Precise Longitudinal Power Monitoring over 2,080 km Enabled by Step Size Selection of Split Step Fourier Method

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Abstract: We propose a step-size optimization scheme of the split-step Fourier method for longitudinal power profile monitoring. We observe only a 1.06-dB root-mean-square error from the theoretical power profile for a 2,080-km transmission link. © 2022 The Author(s)

1. Introduction

Operating an optical transmission system at its maximum data rate under individual channel conditions has become increasingly challenging. One reason is that recent rate-flexible transceivers provide an enormous variety of transmission modes, and thus selecting the best one for a given transmission channel is a daunting task. As another reason, along with the increasing demand for high spectral efficiency (SE), system operators have to preset larger margins in optical signal-to-noise ratio (OSNR) since higher SE signals are typically more susceptible to channel noises, which hinders the full use of the potential capacity.

The first essential step to be commonly taken for these issues is to correctly perceive the transmission link status, such as the longitudinal optical power evolution [1-3], amplifier's gain tilt [4], and filters' frequency detuning [2,3], which can then help to estimate the fiber nonlinearity, OSNR, and filtering penalties. Accordingly, longitudinal signal power profile monitoring has been demonstrated using a correlation method [1], a split-step Fourier method (SSFM) based channel reconstruction [2,4], and a Volterra-based method [3]. However, though these methods reveal multispan power profiles only from online signals with the receiver-side (Rx) digital signal processing (DSP), they suffer from the following issues: (i) obtained profiles have *dead zones* at the beginning and end of spans, where significant deviations from true power profiles are observed; (ii) fiber launch power far higher than the system operational range is required to achieve adequate estimation performance since these methods highly rely on the fiber nonlinearity.

In this work, we present an optimization method of the spatial step size in SSFM for longitudinal power profile monitoring, which has so far adopted the uniform step size, and experimentally assess the different step size selection rules. Results show that the combination of the proposed scheme and the selection rule named the local error method (LEM) [5,6], which employs step size $h(z) \propto P(z)^{-1}$ for asymmetric (A-) SSFM, minimizes the dead zones and provides an excellent agreement between estimated and theoretical power profiles. We observe that this rule achieves root mean square (RMS) errors of 1.06 and 1.68 dB from the theoretical power for 2,080 km at a fiber launch power of 6 dBm and at the system optimum launch power -2 dBm (at 64 GBd), respectively.

2. Spatial Step-Size Selection Rules for SSFM

In this work, power profiles are obtained using A-SSFM for the Manakov equation, where the chromatic dispersion (CD) $\exp(\hat{D}h) = \exp(-i/2 \cdot \beta_2 \ \partial^2/\partial t^2 h)$ and the nonlinear phase rotation (NLPR) $\exp(\hat{N}h) = \exp(i\gamma'(z)h)$ are iteratively applied to the signals E(z,t), where β_2 the group velocity dispersion (GVD), $\gamma'(z) = 8/9 \cdot \gamma P(z)$, γ the nonlinear coefficient, P(z) the signal power, and h the step size. Our estimation target is $\gamma'(z)$ since power profiles can be calculated as $P(z) = 9/8 \cdot \gamma'(z)/\gamma$. We find $\gamma'(z)$ from boundary conditions, i.e., transmitted E(0,t) and received signals E(L,t), in the minimum mean square error (MMSE) criterion as in [2,4,7].

For SSFM simulations, several works discussed its step-size selection rules for a more precise simulation of nonlinear Schrödinger equation (NLSE) with a lower number of fast Fourier transform. The simplest rule is the uniform step size. Another rule is the walk-off method, where the step size is determined to bound the largest GVD difference. This method is the same as the uniform step size when homogeneous fibers are used in a link. The third one is the NLPR method, where NLPR is bounded in each step. In [5], LEM was proposed, where the step size h was selected so that the local error of each step was bounded. The analytical expressions of LEM for both A- and symmetric

(S-) SSFM were derived in [6] as $h(z)^2 \le \varepsilon / \{\gamma P(z) (2\pi | \beta_2 | B_w^2)\}$ and $h(z)^3 \le \varepsilon / \{\gamma P(z) (2\pi | \beta_2 | B_w^2)^2\}$, respectively, where ε is a local error, B_w the simulated bandwidth. However, simply bounding the local error does not guarantee the bounded global error. Thus, under the assumption that the global error is a linear accumulation of local errors in each step, the step size should be taken as $h(z) \propto P(z)^{-1}$ and $h(z) \propto P(z)^{-0.5}$ for A-SSFM and S-SSFM, respectively [6]. In the following experiments, we use homogeneous fibers with constant γ and β_2 in a link, and assume B_w will not change during the propagation; thus, we eliminated these parameters. Note that the former



Fig. 1. (a) Proposed step-size optimization scheme for longitudinal power profile monitoring. (b) Experimental setup.

rule $(h(z) \propto P(z)^{-1})$ is the same as the NLPR method in this situation. Though our method is based on A-SSFM, the latter rule for S-SSFM could also be a candidate since a finer step size is preferable for the sufficient spatial resolution, and for such a small h, S-SSFM can be approximated as A-SSFM. To summarize, in this work, all these rules become the matter of the order p of P(z) and thus we investigated p dependency of power profiles.

3. Step-Size Optimization Method for Power Profile Monitoring

However, these step size selection rules require a priori knowledge of $P(z) \propto 10^{-\alpha z/10}$ and thus loss coefficients α , though the goal of the monitoring is to estimate P(z). To address this issue, we first obtain a power profile under p=0 (uniform step size) and then estimate α around the center of the spans (Fig. 1(a)). In this work, we use powers from 40% to 50% position from the beginning of the span to calculate α . Using estimated α , we recalculate $h(z) \propto P(z)^p$ and the same procedure continues iteratively until the estimated α converges. Similarly, the amplifiers' positions should also be available in advance. Although they are assumed to be known in this work to focus on the effect of the step size selection, their positions can also be estimated from a power profile by taking the second derivative of it.

To assess the different step size selection rules, we conducted 280-km and 2,080-km transmission experiments. Fig. 1(b) shows the setup and the same Tx and Rx are used for both transmissions. In Tx DSP, single-channel PS-64QAM (H=4.347) 64 GBd (400G) with a roll-off factor of 0.2 is generated and emitted from a 4-ch arbitrary waveform generator (AWG) at 120 GSa/s. Tx and Rx laser have a 40-kHz linewidth at 1555.754 nm. On the Rx side, a 256 GSa/s digital sampling oscilloscope (DSO) digitizes the signal and offline Rx DSP is performed. In Rx DSP, frequency offset compensation, CD compensation, polarization demultiplexing, and carrier phase recovery. For power profile monitoring, signals are divided into two paths: one is for the transmitted signal regeneration [1,2], and the other reloads CD on the signals. CD-reloaded signals are then fed into A-SSFM to backpropagate to the transmitter (i.e., digital backpropagation [8]). The global error (square error) is calculated from the output of A-SSFM and regenerated transmitted signals for the gradient descent optimization of $\gamma'(z)$ in A-SSFM. 20 profiles are averaged for each profile.

We prepared two transmission test cases. For the results in Fig.2, we used a straight line 4×70 km and fiber launch power is set to 6 dBm. For the results in Fig. 3, we conducted a 2,080 km transmission using a recirculating loop, in which one circulation consists of 5-span 260 km. In this case, the fiber launch power is varied from -4 to 6 dBm to see the fiber input power dependency. The inline fibers are standard single-mode fibers with α =0.186 dB/km, β_2 =-21.7 ps²/km, and γ =1.11 1/W·km. Amplifiers used in both transmissions are erbium-doped fiber amplifiers (EDFA).

4. Results and discussion

Fig. 2(a) shows power profiles with different step sizes $h(z) \propto P(z)^p = 10^{-p\alpha z/10}$ for the 280-km transmission. For reference, the theoretical power profile measured by an optical time-domain reflectometry (OTDR) is also shown. Note that, to see the capability of estimating the true power, we assume the fiber nonlinear coefficient γ is known (1.11 1/W·km) and thus the absolute power is shown in the vertical axis. For all profiles, the number of split steps is







Fig. 3. (a) Estimated power profile (p=0, -1) for 2,080 km transmission with fiber launch powers Pin of 6 and -2 dBm. (b) Estimation error of obtained profiles in (a) from theoretical power profiles as a function of span number. (c) Histogram of estimation error from theoretical power profiles at Pin -2dBm. (d) Estimation error from theoretical power profile as a function of fiber launch power.

fixed to 560. As shown in Fig. 1, we start from the uniform step size (p=0), which has dead zones at the beginning and end of the spans and there a large deviation from OTDR is observed. From this profile, α of 4 spans are estimated as 0.185, 0.179, 0.205, and 0.187 dB/km, respectively. Then we recalculate $h(z) \propto P(z)^p = 10^{-p\alpha z/10}$. In Fig. 2(b), we show examples of h(z) for various p. Using these h(z), we recaptured profiles as shown in Fig. 2(a). As pdecreases from p=0, the dead zones diminish and the profiles fit the OTDR line except for the last span. For p lower than -1, the profiles again begin to separate from OTDR in the opposite direction. Though a further investigation of the last span's behavior is needed, the estimation performance improvement is clearly observed. The optimum p is around -1 (Fig. 2(c)) and this implies that LEM for A-SSFM or the NLPR method performs well.

Next, we applied our scheme to the 2,080-km transmission. Fig. 3(a) shows the power profiles obtained under p=0 and -1 for fiber launch powers of 6 and -2 dBm. The total number of steps is 1,040. Three things are notable: (i) even in the long-haul case, p=-1 shows a good agreement on the theoretical power, while p=0 generate more dead zones; (ii) the fiber launch power can be estimated from p=-1 with high precision; (iii) the profiles show lower power than theoretical one near the Tx. These tendencies are quantified in Fig. 3(b) and (c). In Fig. 3(b), we can observe that, from the 20 to 40th span near the Rx side, p=-1 exhibit an excellent fitness to the theoretical power with errors below 1 dB, while significant deviations are observed near the Tx side. This deviation is explained as suppression of the noise enhancement. As the Rx signal backpropagates SSFM, noise enhancement occurs in each NLPR block. Since the optimization of $\gamma'(z)$ is based on the MMSE criterion, optimized $\gamma'(z)$ tend to be small to avoid further noise enhancement. Fig. 3(c) shows the histogram of position-wise errors for Pin -2 dBm. p=-1 shows a narrower distribution, and 95% of errors are within 3.47 dB while those for p=0 are within 5.70 dB. Fig. 3(d) shows the fiber launch power dependency of the RMS error. Note that the optimum launch power for the system is -2 dBm. p=-1 provides approximately a 1-dB improvement in the RMS error over p=0 for any launch power. Finally, only 1.06 and 1.68 dB RMS errors are observed for Pin 6 and -2 dBm, while p=0 shows a 2.6 dB error at most.

5. Conclusion

We have presented the spatial step size optimization scheme for the digital longitudinal power profile monitoring and evaluated different step size selection rules. Results show that the rule $h(z) \propto P(z)^{-1}$ provides an excellent agreement between estimated and theoretical power profiles and has achieved 1.06-dB and 1.68-dB RMS errors from the theoretical power profile for a 2,080 km transmission at a launch power of 6 dBm and at the system optimum power -2 dBm at 64 GBd.

6. References

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