Nonlinear Frequency Division Multiplexing: Immune to Nonlinearity but Oversensitive to Noise?

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Abstract: Detection strategies and modulation formats designed for the AWGN channel are not well suited to operate in the nonlinear frequency domain. We study some improved detection strategies and investigate the ultimate performance limitations of NFDM systems that map conventional linear modulations on the nonlinear spectrum.

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1. Introduction

Since the first studies on optical fiber propagation, fiber nonlinearity has been regarded not only as a detrimental effect, but also as an opportunity for all-optical signal processing and regeneration. Key properties of the propagation equation are its integrability via the *nonlinear Fourier transform* (NFT) and the existence of *soliton* solutions [1]. In the context of fiber communications, these properties have fueled the dream of a sort of "transparent" propagation regime, virtually immune to both dispersion and nonlinearity. Soliton systems, in which information bits are carried by solitary waves that propagate unchanged through the fiber, have been deeply studied in the eighties and nineties [2]. More recently, the same dream has been revived and broadened by the proposal of a *nonlinear frequency-division multiplexing* (NFDM) scheme, which, analogously to the well known OFDM for linear channels, uses the NFT to encode information on the *nonlinear spectrum* of the optical signal, relying on its substantial invariance during propagation [3–5]. In recent years, numerous theoretical and numerical investigations and experimental demonstrations of NFDM transmissions have been reported [6–18], though, possibly due to the pioneering nature of the topic, a clear advantage over conventional systems has yet to be demonstrated.

A fundamental difference of NFDM with respect to OFDM is that the NFT, unlike the ordinary Fourier transform, is not a unitary transformation, so that the statistics of amplifier noise, modeled as additive white Gaussian noise (AWGN) in time domain, are not preserved in the nonlinear frequency domain [3, 8, 19, 20]. These modified statistics result in an unfavorable dependence of performance on the signal duration, which practically limits the achievable spectral efficiency [10]. A possible explanation of this negative result comes from the use of detection strategies inherited from conventional communication systems, optimized for the AWGN channel rather than for the actual noise statistics. Indeed, a number of *improved* detection strategies have been proposed to address this issue [12, 14, 21–24] but, unfortunately, without fully solving it. Another possibility is that the linear modulations (e.g., quadrature amplitude modulation (QAM)) that are employed to encode the information on the nonlinear spectrum, while guaranteeing a certain *minimum Euclidean distance* between the waveforms in the nonlinear frequency domain, result in a too small distance in time domain, making the system oversensitive to amplifier noise.

In this paper, we show that the NFDM performance can be significantly improved by modifying the detection strategy. Nevertheless, the main limitation mentioned above remains. This is explained by studying the dependence of the minimum distance between the time-domain waveforms on signal power and duration.

2. System setup and results

Fig. 1. The NFDM system with different possible detection strategies.

The NFDM performance is investigated through simulations using the setup in Fig. 1. The modulation scheme and channel parameters are as in [14]. A 50 GBd 16-QAM signal is mapped to the continuous nonlinear spectrum using the *nonlinear inverse synthesis* method [6, 14]. To account for group velocity dispersion (GVD), information is transmitted in bursts of N_b symbols, separated by $N_z = 2000$ null symbols acting as a guard time. After precompensation of the channel effects and backward NFT (BNFT), the optical signal is obtained through a digital-to-analog converter (DAC) and launched into the fiber. At the receiver (RX), an analog-to-digital converter (ADC) provides the samples of the received signal, from which the transmitted information is extracted by using one of the four detection strategies considered in this work: the conventional *forward NFT* (FNFT) strategy [6] and three improved (yet sub-optimal) strategies based on a *causality* property of the NFT [14], namely, *incremental FNFT* (I-FNFT), *decision-feedback FNFT* (DF-FNFT), and *decision-feedback BNFT* (DF-BNFT) [14, 22, 23]. The FNFT strategy employs a conventional QAM detector (matched filter, symbol-time sampler, and multiple thresholds) after the FNFT and is therefore optimized for an AWGN directly corrupting the nonlinear spectrum. However, even in the absence of propagation, optical amplifier noise is AWGN in time domain, but non-Gaussian and signal-dependent in the nonlinear frequency domain [8, 10, 19], making this strategy highly suboptimal. Thus, the I-FNFT and DF-FNFT strategies exploit the mentioned causality property to reduce the amount of noise that interacts with the signal in the FNFT. Conversely, the DF-BNFT exploits the same causality property to detect symbols directly in the time domain, selecting those that (approximately) minimize the Euclidean distance with the received signal, hence avoiding the detrimental signal–noise interaction caused by the FNFT.

The physical channel is a single-mode fiber of length $L = 2000\,\text{km}$, attenuation $\alpha = 0.2\,\text{dB/km}$, GVD parameter $\beta_2 = -20.39\,\text{ps}^2/\text{km}$, and nonlinear coefficient $\gamma = 1.22\,\text{W}^{-1}\text{km}^{-1}$, along which ideal distributed amplification with spontaneous emission factor $\eta_{\text{sp}} = 4$ is considered. The bandwidth of both the DAC and the ADC is 100 GHz.

Fig. 2(a) shows the best performance of the NFDM system with the described detectors and of a conventional system (i.e., no NFT) employing electronic dispersion compensation (EDC), each evaluated at the corresponding optimal launch power. The performance is reported in terms of Q^2_{dB}, where the Q-factor is evaluated from the bit error rate, measured through direct error counting [14]. The performance is shown as a function of the *rate efficiency* $\eta = N_b/(N_b + N_z)$, which is changed by varying N_b [10, 14]. The proposed I-FNFT, DF-FNFT, and DF-BNFT strategies (in order of increasing complexity and performance) significantly improve the performance of the NFDM system compared to the FNFT strategy, the DF-BNFT one providing a gain of about 6 dB for $\eta \geq 20\%$. Nonetheless, even with the DF-BNFT strategy and differently from conventional systems, the NFDM performance keeps decreasing with the increase of the rate efficiency η.

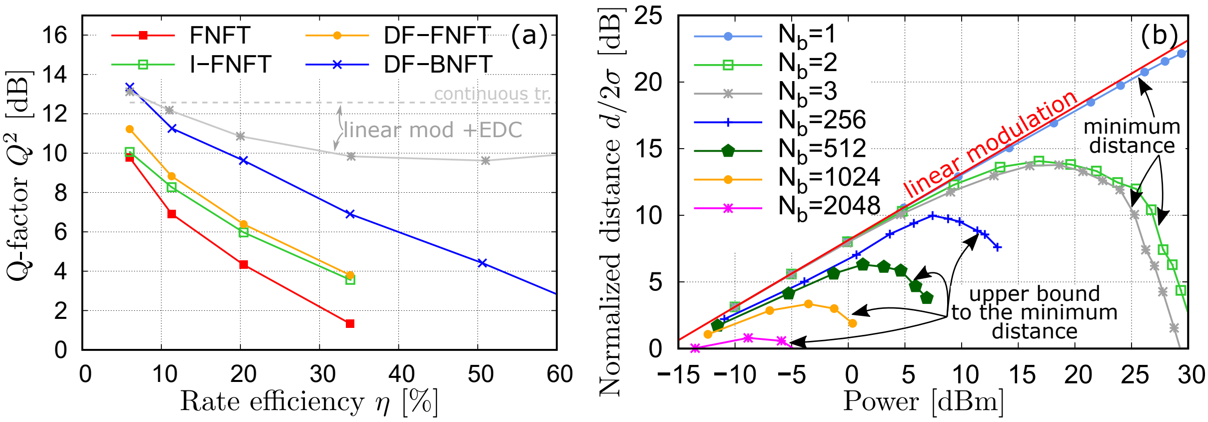

Fig. 2. (a) Performance of NFDM with different detection strategies compared to linear (burst or continuous) modulation; (b) normalized (upper bound to the) minimum distance between transmitted NFDM waveforms.

In order to understand whether this critical behavior is due to the use of a still suboptimal detection strategy (as none of the considered ones is optimal) or, instead, reflects an intrinsic inefficiency of the employed NFDM modulation over an AWGN channel, we investigate the dependence of the minimum Euclidean distance between all the possible waveforms $q(t)$ generated by the NFDM modulator in time domain, where the amplifier noise is indeed AWGN and this metric is strictly related to the system back-to-back (B2B) performance. Fig. 2(b) reports the minimum distance as a function of the signal power (measured without accounting for the guard time [10, 14]) for different block lengths N_b. For $N_b = 1, 2, 3$ the minimum distance is evaluated exactly, while for longer block lengths only an upper bound is computed. The distance is normalized by the equivalent amount of noise generated in the $L = 2000\,\text{km}$ channel considered in Fig. 2(a). For $N_b = 1$, the minimum distance increases monotonically with power, closely lower bounding the linear modulation case (that is valid for any N_b). On the other hand, for $N_b > 1$, the minimum distance curve has a totally different behavior, reaching a maximum at some finite optimum power and then decreasing again. This trend is practically unchanged if the bandwidth of the DAC and the computational accuracy of the BNFT algorithm are increased. Moreover, the upper bounds show that, unlike the linear modulation case, the minimum distance for the NFDM case decreases as the burst length N_b increases, exactly as the Q-factor in Fig. 2(a), denoting an intrinsic limitation of the NFDM technique.

3. Conclusion

We have studied some improved detection strategies to mitigate the detrimental impact of amplifier noise in NFDM systems. Despite the significant gain provided by these strategies, a critical decay of the performance with the increase of the spectral efficiency is still observed. This decay is due to an intrinsic limitation of NFDM, which is virtually immune to nonlinear fiber distortions but oversensitive to amplifier noise. In fact, when mapping a linearly modulated signal (e.g., QAM) onto the (continuous part of) the nonlinear spectrum, while the minimum Euclidean distance in the nonlinear frequency domain is independent of the burst length and increases monotonically with power, the corresponding distance in time domain has a finite peak value that decreases with the burst length.

These results suggest that an improved modulation and coding strategy is required to control the minimum distance between the generated waveforms and make NFDM more efficient in the presence of AWGN.

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