DSP-based Mode-dependent Loss and Gain Estimation in Coupled SDM Transmission

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Abstract: We model analytically the MDG/MDL estimation process in coupled SDM transmission using equalizer coefficients of coherent receivers. We show that estimation errors can be partially compensated in moderate regimes of SNR and MDL/MDG.

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1. Introduction

Space division multiplexing (SDM) enables significant increase in capacity through integration at the component, fiber and system level [1,2]. In recent years, several flavors of SDM have been proposed, including the uncoupled signal transmission over multicore fibers, coupled signal transmission over multimode [3], few-mode multicore [4], or coupled-core fibers [5]. Coupled-core fiber transmission is especially promising because of its tolerance to nonlinearities [5]. Unlike the effects of inter-modal crosstalk and modal dispersion, which are compensated for by multiple-input multiple-output (MIMO) equalizers without reducing the channel capacity, mode dependent loss (MDL) and mode dependent gain (MDG) pose fundamental performance limitations to coupled transmission and can cause outage. In particular, long-haul coupled transmission is fundamentally limited by the MDG generated in optical amplifiers [6,7]. In this context, the DSP-based estimation of MDG by coherent receivers yields a two-fold benefit of assessing the link performance, as well as estimating a per-amplifier MDG performance [3,5].

The MDL/MDG of a link can be computed from the eigenvalues λ_i^2 of operator $\mathbf{HH}^H$, where $\mathbf{H}$ is the channel transfer matrix and $(.)^H$ denotes the Hermitian transpose operator [6,7]. Two possible metrics are usually evaluated; firstly, the peak-to-peak value given by the ratio between the highest and the lowest eigenvalues in dB. The second metric is the standard deviation of the eigenvalues in logarithmic scale. The peak-to-peak metric is more relevant to links with weak channel coupling, whereas the standard deviation metric is more relevant to links with strong channel coupling. In this paper, we focus on the standard deviation metric because of its direct applicability in long-distance links [8]. In DSP-based estimation, the MDL/MDG is usually computed from the inverse of the MIMO equalizer transfer function $\mathbf{W}$, as an estimate of $\mathbf{H}$ [3]. As adaptive MIMO equalizers typically use the minimum mean square error (MMSE) criterion [9], the MDL/MDG estimation accuracy depends on the channel signal-to-noise ratio (SNR). In this paper, we model analytically the influence of the SNR on the DSP-based MDL/MDG estimation process. We show that the DSP-based estimation of MDL/MDG based on MMSE equalizers lead to errors, and that these errors can be corrected in practical scenarios of moderate SNR and MDL/MDG.

2. DSP-based MDL estimation

For a given channel matrix $\mathbf{H}$ and signal to noise ratio (SNR), the transfer matrix of a MIMO MMSE equalizer, $\mathbf{W}_{\text{MMSE}}$, is given by [10]

$$\mathbf{W}_{\text{MMSE}} = \left(\frac{I}{\text{SNR}} + \mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H. \quad (1)$$

DSP-based MDL/MDG estimation usually uses $\mathbf{W}_{\text{MMSE}}^{-1}$ as an estimate of $\mathbf{H}$. Therefore, the eigenvalues λ_i^2 are estimated from the eigenvalues of $\mathbf{W}_{\text{MMSE}}^{-1}(\mathbf{W}_{\text{MMSE}}^{-1})^H$. The relationship between the actual eigenvalues λ_i^2 and the eigenvalues obtained by DSP, $\lambda_{i_{MMSE}}^2$, can be obtained from the eigendecomposition of $\mathbf{W}_{\text{MMSE}}^{-1}(\mathbf{W}_{\text{MMSE}}^{-1})^H$, as

$$\mathbf{W}_{\text{MMSE}}^{-1}(\mathbf{W}_{\text{MMSE}}^{-1})^H = \frac{(\mathbf{HH}^H)^{-1}}{\text{SNR}^2} + \frac{2}{\text{SNR}} + \mathbf{HH}^H = \mathbf{Q}\left[\frac{\Lambda_H^{-1}}{\text{SNR}^2} + \frac{2}{\text{SNR}} + \Lambda_H\right]\mathbf{Q}^{-1}, \quad (2)$$

where Λ_H is a diagonal matrix whose main diagonal has elements λ_i^2, and $\mathbf{Q}$ is a matrix whose columns are the eigenvectors of $\mathbf{HH}^H$. Therefore, the eigenvalues $\lambda_{i_{\text{MMSE}}}^2$, obtained by DSP, and the original eigenvalues λ_i^2, are related by

$$\lambda^2_{i\mathrm{MMSE}} = \left[\frac{(\lambda_i^2)^{-1}}{\mathrm{SNR}^2} + \frac{2}{\mathrm{SNR}} + \lambda_i^2\right]. \tag{3}$$

Fig. 1(a) shows $\lambda^2_{i\mathrm{MMSE}}$ as a function of λ_i^2 for different levels of SNR. As the SNR decreases lower values of λ_i^2 start to raise, affecting the estimation process. If the SNR is known, (3) can be inverted to recover λ_i^2 from $\lambda^2_{i\mathrm{MMSE}}$, resulting in a quadratic equation with two roots

$$\lambda_i^2 = \frac{\left[\mathrm{SNR}^2\,\lambda^2_{i\mathrm{MMSE}} - 2\,\mathrm{SNR}\right] \pm \sqrt{\left[\mathrm{SNR}^2\,\lambda^2_{i\mathrm{MMSE}} - 2\,\mathrm{SNR}\right]^2 - 4\,\mathrm{SNR}^2}}{2\,\mathrm{SNR}^2}. \tag{4}$$

When evaluating (4) at very high SNR values, $\lambda_i^2 \approx \lambda^2_{i\mathrm{MMSE}}$ for the positive factor, and $\lambda_i^2 \approx 0$ for the negative factor. Therefore, in this paper, we use the positive factor. In order to evaluate the estimation error in DSP-based MDL/MDG estimation, we simulate transfer matrices $\mathbf{H}$ obtained by the multisection model presented in [6]. We simulate K = 100 spans of $L_{\mathrm{span}} = 50$ km, yielding a total length of 5,000 km. The group delay standard deviation is set to 3.1 ps/$\sqrt{\mathrm{km}}$ [5]. The MDG of the link is controlled by the per-amplifier MDG standard deviation σ_g. We simulate 1,000 frequency bins over a bandwidth of 240 GHz to capture the effect of frequency diversity [11], corresponding to eight realizations of 30-GHz channels. The estimated link MDL/MDG standard deviation σ_{mdl} is calculated in dB for each $\mathbf{H}$ matrix, and then averaged over the 1,000 realizations. Fig. 1(b) shows contour plots of the estimation error in dB for a wide range of SNRs and MDL/MDG standard deviations σ_{mdl}. Error estimates are computed as the absolute difference between the actual and estimated σ_{mdl}. Fig. 1(b) shows the estimation error without correction. At an SNR = 10 dB, an error higher than 1 dB is observed for $\sigma_{\mathrm{mdl}} > 4$ dB. Fig. 1(c) shows the estimation error with correction, improving the estimation process. Here, at SNR = 10 dB, an error higher than 1 dB is achieved only for $\sigma_{\mathrm{mdl}} > 7$ dB. For SNR $\geq$ 19 dB the correction factor provides an estimation error below 0.5 dB over the entire range of evaluated σ_{mdl} .

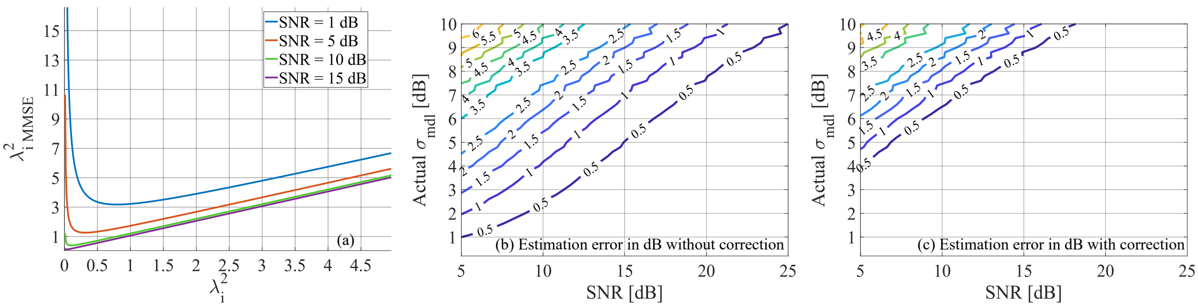

Figure 1. (a) Eigenvalues estimated by DSP, $\lambda^2_{i\mathrm{MMSE}}$, as a function of the actual eigenvalues λ_i^2. (b) Estimation error (in dB) without correction. (c) Estimation error (in dB) after correction by the positive factor of (4).

3. Simulation results

We model a coupled transmission system with $N_m = 6$ spatial modes, having two polarization modes each, as indicated in Fig. 2. At the transmitter, $2N_m$ independent binary sequences are mapped into 400,000 16-QAM symbols at 30 Gbaud. The complex constellations are fed into root-raised cosine (RRC) shaping filters with 0.01 roll-off factor, generating an output signal at 8 samples/symbol. The shaped signals are then sent to I/Q Mach-Zehnder modulator (MZM) models for electro-optical conversion. Finally, the $2N_m$ optical signals are launched into the transmission fiber model with strong mode coupling. The fiber is modelled using the multisection scheme presented in [6], with 1,000 frequency bins spread over 240 GHz (note that the simulation bandwidth is 30 GHz times 8 samples per symbol, yielding 240 GHz). The resolution of the channel in frequency domain is adjusted by replicating channel matrices between simulated frequency bins. The fiber parameters are the same as in Section 2. After propagation, the received signals are converted from the optical to the electrical domain by the receiver front-end model. The electrical signals are downsampled at two samples per symbol. The output signals are fed into the MIMO equalizer for source separation and equalization. 12×12 MIMO equalization is carried out by 144 finite impulse response filters with 100 taps each, updated by a fully supervised least mean squares (LMS) algorithm.

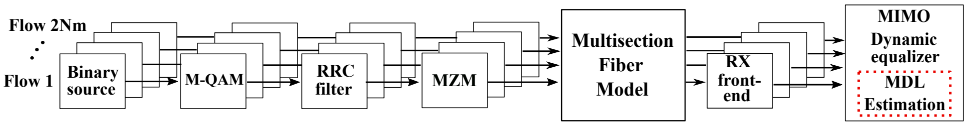

Figure 2. Simulation setup of coupled signal transmission.

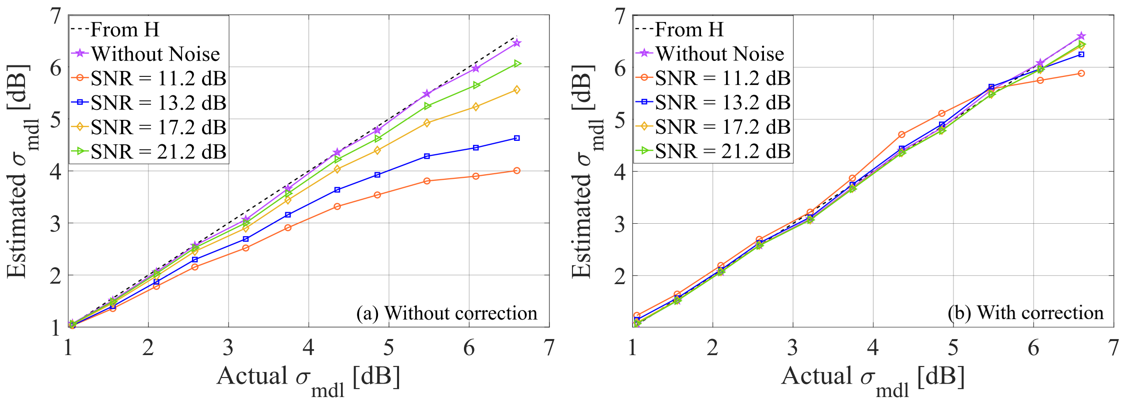

Figure 3. MDL/MDG standard deviation σ_{mdl} estimated from the equalizer coefficients W_{MMSE}^{-1} versus actual σ_{mdl} at different SNRs. (a) Without correction. (b). With positive correction factor.

Fig.3(a) shows σ_{mdl} estimated by DSP as a function of the actual value. In the absence of noise, the σ_{mdl} estimated from the equalizer coefficients tracks the actual σ_{mdl} with negligible error. As the SNR decreases, the estimation error increases for higher values of σ_{mdl}, underestimating the actual values. Fig.3(b) shows that DSP-based MDL estimation can be significantly improved by means of the positive correction factor derived in (4), resulting in a small residual error in the investigated range of values.

4. Conclusion

Space-division multiplexing using coupled transmission is a promising alternative for future high-capacity optical interconnection. In these systems, MDL/MDG reduce the average capacity and can cause outages. Therefore, MDL/MDG estimation carried out by the coherent receiver is a useful tool for link assessment and troubleshooting. In this paper we show analytically that MDL/MDG estimation carried out directly from the dynamic equalizer coefficients are prone to errors. Based on the transfer function of an MMSE equalizer, we calculate a correction factor that improves the estimation process in moderate levels of MDG/MDL and SNR. Lastly, we validate the method by Monte-Carlo simulation of waveforms launched into a coupled transmission model and received by an LMS dynamic equalizer. The results confirm the applicability of the method in practical transmission scenarios.

Acknowledgments

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References

1. P. J. Winzer *et al.*, "Optical MIMO-SDM system capacities," Proc. of OFC, paper Th1J.1, (2014).
2. S. O. Arik *et al.*, "MIMO signal processing for mode-division...," IEEE Sig. Proc. Magazine, **31**, pp. 25-34, (2014).
3. J. van Weerdenburg *et al.*, "138 Tbit/s transmission over 650 km graded-index...," Proc. of ECOC, pp. 1-3, (2017).
4. R. G. H. van Uden *et al.*, "Ultra-high-density spatial division multiplexing...," Nature Photonics, **8**, pp. 865, (2014).
5. R. Ryf *et al.*, "Coupled-core transmission over 7-core fiber," Proc. of OFC, paper Th4B.3, (2019).
6. K.-P. Ho *et al.*, "Mode-dependent loss and gain: statistics and...," Optics express, **19**, pp. 16612-16635, (2011).
7. P. J. Winzer *et al.*, "MIMO capacities and outage probabilities in ...," Optics express, **19.17**, pp. 16680-16696, (2011).
8. K. Choutagunta *et al.*, "Characterizing mode-dependent loss and...," J. Lightw. Technol., **36**, pp. 3815-3823, (2018).
9. M.S. Faruk *et al.*, "Digital signal processing for coherent...," J. Lightw. Technol., **35**, pp. 1125-1141, (2017).
10. M. R. McKay *et al.*, "Achievable sum rate of MIMO MMSE...," IEEE T. Inform. Theory, **56**, pp. 396-410, (2010).
11. K.-P. Ho *et al.*, "Frequency diversity in mode-division...," J. Lightw. Technol., **29**, pp. 3719-3726, (2011).