An Optimized Full-spectrum Modulated NFDM System by Combining Geometric Shaping and Linear Minimum Mean Square Error Estimator

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Abstract: We elaborately design a full-spectrum modulated NFDM system with *b*-scheme. A 1120km transmission with BER $< 3.8 \times 10^{-3}$ at 103.75 Gbps is achieved through geometric shaping (GS) and linear minimum mean square error (LMMSE) estimator.

1. Introduction

With the development of optical fiber communication systems toward ultra-high rate and ultra-large capacity, the launching power will inevitably increase, which makes the Kerr nonlinearity in optical fibers become the most important factor limiting the increase in the capacity of optical fiber systems [1]. In the seminal work [2], nonlinear frequency division multiplexing (NFDM) was proposed as a very promising technique to solve the nonlinear problem of the optical fiber. The highlight of NFDM is that the fiber nonlinearity is taken as the system design factor, and the nonlinear transmission of signals in the optical fiber is linearized in the so-called nonlinear spectral domain (NSD) by the mathematical tool of nonlinear Fourier transform (NFT).

The signal can be decomposed into two types of spectra in the NSD by ordinary NFT: the discrete spectrum (DS) corresponding the solitonic part and the continuous spectrum (CS) representing the dispersive part of the pulse. Therefore, according to which part of nonlinear spectrum is exploited, NFDM systems can be divided into DS modulation, CS modulation and full spectrum (FS) modulation, i.e. combined modulation of DS and CS. In order to further improve data rates and spectral efficiency (SE) by exploiting all available degrees of freedom, FS modulation were studied in [3-6]. Although the FS modulation with *b*-scheme is demonstrated in [5], the analysis is not comprehensive. Moreover, low-order modulation formats are adopted in DS of FS and there is a lack of corresponding optimization and equalization schemes.

In this paper, we design a FS modulated NFDM system with *b*-scheme in detail, where the DS adopts the high-order modulation. Simultaneously, through the geometric shaping (GS) and linear minimum mean square error (LMMSE) estimator, the system performance is effectively improved.

2. Design for the FS system

The block diagram of the system is shown in Fig.1 and fiber parameters are the same as those adopted in our work [7].

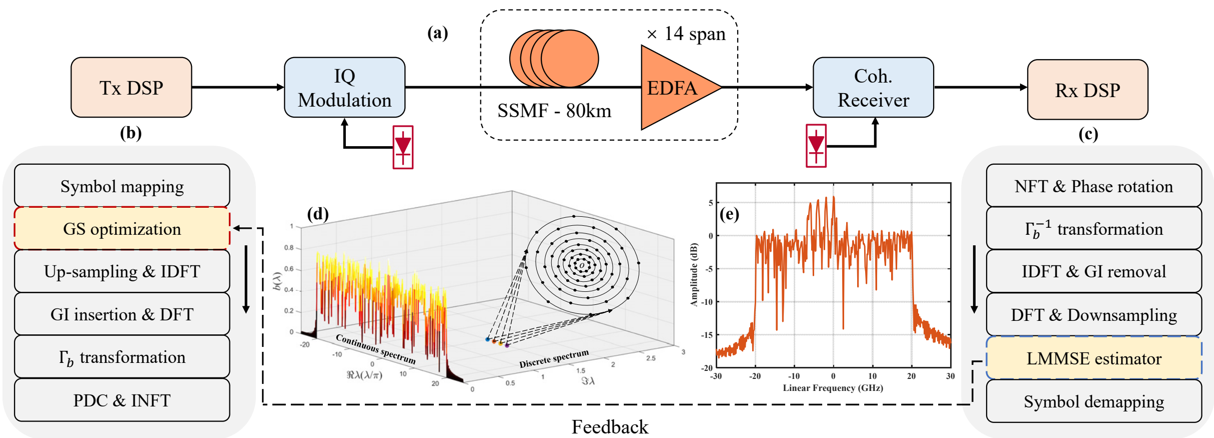

Fig. 1. (a) The diagram of coherent optical fiber communication system; (b) Tx DSP; (c) Rx DSP; (d) The schematic diagram of the FS; (e) The linear spectrum of the FS signal.

We use the OFDM-type modulation to construct the CS, as done in [7]:

$$b_c(\lambda) = Ae^{-2j\lambda^2\mathcal{L}}\Gamma_b\left(\sum_{k=-N_c/2}^{N_c/2-1} c_k \frac{\sin(\lambda T_0/T_s + k\pi)}{\lambda T_0/T_s + k\pi}\right), \tag{1}$$

where A is the power control parameter, λ is the so-called nonlinear frequency, $e^{-2j\lambda^2\mathcal{L}}$ is the pre-compensation term and Γ_b is the exponential scaling in [8]. $N_c = 128$ is the number of subcarriers, c_k is the symbols drawn from 32QAM constellation, $T_0 = 3.2ns$ is the useful block duration and $T_s = 1ns$ is the normalized time parameter. In order to avoid the inter-symbol interference (ISI), we take guard interval GI $= 3.2ns$, which makes the raw data rate for the CS reach 100 Gbps.

In the discrete part of FS, we set the following four eigenvalues:

$$\lambda_k = 2(k-1)\pi + 1.5j, \qquad k = 1,2,3,4. \tag{2}$$

As shown in [9,10], APSK is more suitable for DS than QAM. On each discrete eigenvalue, the high-order 64APSK is adopted, resulting the raw data rate of 3.75 Gbps for the DS. Fig. 1(d) shows the schematic diagram of FS and Fig. 1(e) draws the linear spectrum of the FS modulated signal. The corresponding relationship between the nonlinear frequency and linear frequency is $\lambda = -\pi f$. We can see that the introduction of discrete eigenvalues causes the linear spectrum to appear peaks at the corresponding real-valued frequencies.

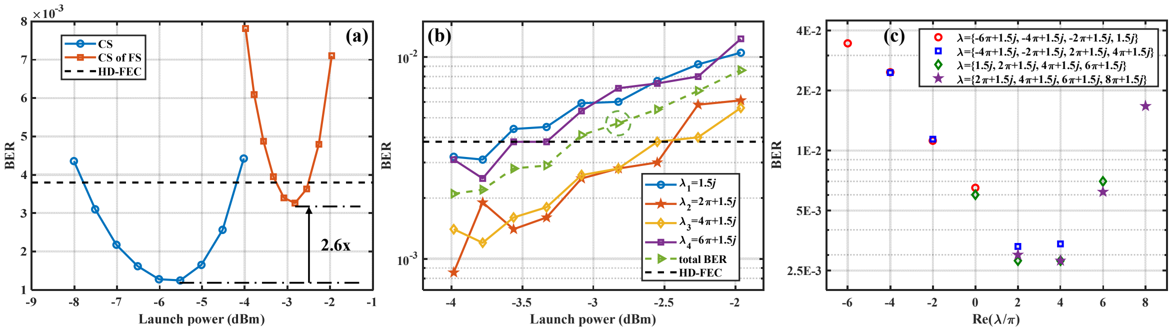

Fig. 2. (a) The BER performance for CS with and without DS after 1120km transmission (b) The BER performance for DS of FS after 1120km transmission; (c) The BER versus different sets of eigenvalues at the optimal power.

The BER versus launch power for CS with and without DS and for DS of FS after 1120km transmission are plotted in Fig. 2(a) and Fig. 2(b) respectively. Note that the launch power $P = P_d + P_c$, where P_d represents the power of DS, which remains unchanged, and P_c represents the power of CS, which is controlled by A in (1). Fig. 2(a) indicates that after multiplexing four eigenvalues, the BER for CS increases to 2.6 times of the original at the optimal power, but still remains below the HD-FEC threshold of 3.8×10^{-3}. From Fig. 2(b), we can see that the BERs of the four eigenvalues increase with the increase of the launch power. We speculate that the reason is that as the power increases, the energy of DS is submerged in the CS, and its characteristics are difficult to maintain after being affected by noise. Fig. 2(c) shows the BER performance of different sets of eigenvalues at the optimal power, which is also the basis for our selection of the four eigenvalues in (2).

3. Optimization for the FS system

As depicted in Fig. 2(b), the total BER of the DS at the optimal power is above the threshold, so we need to reduce it below the threshold by means of optimization. Considering the correlation between the coefficients of DS [11], we

Table 1. Correlation coefficients between the deviations

Correlation coefficients	$\Delta\Gamma_{b,1}$	$\Delta\theta_{b,3}$	$\Delta\Gamma_{b,3}$	$\Delta\theta_{b,4}$	$\Delta\Gamma_{b,4}$
$\Delta\lambda_{I,1}$	0.3235	-	-	-	-
$\Delta\lambda_{R,3}$	-	0.3955	-	-	-0.2187
$\Delta\lambda_{I,3}$	-	-	-0.3754	-	-
$\Delta\lambda_{R,4}$	-	-	-	0.5715	-
$\Delta\lambda_{I,4}$	-	-	-	-	-0.6218

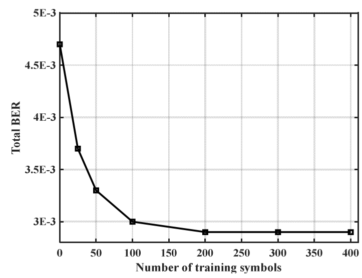

Fig. 3. Total BER versus the number of training symbols at optimal power for LMMSE estimator.

first apply the LMMSE estimator to optimize the system performance. Before applying the LMMSE estimator, we need to calculate the table of correlation coefficients. It was originally supposed to be 16×8 in size, here we remove the correlation coefficients with absolute values less than 0.2, the size of the table is reduced to 5×5, as shown in Table 1. It shows that except for λ_2, the other three eigenvalues are needed to apply LMMSE estimators. Fig. 3 shows that 200 training symbols are enough for each eigenvalue to estimate the coefficients of the LMMSE estimator.

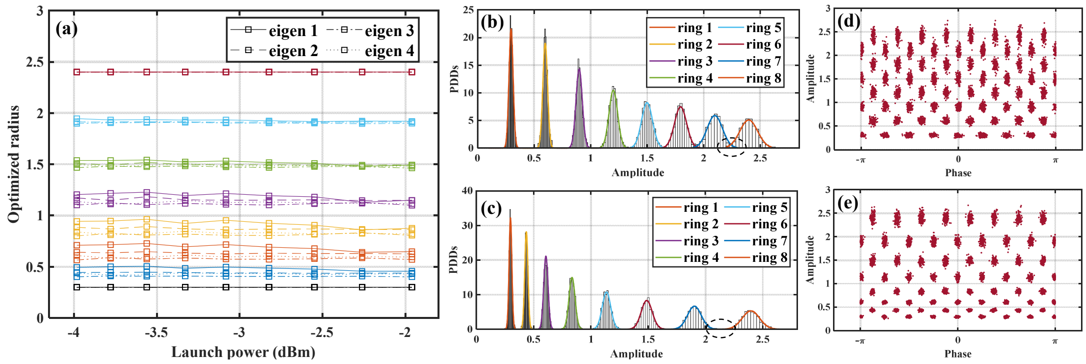

Fig. 4. (a) The optimized radiuses of each eigenvalue at different launch power; (b-c) PDDs for each ring on λ_4 with Norm-64APSK and GS-64APSK after LMMSE respectively; (d) The constellation on λ_4 with Norm-64APSK before LMMSE; (e) The constellation on λ_4 with GS-64APSK after LMMSE.

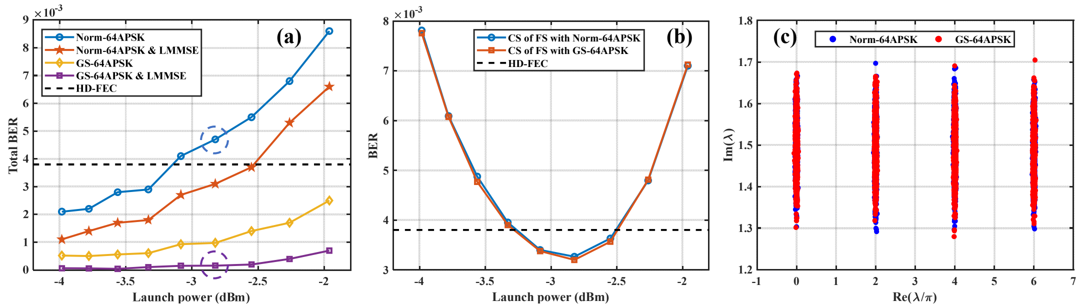

Fig. 5. (a) The total BER of DS with different schemes; (b) The comparison of BER performance for CS of FS with Norm-64APSK and GS-64APSK; (c) The distribution of eigenvalues with Norm-64APSK and GS-64APSK at Rx.

In order to further improve the system performance, we extend the work in [10] to the DS of multi-eigenvalues under FS modulation. Fig. 4(a) depicts the optimized radiuses of each eigenvalue at different launch power after geometric shaping (GS) optimization. We can see that the optimal radius of each ring fluctuates slightly with power, but there are differences between different eigenvalues. Therefore, we can perform GS optimization for each eigenvalue at optimal power, and then apply to all power values. Fig. 4(b-c) draw the probability density distributions (PDDs) for each ring on λ_4 before and after GS. It can be seen that after GS, there is no overlap between the distributions, leading to better BER performance. Fig. 4(d-e) show the constellations before and after GS. As shown in Fig. 5(a), the best BER performance can be achieved by applying GS and LMMSE, which is far below the HD-FEC threshold. Fig. 5(b) shows that the performance of CS before and after GS is basically unchanged. Fig. 5(c) depicts the distribution of eigenvalues at Rx before and after GS.

4. Conclusion

In this paper, a FS-modulated NFDM system with *b*-scheme is built by simulation, where CS using 128 subcarriers with 32QAM and DS adopting 4 eigenvalues with 64APSK. Through GS and LMMSE, we achieved 103.75 Gbps transmission over 1120km with BER below HD-FEC threshold.

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