Low-Complexity Equalizer based on Volterra Series and Piecewise Linear Function for DML-based IM/DD System

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Abstract: We propose and demonstrate a low-complexity equalizer specifically designed for DML-based IM/DD system using Volterra series and piecewise linear function. The proposed equalizer performs similarly to the Volterra equalizer, but reduces the complexity by >90%.
OCIS codes: (060.4510) Optical communications; (060.2360) Fiber optics links and subsystems

1. Introduction

Due to the unabated growth of data traffic in wireless communications and datacenter-centric applications, it is important to implement short- and intermediate-haul optical transmission systems operating 50 Gb/s and beyond in a cost-effective manner [1]. A popular approach is to utilize the simple intensity-modulation (IM)/direct-detection (DD) scheme along with a multi-level modulation format such as 4-ary pulse amplitude modulation (PAM-4). This approach can be realized in a highly cost-effective manner by using a directly modulated laser (DML). Compared to externally modulated transmitters employing electro-absorption modulated lasers or Mach-Zehnder modulators, DMLs feature low cost, high output power, low power consumption, and small footprint [2]. However, a major technical problem associated with the DML-based IM/DD system is the severe waveform distortions induced by the interplay between the DML's frequency chirp and fiber chromatic dispersion. These waveform distortions are inherently nonlinear due to square-law detection of DD receiver and limit the maximum transmission distance over dispersive fiber [3], [4].

Volterra series theory offers a popular way to design an equalizer to compensate for nonlinear distortions [5], [6]. However, the Volterra nonlinear equalizer (VNLE) suffers from enormous implementation complexity. As the memory length increases, its implementation complexity grows rapidly, and thus hinders its application to cost-sensitive IM/DD systems. A couple of simplified VNLEs have been proposed to lower their complexity. For example, the diagonally-pruned (DP) VNLE sets the coefficients on the diagonals far from the main diagonal in the coefficient matrix to be zero [6]. The polynomial nonlinear equalizer (PNLE), where the coefficient matrix is a diagonal matrix, contains only the self-beating terms (i.e., $x_k \cdot x_k$, where x_k is the discrete-time sample of the equalizer's input at time index k) and sets the coefficients of all the cross-beating terms (i.e., $x_k \cdot x_{k-i}$, where $i \neq 0$) to be zero. Even though these equalizers reduce the implementation complexity, they are merely designed to reduce the number of terms included in the equalizer without considering the system characteristics. Thus, they are bound to sacrifice the performance considerably in many systems.

In this paper, we propose a low-complexity nonlinear equalizer specifically designed for DML-based IM/DD systems. Since the adiabatic chirp of DML produces the beating terms between the signal and the time integral of the signal after transmission over dispersive fiber, we group some beating terms (having similar coefficients) and reduce the number of coefficients to be determined in the VNLE [5]. We also note that the adiabatic chirp of DML makes the high-intensity signal travel faster than the low-intensity one under anomalous dispersion. Thus, this distortion can be compensated by utilizing a piecewise linear (PWL) function [7], [8]. We show through experiment that the proposed nonlinear equalizer performs similarly to the full 2nd-order VNLE, but its implementation complexity can be reduced by a factor of 10.

2. Principle

The 2nd-order VNLE can be expressed as [5], [6]

$$y(n)=\sum_{m=0}^{L_1-1} g_1(m)\,x(n-m)+\sum_{w=0}^{L_2-1}\sum_{k=0}^{L_2-1-w} g_2(k,w)\,x(n-k)\,x(n-k-w) \tag{1}$$

where $x(n)$ and $y(n)$ is the n-th received and recovered sample, respectively. Also, L_p and g_p are the p-th order memory length and equalizer coefficient, respectively, where p=1, 2. The first term on the right-hand side of (1) denotes a linear feedforward equalizer (FFE), and the second term represents a quadratic equalizer. The required number of taps of this quadratic nonlinear equalizer is $L_2(L_2+1)/2$. Thus, the implementation complexity of VNLE increases rapidly with the memory length, L_2.

The 2nd-order DP-VNLE has W non-zero diagonals in its coefficient matrix and can be expressed as

$$y(n)=\sum_{m=0}^{L_1-1} g_1(m)x(n-m)+\sum_{w=0}^{W-1}\sum_{k=0}^{L_2-1-w} g_2(k,w)x(n-k)x(n-k-w) \quad (2)$$

where $W \leq L_2$. The required number of coefficients in the nonlinear terms is $W(2L_2\text{-}W\text{+}1)/2$. Thus, the pruning reduces the complexity of nonlinear equalizer noticeably. However, when we apply the DP-VNLE to a DML/DD system, one practical problem is that the required number of diagonals, W, should be large (e.g., >5) due to a large delay spread of fiber channel when combined with DML's frequency chirp. This leads to a huge implementation complexity. We have recently shown that the interplay between fiber chromatic dispersion and adiabatic chirp of DML produces the beating terms between the signal and the *time integral* of the signal after transmission, which in turn, makes the coefficients of those beating terms nearly identical [5]. On the other hand, it is well-known that the adiabatic chirp shifts the frequency of the DML's output proportional to the optical intensity. Thus, the high-intensity signal has blue chirp and travels faster than the low-intensity signal (having red chirp) in the presence of anomalous dispersion. One effective way to compensate for this intensity-dependent waveform distortions is to utilize the PWL equalizer [7], [8]. Based on these findings, we propose the following nonlinear equalizer for DML/DD systems.

$$y(n)=\sum_{m=0}^{L_1-1} g_1(m)x(n-m)+\sum_{l=0}^{L_{v,2}-1} g_v(l)x(n-l)\sum_{w=0}^{Q-1}x(n-l-w)+\sum_{k=0}^{L_{p,2}-1} g_p\left\{\mathrm{D}\left[x(n-k)\right],k\right\}x(n-k) \quad (3)$$

where $L_{v,2}$ and $L_{p,2}$ are the nonlinear memory lengths of the Volterra and the PWL equalizers, respectively. Q is the accumulation depth ($2 \leq Q \leq L_2$). Also, the decision operation, D[$x(n)$], returns an index i when the signal amplitude ranges from λ_{i-1} to λ_i. The proposed equalizer comprises three equalizers: linear FFE [first term in (3)], neighbor-grouped VNLE (second term), and PWL equalizer (third term), as shown in Fig. 1. The PWL can be implemented by using a finite impulse response (FIR) filter. In this case, we can select g_p from a group of coefficients using the selection signal provided from a decision circuit. In our proposed equalizer, the number of the nonlinear coefficients is $L_{v,2}$+$L_{p,2}$. Therefore, the proposed equalizer has a lower implementation complexity than the DP-VNLE. It is also worth mentioning that the coefficients of the PWL equalizer are linear with respect to the output [7], and thus the coefficients can be obtained readily using the adaptive linear algorithm, such as the least mean square (LMS).

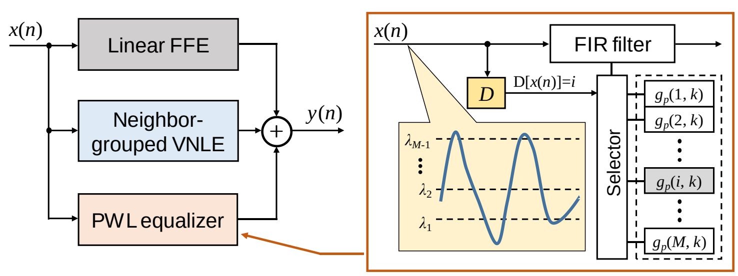

Fig. 1. Block diagram of the proposed equalizer

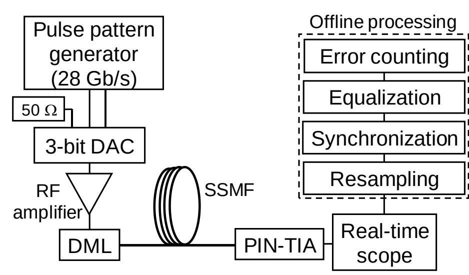

Fig. 2. Experimental setup

3. Experimental setup and results

We evaluate the performance of the proposed nonlinear equalizer on a 56-Gb/s PAM-4 link operating at the C band. Fig. 2 shows the experimental setup. We first generate a PAM-4 signal operating 56 Gb/s and feed it directly to a DML operating at 1556.6 nm. The DML is biased at 85 mA and emits the output power of 9.5 dBm. The 3-dB bandwidth is measured to be 24.6 GHz. After transmission over standard single-mode fiber (SSMF), the PAM-4 signal is detected by using a PIN-TIA detector (bandwidth = 33 GHz). Then, the received signal is digitized at 80 Gsample/s using a real-time oscilloscope. Finally, the captured waveforms are processed offline. The post-detection signal processing includes resampling, synchronization, half-symbol-spaced electrical equalization, and direct error counting. The LMS algorithm based on training sequences is adopted to determine the tap coefficients. The coefficients remain unchanged after the algorithm converges. L_1 is set to be 13, and all the nonlinear taps length (i.e., $L_{v,2}$, $L_{p,2}$ and L_2) are set to 11. We optimize the value of Q at different distances, and set the number of thresholds for PWL equalizer to be 3 (i.e., M=3).

Fig. 3 shows the bit-error ratio (BER) performance of the 56-Gb/s PAM-4 system. In the back-to-back case (i.e., 0 km), all the equalizers perform similarly and no BER floor is observed. This is because the system performance is mainly limited by linear distortions (e.g., non-flat frequency response of DML). Fig. 3(b) shows the BER performance measured after 25-km transmission. In this case, the system performance is limited by the nonlinear waveform distortions arising from the interplay between the DML's adiabatic chirp and fiber dispersion. Thus, the VNLE

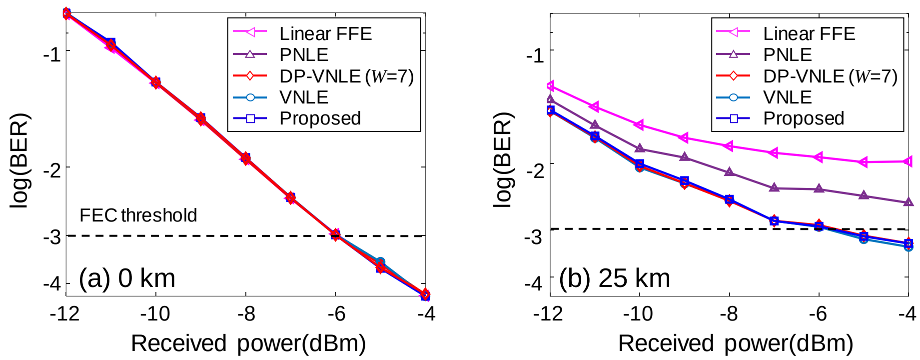

Fig. 3 BER performance of 56-Gb/s PAM-4 signal after (a) 0- and (b) 25-km transmissions.

	Number of multipliers
FFE	L_1
Proposed	$L_1 + 3L_2$
DP-VNLE	$L_1 + W(2L_2 - W+1)$
VNLE	$L_1 + L_2(L_2+1)$

Table I. Complexity of equalizers

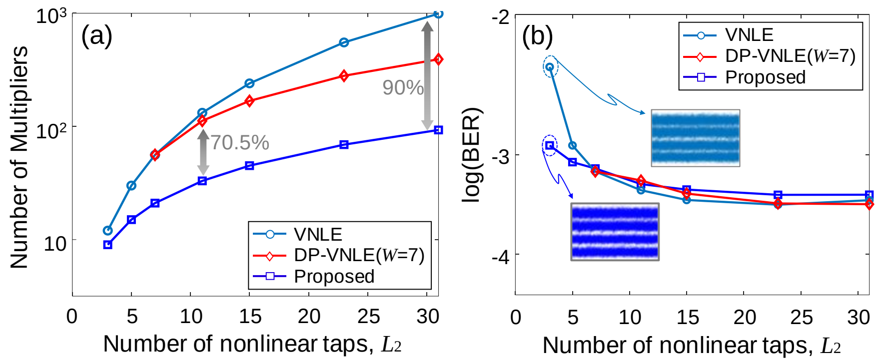

Fig. 4 (a) Comparison of implementation complexity (L_1=0). (b) BER performance after 25-km transmission. The insets in (b) are the scatter diagrams when L_2=3.

improves the BER considerably, compared to the linear FFE. The PNLE brings a relatively small performance improvement, and does not reach the FEC threshold of 10^{-3}. The DP-VNLE can improve the BER performance over PNLE by introducing some crossing-beating terms, and achieves almost the same performance as the VNLE when W =7. It should be noted that this W value is quite large due to a lot of cross-beating terms, resulting in high complexity in DP-VNLE (which will be shown later). When the proposed equalizer is adopted, we can achieve a performance similar to the VNLE. This confirms that the proposed equalizer is highly effective in compensating for the nonlinear distortions arising from the DML's chirp and fiber dispersion.

Table I summaries the complexity of equalizers in terms of the number of multipliers. We can observe that the implementation complexity of the proposed equalizer increases linearly with L_2. Fig. 4(a) shows the number of multipliers required to implement the nonlinear equalizers. It shows that the proposed equalizer reduces the complexity considerably compared to other nonlinear equalizers. For example, it reduces the number of multipliers by >70%, compared to DP-VNLE (W=7) when L_2=11. Also, we also achieve an order of magnitude reduction in complexity vis-à-vis the VNLE. Fig. 4(b) shows the BER performance versus L_2. We can see that all the three equalizers exhibit almost the same performance when $L_2 \geq 7$. It is interesting to note that the proposed equalizer outperforms the VNLE when L_2 is small (e.g., <7). This confirms that the proposed equalizer specifically designed for DML-based IM/DD system compensates for the nonlinear distortions arising from DML's chirp and fiber dispersion very effectively.

4. Conclusions

We have proposed a low-complexity nonlinear equalizer specifically designed for DML-based IM/DD systems. It not only compensates for the waveform distortions induced by DML's adiabatic chirp and fiber dispersion, but also reduces the implementation complexity significantly in comparison with the conventional Volterra equalizers.

Acknowledgment: This work was supported by IITP grant funded by the Korean government (No. 2016-0-00083).

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