Nonlinear Pre-Distortion in DML-based OFDM Transmission Enabled by Low-Complexity Sparse Volterra Filtering

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Abstract: A nonlinear pre-distorter is proposed in DML-based OFDM transmission. The complexity of pre-distorter is reduced by $>90\%$ using the ℓ_0-regularization, ℓ_1-regularization, or re-orthogonalization, and the data rate is still increased by $>50\%$ after ≥ 150-km fiber.

1. Introduction

Despite the superior performance of coherent detection, the intensity modulation and direct detection (IMDD) scheme implemented by directly modulated lasers (DMLs) is preferred for short/medium-reach networks, due to its cost-effectiveness, compact size, high output power and low power consumption [1]. Besides, orthogonal frequency division multiplexing (OFDM) provides high spectral efficiency, resilience against inter-symbol interference, and scalable bit rates. Unfortunately, the presence of adiabatic chirp in DMLs combined with dispersion leads to serious nonlinear distortion, which greatly limits the capacity of the DML-based OFDM transmission, thereby requiring nonlinear compensation techniques, such Volterra filtering [2,3]. Depending on being located at the receiver or the transmitter, the techniques to compensate for nonlinear distortion can be classified into post-compensation [2] or pre-distortion [3], respectively. Although it is easier to adaptively equalize signals using a post-compensator, a pre-distorter can share the equalization functionality to reduce the complexity of receivers, which is particularly important in an access system. Besides, in spite of the extraordinary ability of Volterra filtering to restore distorted signals, its practical applicability is limited by the high computational complexity. There are several schemes of sparse identification to overcome this issue, such as the ℓ_1-regularization [4], the ℓ_0-regularization [5], and the re-orthogonalization [6,7]. However, the effectiveness of a pre-distortion technique based on sparse Volterra filtering has not been investigated in a DML-based OFDM dispersive transmission.

This work employs a sparse Volterra filter to realize nonlinear pre-distortion in the DML-based OFDM transmission, and the proposed pre-distorter is specially designed to prevent significant changes in the powers of transmitted subcarriers. To make the pre-distorter practical, the sparsity of filter is realized by applying ℓ_0-regularization, ℓ_1-regularization, or re-orthogonalization. Among the three schemes, the experiment indicates that the re-orthogonalization slightly outperforms the others. At the limited expense of reducing the achievable data rate, it is possible to decrease the complexity of pre-distortion by up to 90%. Specifically, the total number of multiplications (i.e., a measure of computational complexity) can be limited to <100 even after 175-km transmission, thereby showing its practical feasibility. Meanwhile, the sparsity only reduces the data rate by 3.4%, still achieving the improvement of $>50\%$ in the achievable data rate.

2. Concepts of pre-distortion and sparsity

It is known that the pth-order pre-distorter of a nonlinear system is identical to its pth-order post-compensator [8]. Thus, interchanging a system and its pth-order inverse does not affect the ability to remove the nonlinearity of $\leq p$th order. As a result, as shown in Fig. 1, it is possible to utilize the received samples of training signals $r[n]$ after transmission to build a Volterra filter as a post-compensator or pre-distorter. Note that $s[n]$ and $y[n]$ in Fig. 1 stands for the nth samples of ideal and output signals, respectively. To build a post-compensator, the Volterra filter aims to perform $\mathbb{Q}\{r[n]\} \approx s[n]$; however, the possibility of significantly altering the signal power by $\mathbb{Q}\{\cdot\}$ is undesired for pre-distortion. Therefore, we propose to firstly apply a feed-forward equalizer (FFE) to equalize the frequency response of $r[n]$; i.e., $\mathbb{L}\{r[n]\} = x[n]$ in Fig. 1. In this case, the channel-induced frequency-selective fading or gain would be eliminated to some extent before entering the following Volterra filter, making it possible to mitigate nonlinear distortion alone without significantly varying the relative powers of subcarriers. Note that the FFE is required only in the training procedure and unnecessary in pre-distortion.

The function of Volterra filter can be written as [9],

$$y[n] = \mathbb{Q}\{x[n]\} = \sum_{p=1}^{P}\sum_{l_1=0}^{L-1}\sum_{l_2\geq l_1}^{L-1}\cdots\sum_{l_p\geq l_{p-1}}^{L-1} w(l_1,\ldots,l_p)\prod_{i=1}^{p}x[n-l_i] = \mathbf{x}_n^{\mathsf{T}}\mathbf{w} \tag{1}$$

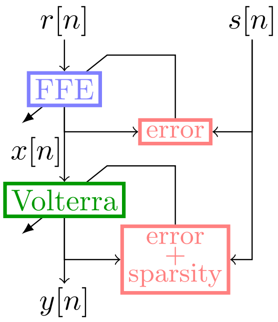

Fig. 1: Training procedure of the proposed pre-distorter

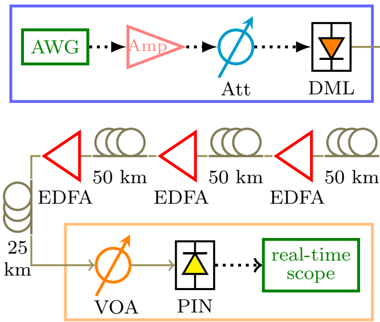

Fig. 2: Experiment setup

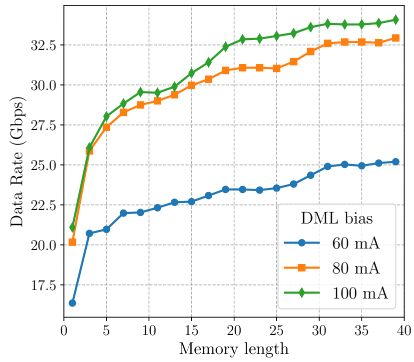

Fig. 3: The data rate after 150 km versus the memory length of filter

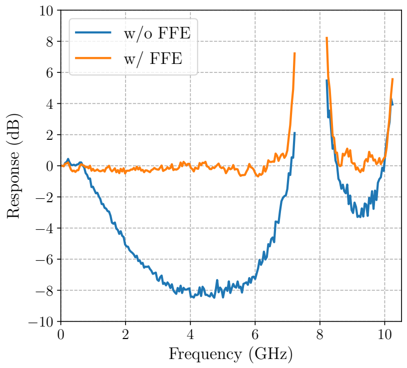

Fig. 4: The responses of the pre-distorters with and without the FFE

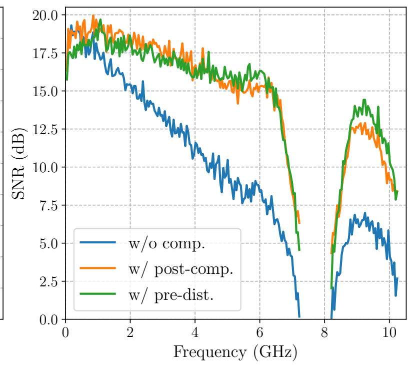

Fig. 5: The SNR with or without nonlinear compensation after 150-km fiber

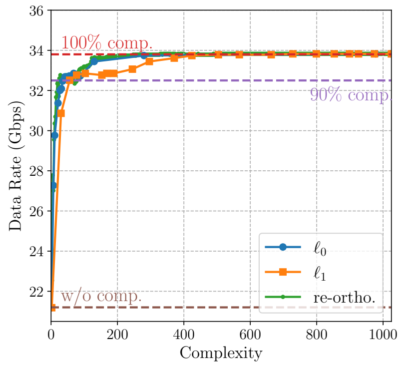

Fig. 6: The achievable data rate versus the complexity of sparse Volterra filters

where $w(l_1,\ldots,l_p)$ denotes the weight of a pth-order tap; P is the order of filter ($P=2$ in this work); L is the memory length, and $\mathbf{x}_n$ and $\mathbf{w}$ represent the input vector and weight vector, respectively, composed of all inputs $\prod_{i=1}^{p} x[n-l_i]$ and weights $w(l_1,\ldots,l_p)$. Note that the total number of multiplications in (1) could characterize the computational complexity, and it is $M(P,L)=\sum_{p=1}^{P} p\binom{L+p-1}{p}$ [9]. To realize the sparsity, the ℓ_0- or ℓ_1-regularization can be applied; that is, the ℓ_0-norm or ℓ_1-norm of $\mathbf{w}$ (i.e., $\|\mathbf{w}\|_0$ or $\|\mathbf{w}\|_1$) is added into the cost function during training, and the more weight of the norm will tend to remove more taps of less importance [4,5]. In contrast, the re-orthogonalization by the Gram-Schmidt decomposition of training matrix $[\mathbf{x}_1,\mathbf{x}_2,\cdots]^{\mathsf{T}}$ could identify the taps of more importance to the equalization [6,7]. Through sorting the taps in terms of their importance, it is possible to realize the sparsity of any degree.

3. Experiment and Discussion

Fig. 2 plots the experiment setup of the DML-based transmission over 0–175-km fiber. The electrical OFDM signal was generated by an arbitrary waveform generator (AWG) with 92-GS/s sampling rate. The FFT size was 2048; 233 subcarriers were used to carry 16 quadrature amplitude modulation (QAM) data, and the signal bandwidth was $\sim$10.5 GHz. The 10-GHz DML was operated at around 1550 nm. After fiber transmission and direct-detection using a 10-GHz PIN receiver with the received optical power of -4 dBm, the detected signal was recorded by a digital oscilloscope with 40-GS/s sampling rate. An off-line DSP program, including the standard one-tap equalization, was then applied for the demodulation of OFDM signals. For comparison, a post-compensator based on Volterra filtering was also realized; in this case, the DSP program included the post-compensation before the one-tap equalization. It should be noted that the sampling rate was fixed at 92 GS/s regardless of applying post-compensation or pre-distortion. After fiber transmission, the memory length L of Volterra filtering was optimized to carry out the maximum achievable data rate in the training procedure, which was estimated by the bit-loading technique at the bit-error rate of 3.8×10^{-3}. Fig. 3 shows the achievable data rate as a function of the memory length after 150-km fiber. The data rate in Fig. 3 is about optimized with $L\geq 31$; thus, L was 31 in this case. For the transmission over 50, 100, and 175 km, the values of L were determined in the similar manner: 27, 29, and 35, respectively. In addition, Fig. 3 also shows that a higher bias current of DML results in a better performance due mainly to the higher bandwidth; thus, the bias current was set to 100 mA in this work.

Fig. 4 shows the frequency responses of the proposed pre-distorters designed for the transmission over 150 km. When the FFE in Fig. 1 is omitted, the Volterra filter would be trained to alter the powers of subcarriers significantly. For instance, the pre-distorter without FFE decreases the powers at $\sim$4 GHz due to the chirp-induced gain after transmission. By contrast, the pre-distorter would keep the response flat, except for the frequencies

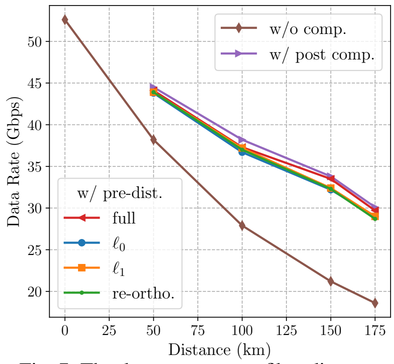

Fig. 7: The data rates versus fiber distance

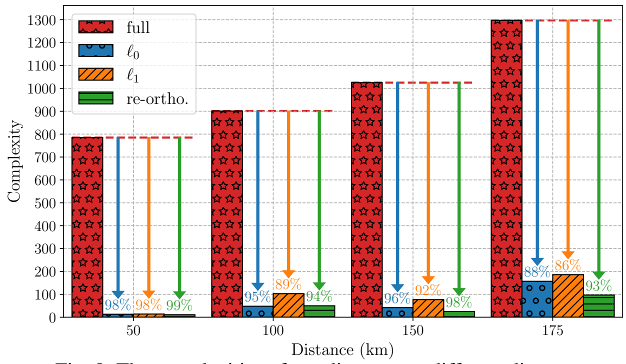

Fig. 8: The complexities of pre-distorters at different distances

around the sever fading dip at ∼8 GHz. In addition, Fig. 5 shows the signal-to-noise ratio (SNR) after 150-km fiber using the post-compensator or pre-distorter. Compared to the case without nonlinear compensation, the two nonlinear equalizer can realize the improvement of $>$ 8.6 dB in SNR. The similar SNRs in Fig. 5 indicates that the pre-distorter can perform similarly in terms of eliminating nonlinear distortion. Based on the bit-loading technique, the achievable data rates would be 33.8 and 33.5 Gbps using the post-compensator and pre-distorter, respectively. In this case, if the sparsity schemes are applied to the post-compensator, as shown in Fig. 6, the achievable data rates will decrease slowly as decreasing the complexity at the beginning. However, it drops quickly with the complexity of $<\sim$ 100; in this situation, the ℓ_0-regularization and re-orthogonalization reveal a better ability to balance complexity and performance. To keep the loss of data rate improvement to 10% (i.e., achieving 90% compensation), Fig. 6 indicates that all three schemes can make it with the complexity of $<$ 100. Using various nonlinear compensation, Fig. 7 compares the achievable data rates over different transmission distances. Compared to the full pre-distortion, the post-compensation could achieve a slightly better performance. This is probably caused by the difference in nonlinear characteristics between training and testing. When applying the sparsity schemes, of which the complexities are set to keep 90% compensation, the measured results in Fig. 7 confirm the ability of sparse pre-distorters: the data rates with spare pre-distorters indeed agree with the expected values. For instance, the achievable data rates after 150 km are 32.2, 32.4, and 32.3 Gbps, respectively, using the ℓ_0-regularization, ℓ_1-regularization, and re-orthogonalization. Fig. 8 summaries the complexities of pre-distorters at various distances. The complexities of full pre-distorters are $M(2,27)$, $M(2,29)$, $M(2,31)$ and $M(2,35)$ after 50, 100, 150, and 175 km, respectively. The ℓ_1-regularization performs the least reduction in complexities among thee three schemes; the re-orthogonalization demonstrates the most reduction in complexities and can keep the complexities below 100 for all distances, revealing the practical feasibility of a sparse pre-distorter in the DML-based transmission system.

4. Conclusions

This work proposes the Volterra-based pre-distorter with sparsity to eliminate the severe nonlinear distortion in DML-based OFDM transmission without the need for nonlinear post-compensation. The proposed pre-distorter is designed to prevent a significant change in the spectrum of pre-distorted signals. The pre-distortion with sparsity can still achieve the improvement of $>$ 50% in the data rate after $\geq$ 150-km dispersion-uncompensated fiber. The work also compares three schemes for the sparsity, and the re-orthogonalization could slightly outperform the regularization schemes in terms of the ability to lower the complexity. Using the re-orthogonalization, the complexities are reduced by $>$ 90% and kept below 100 for the distance in the range of 50–175 km.

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