Message Passing: Towards Low-Complexity, Global Optimal Routing and Wavelength Assignment Solutions for Optical Networks

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Abstract: We introduce a polynomial-time distributed message passing algorithm for routing and wavelength assignment. Exact global solutions are obtained for small-scale networks and improvements are demonstrated on network scales beyond the reach of established global algorithms.

1. Introduction

Wavelength division multiplexing (WDM) networks underpin global communication infrastructures, vital for today's information society. One of the key problems in implementing dynamic optical WDM networks is the routing and wavelength assignment (RWA) under dynamic traffic scenarios [1, 2], where path and wavelength pairs (lightpaths) must be assigned to satisfy the demand of incoming traffic requests. This is a difficult optimisation problem, as the sequence in which the traffic requests arrive has a significant impact on the network performance (e.g. blocking probability, resource utilisation, etc.) [2]. Previously, in order to tackle this, heuristic routing strategies such as random wavelength allocation, load balancing and wavelength packing were used in dynamic RWA scenarios [1]. However, these strategies tackle traffic requests one-by-one, consistently leading to sub-optimal solutions. A solution to this is batch processing [3], which significantly improves packing performance compared to the one-by-one approach, although the optimal routing of a traffic batch is also an NP-hard problem [3]. The computational complexity of global optimisation methods, such as integer linear programming (ILP), grow exponentially as the network scale increases. Alternatively, local heuristics offer lower computational complexity but cannot ensure the global optimality of the solutions found [4]. In addition, traditional global optimal approximation methods such as meta-heuristics, search the solution space by constructing random solutions based on their own policies, which cannot ensure the optimality in reasonable time.

Message passing (MP), also known as belief–propagation or the cavity method [5, 6], has been shown to be a polynomial time global approximation method for static routing problems in large-scale networks [7–9]. It is based on iteratively passing conditional probabilities between neighbouring variables until convergence to facilitate the calculation of optimal solutions. In this paper, we describe a new MP-based algorithm, in conjunction with batch processing, for solving the dynamic RWA problem. Simulation results show that MP achieves the same global solutions as ILP in reduced computing times for small graph instances, as well as having significant advantages in both blocking probability and resource utilisation compared to baseline methods, including first-fit k-shortest-path (FF-kSP), k-shortest-path first-fit (kSP-FF) [10] and adaptive shortest path (A-SP) [1], for large graph instances (≈ 100 nodes).

2. Message Passing Method

In this work, we have constructed the RWA problem as a pure edge-disjoint routing problem in a logical network, as shown in Fig. 1. The number of nodes, edges, traffic requests and wavelengths in the original WDM network are denoted as N, E, M and Q, respectively. All original nodes and edges are duplicated Q times, representing the paths available on different wavelengths. Virtual source and destination nodes (e.g. μ_1 in Fig. 1) are introduced for every traffic request ($\{\mu_1, \ldots, \mu_M\}$), which connect to the corresponding nodes on all wavelengths.

An MP model is developed to minimise the total path length over the previously constructed logical network. Other objectives and a variable number of wavelengths per edge can be easily accommodated within the same framework. The general idea of MP is to map the global optimisation problem onto localised inference tasks to obtain a common optimal solution. This distribution of tasks is key to reducing computational complexity and

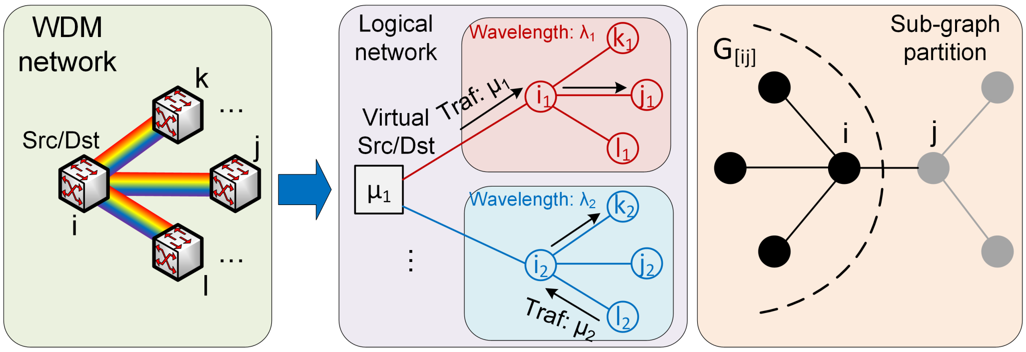

Fig. 1: Demonstration of message passing.

allows for the algorithm to scale well to larger networks. The MP model could be described as follows. Firstly, a variable $\mu_{i,j}^{\lambda}$ is introduced to represent the traffic request passing an edge (i,j) on wavelength λ. It takes a value within $\{0,\pm\mu_1,\ldots,\pm\mu_M\}$, which represents the traffic index (negative values means the traffic passes inversely). The message $\phi_{i\to j}^{\lambda}(\mu)$ is related to the probability of passing message μ from nodes i to j using wavelength λ given similar probabilities provided from the sub-graph $G_{[ij]}$, a sub-tree component of the logical network (when the network is not a tree, it is an approximation), as shown in Fig. 1. The process is iterative and once it converges, the objective function of MP can be obtained as described in Eq. (1), where $w_{i,j}$ represents the weight of edge (i,j). δ_x^y is the Kronecker delta such that $\delta_x^y=1$ only if $x=y$; and $\delta_x^y=0$ otherwise.

$$\frac{1}{Q\cdot E}\sum_{(i,j),\lambda}\min_{\mu}\left[\phi_{i\to j}^{\lambda}(\mu)+\phi_{j\to i}^{\lambda}(-\mu)+w_{i,j}(\delta_{\mu}^{0}-1)\right] \tag{1}$$

$$\begin{cases}\phi_{i\to j}^{\lambda}(0)=\min\limits_{\substack{\text{Matching}\\ \text{Pairs: }\vec{\mu}_{\partial i\setminus j}}}\sum\limits_{k\in\partial i\setminus j}\phi_{k\to i}^{\lambda}(\mu_{k,i}^{\lambda}),\\[2ex] \phi_{i\to j}^{\lambda}(\mu)=w_{i,j}+\min\limits_{k\in\partial i\setminus j}\Big[\phi_{k\to i}^{\lambda}(\mu)+\\ \qquad\min\limits_{\substack{\text{Matching}\\ \text{Pairs: }\vec{\mu}_{\partial i\setminus j,k}}}\sum\limits_{l\in\partial i\setminus j,k}\phi_{l\to i}^{\lambda}(\mu_{l,i}^{\lambda})\Big],\ \text{for }\mu\neq 0.\end{cases}
\qquad
\begin{cases}\phi_{\mu_n\to i}^{\lambda}(0)=\min\limits_{\lambda'\neq\lambda}\Big[\phi_{i\to\mu_n}^{\lambda'}(-\mu_n)+\sum\limits_{\lambda''\neq\lambda,\lambda'}\phi_{i\to\mu_n}^{\lambda''}(0)\Big],\\[2ex] \phi_{\mu_n\to i}^{\lambda}(\mu_n)=w_{\mu_n,i}+\sum\limits_{\lambda'\neq\lambda}\phi_{i\to\mu_n}^{\lambda'}(0),\\[2ex] \phi_{\mu_n\to i}^{\lambda}(\mu)=\infty,\ \text{for }\mu\neq 0,\mu_n.\end{cases} \tag{2}$$

The self-consistent MP iterative equations for intermediate nodes (left) and virtual source/destination nodes (right) are listed in Eq. (2). For each node $i\in N$, we define $\partial i\setminus j$ as the adjacent node set of node i except node j, and $\vec{\mu}_{\partial i\setminus j}$ as an allocation of traffic requests on associated edges of node i except edge (i,j). "Matching Pairs: $\vec{\mu}_{\partial i\setminus j}$" refers to the condition that every lightpath in $\vec{\mu}_{\partial i\setminus j}$ comes and leaves node i without passing through edge (i,j). The equations in the left set of Eq. (2) describe the scenarios of traffic μ not passing through edge (i,j) (top) and passing through edge (i,j) on wavelength λ (bottom), corresponding to the lightpath situations of λ_2 and λ_1 in Fig. 1, respectively. While the right set of Eq. (2) refers to the virtual source/destination node selecting the same single wavelength λ at the start/end of a lightpath for traffic μ_n. The messages $(\phi_{i\to j}^{\lambda}(\mu))$ only relates to the messages from its adjacent nodes. Thus, by passing the messages iteratively, the routing information spreads across the whole network, leading to an (near-)optimal solution upon convergence. Moreover, the computational complexity of MP is greatly reduced from $O(2^{Q\cdot M\cdot E})$ to $O(MQ(\frac{M}{N}+N+Q))$ compared to ILP.

3. Results and Discussions

To verify the efficiency and optimality of the MP method, we firstly calculated the minimum number of wavelengths ($Q_{\min}$) needed for all-to-all traffic requests on 4 real core networks: NSFNet (NSF), Google B4 (GB4), DTAG/T-system (DTAG) and BT-Core (BT) networks [11, 12]. Table 1 shows that MP achieves the same results ($Q_{\min}$ and total lightpath length L) as the ILP, yet on average takes only 61% of the ILP's computing time.

To further explore the performance of MP, we conducted a dynamic RWA simulation using a 100-node 130-edge graph, generated with the SNR-BA model [13]. For the physical layer, we assumed a full C-band (1530-1570 nm) transmission with 156 wavelengths (32 GHz Nyquist-spaced) on all fibre links. A Poisson traffic model [14] was implemented, with the demand distributed uniformly among all node pairs. For each network load (Erlang per wavelength), we generated 10 sets of 10,000 requests with a batch size of 200. Thereafter, FF-kSP, kSP-FF and A-SP are used as baseline dynamic RWA methods for comparison.

As shown in Fig. 2(a), MP achieved the lowest blocking probability under all network loads. Compared to one of the widespread, and in some cases, commercially implemented method kSP-FF, MP achieved 8.9% blocking probability reduction at the loads of 3-10 Erlang per wavelength (Fig. 2(a) inset). At the highest load of 10 Erlang per wavelength, the average blocking probability of MP is 2.6% less than A-SP (best of the baseline methods), which is a 9.4% relative enhancement. To understand how the routing methods use the resources (wavelengths) of the network, the resource utilisation rate was defined as $\frac{\sum_e \lambda_e}{Q \cdot E}$, where λ_e is the number of wavelengths used on edge e. As shown in Fig. 2(b), the kSP-FF and FF-kSP heuristics both end up under-utilising the network resources, since they have a limited number of k-shortest paths to choose from. A-SP is able to find the shortest path available over all the wavelengths, yet still limited by the one-by-one lightpath allocation approach. The MP method takes the whole batch into account and finds the minimum total path length of that batch, whilst still attempting to allocate as many requests as possible. This allows for better lightpath allocations that reduce 9.4% relative blocking, whilst still using 2.6% relative less wavelengths at the same time.

Table 1: Static RWA results

Network		NSF	GB4	DTAG	BT
Nodes		14	12	14	22
Edges		21	19	23	35
ILP	$Q_{\min}$	13	16	14	39
	L	195	153	218	697
	T(sec)	45	16	20	273
MP	$Q_{\min}$	13	16	14	39
	L	195	153	218	697
	T(sec)	7	9	17	239

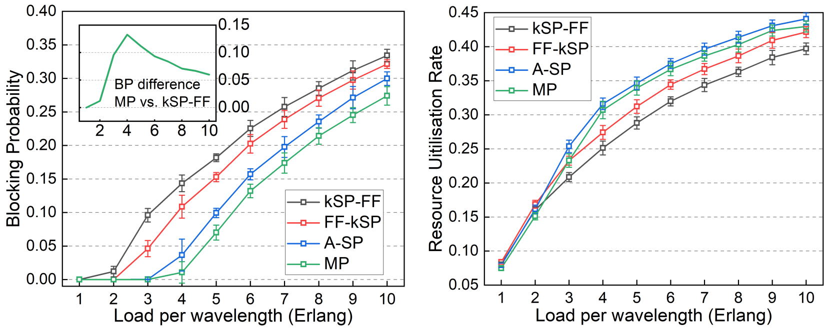

Fig. 2: (a) Blocking Probability (left), (b) Resource Utilisation Rate (right) – 95% CI

4. Conclusions

We proposed a new scalable probabilistic algorithm for dynamic RWA in optical networks. Unlike the traditional RWA methods, which are either non-scalable in the search for a global optimum (e.g., ILP) or find local optima in a greedy manner (e.g., heuristics or meta-heuristics), MP converts the global optimisation problem into distributed local optimisation, solved by iteratively passing self-consistent probabilistic messages. This facilitates obtaining an approximate global RWA solution in polynomial time. MP was shown to give exact global solutions for small scale networks compared to ILP and offer a 9.4% relative reduction in blocking probability than the best performing heuristic in a large scale network (100 nodes). Work is ongoing to support the re-routing functionality, critical for adaptive, dynamic RWA applications, which has the potential to keep the network operating at near-optimal states.

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