral-efficiency fiber-optic communications," JLT 30 (2012).
- 11. A. Alvarado et al., "Achievable information rates for fiber optics: Applications and computations," JLT 36 (2018).
- 12. A. Carena et al., "EGN model of non-linear fiber propagation," Opt. Express 22 (2014).
- 13. K. Gümüs et al., "End-to-end learning of geometrical shaping maximizing generalized mutual information," in OFC, (2020).
- 14. T. A. Eriksson et al., "Comparison of 128-SP-QAM and PM-16QAM in long-haul WDM transmission," Opt. Express 21 (2013).