Solving Vertex Cover Problem using Quadrature Photonic Spatial Ising Machine

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Abstract: In this paper, we solve the 1600-vertex cover problem by a novel quadrature photonic spatial Ising machine. Our work suggests flexible combinational optimization problem solving for Ising models with external magnetic field. © 2022 The Author(s)

1. Introduction

The innovative application-specific hardware in the platforms of quantum mechanics [1], memristors [2] and photonics [3–5] are attracting enormous attentions in order to solve large-scale computation-intensive problems that cannot work out timely and efficiently by conventional electronic architectures in the post-Moore era. Combinatorial optimization, most of which are classified as non-deterministic polynomial (NP) hard, is one of important but computational intractable problems and find their critical applications in artificial intelligence, scheduling, finance prediction and *etc* [6, 7]. However, typical NP hard problems cannot be tackled in the conventional computing architecture with a reasonable cost as training data and adjustable parameters grow exponentially for a real-world challenge. Ising machine, viewed as an efficient NP problems solver, is originated from Ising model describing a dynamics physical system that evolve towards the minimum Hamiltonian in (1) by continuous change of spin binary state $x_l \in \{1, -1\}$.

$$H = -\sum_{<l,k>} J_{l,k} x_l x_k - \sum_k h_k x_k \tag{1}$$

where $J_{l,k}$ is the interaction between spins and h_k is spin interact with external magnetic field. Moreover, photonic Ising machines, with spin state encoded to phase or intensity of light, have been proposed by leveraging optical parametric oscillators [8, 9], integrated linear photonics [10], and spatial field manipulation [11]. However, current photonic Ising machine accelerators are all absent of external magnetic field in Ising model due to physical limitation, which narrow the freedom to map various optimization applications.

In this paper, we propose a quadrature photonic spatial Ising machine (Q-SIM) by synchronous phase manipulation in three sections. Vertex cover problem solution with 1600-vertex has been solved in simulation with 4-vertex demonstrated in experiment, which is the first time to solve vertex cover problem through a photonic Ising machine.

2. Principle

Architecture of quadrature photonic spatial Ising machine is shown in Fig. 1(a). A beam of light is three-part phase modulated by SLM. Spatial phase with values of $\{0, \pi\}$ and $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$ is modulated in the first two parts, leaving last part a fixed phase. The central intensity from spatial intensity distribution can be obtained at the charge-coupled device (CCD) image sensor after two-dimensional Fourier transform of the emergent light as

$$I(0,0) = (x^T\xi + y^T\eta + z^T\sigma)(\overline{\xi^T x + \eta^T y + \sigma^T z}) \tag{2}$$

where x, y and z are the three Ising spin value with ξ, η and σ as different modulated intensities respectively. As shown in Fig. 1(c), two Ising models with designed relation are constructed and updated in a synchronous state. The relation matrix $A = diag(a_1, a_2, ..., a_N)$, determining the spin value of second Ising model, is a unitary diagonal matrix. The spin value vector of the first and second Ising model are denoted by x and y respectively. By utilizing Eq.(2) represents an Ising model with

$$J_{l,k} = 2\xi_l\xi_k + 2a_l\overline{a_k}\eta_l\eta_k \tag{3}$$

$$h_k = 2\xi_k \sum_k z_k\sigma_k \tag{4}$$

The extension including external magnetic field will contain various NP hard problems, such as vertex cover problem and traveling salesman problem. For solving those NP hard problems, we firstly tune ξ_k to satisfy the h_k value. With fixed ξ_k, η_l can be tuned to achieve different values for the required matrix.

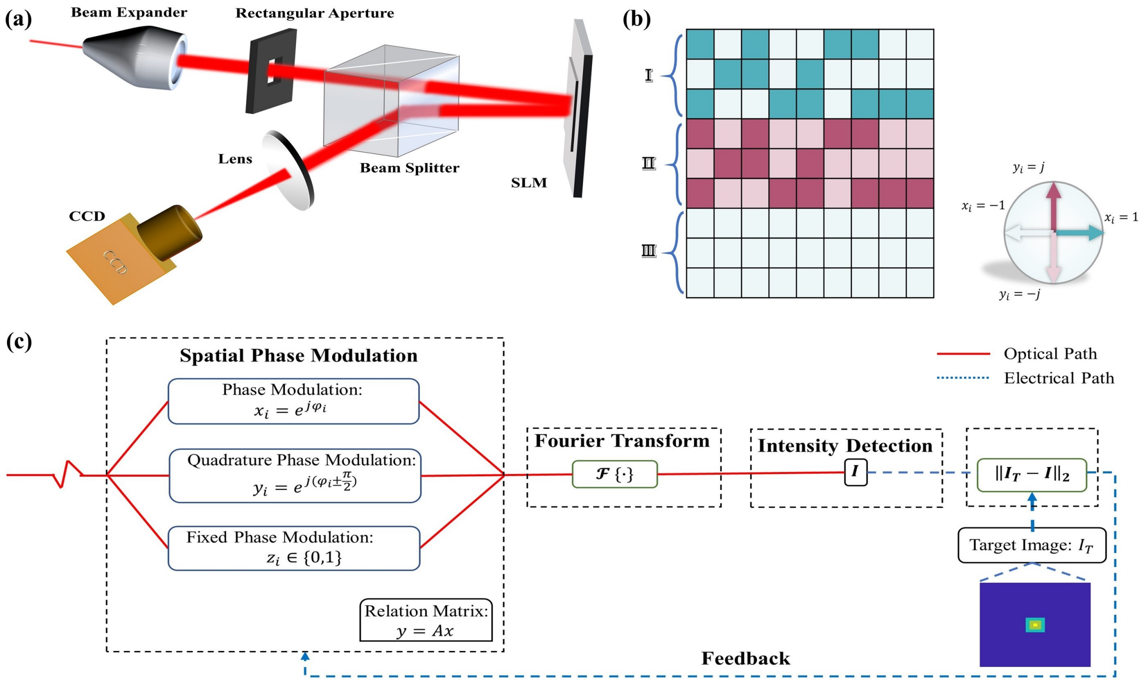

Fig. 1. **Architecture of quadrature photonic spatial Ising machine.** (a) Experimental setup. (b) A three-section phase map(c) The flow chart of optimization process.

3. Experimental and simulation results

Experimental setup is shown in Fig. 1(a). Intensity modulated incident beam with $\lambda = 632.8nm$ and output power of $1.6mW$ shines on a reflective SLM of HOLOEYE PLUTO-NIR-011 with 1920×1080 pixels. Thereby, spatial phase of an expanded laser beam with rectangular spot is modulated by $\{0, \pi\}$ or $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$. After spatial phase modulation, intensity profile of this beam is transformed through two-dimensional Fourier optics with focal length of $15cm$ and detected on CCD camera with frame rate of $40Hz$ and quantization bit of 8. For controlling feedback to search ground state of arbitrary Ising model, we randomly flip every spin to approach a target image with intensity decreasing stepwisely from the center to the outside. Due to the slow speed of SLM and CCD, the iteration time for the experiment is set to $1s$ with five times intensity detection in order to obtain an averaging intensity output. Specifically, for each iteration, the difference $\triangle d$ between detected intensity image I and target image I_T is calculated and simulated annealing algorithm with Metropolis probabilistic model [12] to find ground state. In each iteration, changed spatial phase is accepted with the probability as

$$P = min(1, e^{\frac{\triangle d}{T}}) \tag{5}$$

We firstly load an Ising model with 16 spins on SLM. For introducing external magnetic field, three-part phase modulated on incident light. The relation between the first and second phase modulation determined by a relation matrix $diag(j, j, ...j)$. And the third fixed modulated phase is consist of π and 0 that number of π is 6 and 10 to get $h_k = 8$ and $h_K = -8$ respectively. Fig. 2(a) shows the convergence of energy in each iteration and the illustration are the final phase maps under different external magnetic field.

Vertex cover problem of finding a vertex subset to contain at least one vertex of edge set is a NP hard problem. In vertex cover problem, vertex is corresponding to spin and binary spin value determines whether vertex is in the subset. By the map between vertex cover problem and Ising model [7], the Hamiltonian can be described as Eq.6 when the graph is complete

$$H = A\sum_{l,k} x_l x_k + (B - NA)\sum_k x_k \tag{6}$$

where A and B are set to prevent some subset is invalid. Then solving vertex cover problem can be mapped to find ground state of Ising model with external magnetic field. By Eq.3 and Eq.4, we can set

$$A = 2(\xi_l \xi_k \pm \eta_l \eta_k) \tag{7}$$

$$B - NA = 2\xi_k \sum_k z_k \sigma_k \tag{8}$$

In order to validate the solution, We construct a vertex cover problem that scale is 4 and $A = 4$ and $B = 8$ for demonstration. In experiment, spatial phase is directly modulated for a beam from laser source after expanding and shaping by a rectangular aperture. Relation matrix is set to be $diag(j, j, ..., j)$. A fixed phase of π is placed on the third section of SLM in order to obtain the last part of (1). During iterations, the probability of each spin configuration is shown in Fig. 2. Because of a complete graph, the optimal solution is obvious as only one $|\downarrow>$ for any spin configurations. Fig. 2(b) demonstrates that our Q-SIM can complete the task with a high probability. A lager-scale vertex cover problem is simulated as shown in Fig. 2(c). In order to verify the solution, a complete graph is used for evaluation and Fig. 2(c) shows the Q-SIM is able to solve a 1600-vertex cover problem with high probability.

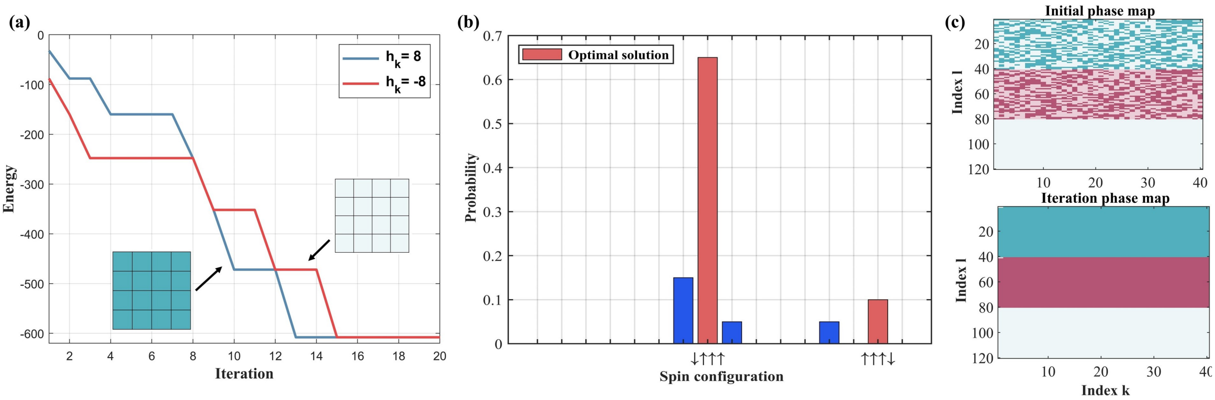

Fig. 2. **Experimental and simulation results.** (a) Ground state search. (b) A 4-vertex cover problem solving (c) A 1600-vertex cover problem

4. Summary

To conclude, we propose a novel quadrature photonic spatial Ising machine to bridge over a chasm between NP hard problems and photonic Ising accelerator. The proposed architecture is able to run more optimization problems by configuring external magnetic field in Ising model. Vertex cover problem have been successfully implemented in quadrature photonic spatial Ising machine. Our work pave a bright way for problem solution by large-scale photonic spatial Ising machine.

5. Acknowledgement

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