# Advanced Nonlinear Perturbation Theory in Coherent WDM Systems

**Amirhossein Ghazisaeidi**
*Nokia Bell Labs, route de villejust Paris-Saclay, 91620, Nozay, France*
*Amirhossein.ghazisaeidi@nokia-bell-labs.com*

**Abstract:** We review the theoretical efforts to develop models to analyze fiber-optic coherent systems using perturbation analysis. We start with models for the nonlinear signal-signal distortions and continue to address nonlinear signal-noise interactions and SOA-induced distortions.

## 1. Introduction

Terrestrial and submarine fiber optic transmission systems based on wavelength division multiplexing (WDM) and coherent detection provide the backbone of data traffic infrastructure around the world. To efficiently design and scale these links, system architects require accurate and fast models of system performance analysis. The propagation of the electromagnetic fields through optical fibers is governed by the nonlinear Schrodinger equation (NLS) [1], which, although integrable in special cases [2,3], does not admit general analytical solutions in the presence of chromatic dispersion, loss and amplified spontaneous emission (ASE) generated by in-line amplifiers. The system performance of such systems, quantified by the received signal-to-noise ratio (SNR), is limited by the ASE noise and the nonlinear distortions due to fiber Kerr nonlinear coefficient [4]. Over the past three decades substantial progress has been made to develop approximate analytical and semi-analytical models of the system performance of fiber-optics systems, using the well-known perturbation analysis of NLS. In this paper, we briefly review these efforts, and highlight the most recent advances in system modeling.

## 2 Nonlinear signal-signal distortions

In absence of ASE noise and random polarization fluctuations, the nonlinear distortions are deterministic and result from the four-wave mixing (FWM) among photons from various WDM channels. These signal-signal distortions can be either compensated, or else treated as if they were unstructured random noise. In the latter case the time average of the intensity of these distortions is considered as an equivalent nonlinear noise variance and is added to the ASE noise. Moreover, the presence of fiber-dispersion mixes the various time samples of the fields, hence, the nonlinear distortions can be approximately, but efficiently, characterized by Gaussian statistics due to central limit theorem. In order to analytically compute the variance of these equivalent Gaussian random variables, either time-domain or frequency-domain first-order regular perturbation theory (FRP) can be applied to the NLS equation, and then ensemble averaging over the unknown information symbols can be carried out (ergodicity is assumed). Refs. [5-8] present some of the pioneering works in this context. The first-full-fledged attempts to compute the total nonlinear variance of multi-span EDFA-amplified systems were presented in [9, 10] by applying the FRP in the frequency domain. In [10] the total WDM signal at the channel input was approximated by fine-grid discrete spectral lines modulated by circular complex Gaussian random variables, and then all the FWM terms among these lines were computed at the channel output, after propagation, and ensemble averaged to derive what is now known as the Gaussian noise (GN) model for the nonlinear variance. The GN model is fast, and the underlying assumptions are justified for long-distance dispersion-unmanaged systems; however, the assumption of Gaussian statistics at the channel input is too restrictive, in the sense that modulation-dependence of nonlinear variance is not captured by the GN model especially in the first spans and/or for low-dispersion systems. An alternative approach, based on calculating the FWM terms among optical pulses of various WDM channels using perturbation analysis in the time domain was pioneered in [6], and later on, was extended to a full model for signal-signal nonlinear distortions in [11-14]. This time-domain *pulse collision* approach has many advantages: it successfully captures the modulation-format dependent corrections and is valid even at short distances and low-dispersion and/or dispersion/managed systems. It allows the computation of the whole auto-correlation function of the signal-signal distortions [15]. It also provides the theoretical ground for efficient nonlinear equalizers [16-19]. Nonlinear distortions in spatially division multiplexed systems have been analyzed using this method [20-21]. Finally, it can be adapted to capture other more advanced nonlinear impairments beyond the signal-signal distortions, as will be discussed below. On the other hand, the final expressions for the variance and autocorrelation terms are in integral forms, which should be evaluated numerically by Monte Carlo integration. Although this computation is fast and efficient and is trivially parallelizable, compared to approximate closed form expression of GN model it is still one to two orders of magnitude slower.

In the mean while the GN model was also upgraded to the enhanced GN (EGN) model [22], which fixes the inaccuracies of GN model, but, like the pulse collision approach, is a semi-analytical model, *i.e*., requires numerical integration in the last step. Alternative approaches in analyzing the signal-signal distortions can be found in [23-25]

**3. Nonlinear signal-noise interactions**

Time domain perturbation techniques can be successfully applied to investigate nonlinear signal-noise interactions (NSNI), *i.e.*, the FWM between the signal photons and the distributed ASE [26, 27]. In particular, in [27] the pulse collision theory is extended to develop a computationally-efficient model of the NSNI. In the majority of modern links, and in the absence of nonlinear equalizers, the NSNI is negligible compared to signal-signal distortions; however, NSNI sets the fundamental limit on the performance of ideal nonlinear zero-forcing equalizer, *i.e.*, full-field digital back propagation [28] with arbitrarily fine spatial and temporal resolution. In the absence of copropagating ASE noise, such an equalizer forces the total signal-signal distortions to zero at the receiver side; however, the presence of copropagating ASE photons along with the signal triggers signal-noise FWM both in the physical link and inside the nonlinear equalizer. Noise analysis of nonlinear zero-forcing equalizer is theoretically important in that it provides improved lower bounds on the nonlinear Shannon capacity, which is still unknown as of today. It has been proved that the well-known linear Shannon capacity formula sets an upper bound on the nonlinear Shannon capacity [29]. Various lower bounds exist in the literature [30, 31]. In [27] and [32] we computed the zero-forcing lower bound limit of nonlinear Shannon capacity, assuming two-dimensional complex Gaussian symbols. Note that computing this limit via numerical simulations of nonlinear propagation is prohibitively complex and time-consuming even when GPUs are used. The rapidly computed lower bounds in [27,32], together with the Shannon linear capacity formula as the upper bound significantly reduce the uncertainty zone where the true unknown nonlinear Shannon capacity lies.

**4. Beyond EDFA amplification**

In all previous works discussed so far, the only optical amplification mechanism was that of lumped erbium-doped fiber amplifiers (EDFA) placed at the end of each span. The ever-increasing need for bandwidth has rekindled interest in ultrawideband systems (UWB). The first generation of UWB systems, based on C+L-band EDFA amplifiers or hybrid EDFA-Raman amplifiers have bandwidth up to 10 THz. Recently, UWB semiconductor optical amplifiers (SOA) have been proposed as in-line optical amplifiers capable of providing lumped gain over S, C, and L bands [33, 34]. The presence of optical amplifications other than EDFA poses new challenges for system performance analysis.

The mere presence of optical signal over bandwidths as large as 10 THz or more, triggers stimulated Raman scattering (SRS) in the optical fiber [1], which progressively tilts the spectrum, due to energy transfer from short wavelengths to long wavelengths and modifies the total variance of nonlinear distortions. Ref [35] extends the GN model to include the SRS.

Contrary to EDFAs, the SOAs have a rich nonlinear behavior, which makes them attractive building blocks for optical signal processing applications, but this enhanced nonlinear behavior is an inconvenience when it comes to using them as in-line optical amplifiers. Although the UWB in-line SOAs introduced in [33, 34] have improved nonlinear performance compared to conventional devices, it is of utmost importance to accurately model the residual impact of SOA nonlinear amplification on the total system performance. The carrier lifetime of the SOA is between tens of pico-seconds and one nano-second, whereas that of EDFA is of the order of mili-second; therefore, contrary to the EDFA the gain of the SOA is signal-dependent and fluctuating. Moreover, the phase and intensity of the optical field amplified by an SOA are coupled through a parameter called linewidth enhancement factor. Finally, there is a correlation between the nonlinear distortions generated in the SOA and in the fiber spans. In [36] the pulse collision theory was extended to a rigorous model for the analysis of nonlinear signal-signal distortions in SOA-amplified coherent WDM systems.

**5-Conclusion**

We reviewed the theoretical investigations dedicated to understanding and construction of fast and efficient system performance analysis models for multi-span fiber-optic transmission systems based on coherent WDM technology. Although the nonlinear propagation through an optical fiber is accurately modeled by nonlinear Schrödinger (NLS) equation, the general analytical solutions do not exist, and numerical simulations are time-consuming. We reviewed the accurate approximate analytical treatments based on various flavors of perturbation-analysis of NLS, to model not only the nonlinear signal-signal impairments, but also more delicate impairments stemming from nonlinear interaction between signal and spontaneous noise or from nonlinear gain dynamics of ultrawideband semiconductor optical amplifiers.

## 3. References

[1] G. P. Agrawal, ``Nonlinear Fiber Optics", Academic Press, 2007.
[2] M. I. Yousefi and F. R. Kschischang, ``Information Transmission Using the Nonlinear Fourier Transform, Part I: Mathematical Tools, " Trans Information theory, vol. 60, no. 7, pp. 4312-4328, (2014).
[3] V. Aref, et al., ``Demonstration of Fully Nonlinear Spectrum Modulated System in the Highly Nonlinear Optical Transmission Regime, " Proc. ECOC, PD.3.4, Dusseldorf, Germany, (2016).
[4] R. J. Essiambre, et al., ``Capacity Limits of Optical Fiber Networks, " JLT, vol. 28, no. 4, (2010).
[5] A. Splett, et al. ``Ultimate transmission capacity of amplified optical fiber communication systems taking into account fiber nonlinearities, " Proc. ECOC, vol. 2, pp.41-44 (1993).
[6] A. Mecozzi, et al., ``System impact of intra-channel nonlinear effects in highly dispersed optical pulse transmission, " PTL,. vol. 12, no. 12, pp. 1633-1635, (2000).
[7] J. Tang et al., ``The channel capacity of a multispan DWDM system employing dispersive nonlinear optical fibers and an ideal coherent optical receiver, ", JLT, vol. 20, no. 7, pp. 1095-1101, (2002).
[8] A. Vannucci, et al.``The RP method: A new tool for the iterative solution of the nonlinear Schrodinger equation, " JLT, vol. 20, no. 7, pp. 1102–1112, (2002).
[9] X. Chen and W. Shieh, ``Closed-form expressions for nonlinear transmission performance of densely spaced coherent optical OFDM systems, " Opt. Express., vol. 18, pp. 19039–19054, (2010).
[10] P. Poggiolini, et al., ``Analytical Modeling of Nonlinear Propagation in Uncompensated Optical Transmission Links, " PTL, vol. 23, no. 11, pp. 742-744, 2011.
[9] P. Poggiolini, et al.``The GN-Model of Fiber Non-Linear Propagation and its Applications, " JLT, vol. 32, no. 4, pp. 694-721, (2014)
[11] A. Mecozzi and R. J. Essiambre, ``Nonlinear Shannon Limit in Pseudolinear Coherent Limit, " JLT, vol. 30, no. 12, pp. 2011-2014 (2012).
[12] R. Dar, et al., ``Properties of nonlinear noise in long dispersion-uncompensated fiber links, " Opt Express, vol. 21, no. 22, pp. 25685-25699, (2013).
[13] R. Dar, et al., ``Inter-Channel Nonlinear Interference Noise in WDM Systems: Modeling and Mitigation, " JLT, vol. 33, no. 5, pp. 1044-1053, (2015).
[14] R. Dar, et al., ``Pulse Collision Picture of Inter-Channel Nonlinear Interference in Fiber-Optic Communications, " JLT, vol. 34, no. 2, pp. 593-607, (2016).
[15] O. Golani, et al., ``Modeling the Bit-Error-Rate Performance of Nonlinear Fiber-Optic Systems, " JLT, vol. 34, pp. 3482-3489 (2016).
[16] J. C. Cartledge et al., ``Digital signal processing for fiber nonlinearities, " Opt. Express, vol. 25, no. 3, pp. 1916-1936, (2016).
[17] Z. Tao, et al. ``Multiplier-Free Intrachannel Nonlinearity Compensating Algorithm Operating at Symbol Rate, " JLT, vol. 29, no. 17, pp. 2570, (2011).
[18] A. Ghazisaeidi and R. J. Essiambre, ``Calculation of Coefficients of Perturbative Nonlinear Pre-Compensation for Nyquist Pulses, " Proc. ECOC, We.1.3.3, Cannes, France, (2014).
[19] R. Dar and P. J. Winzer, ``Nonlinear Interference Mitigation: Methods and Potential Gain, " JLT. (2017).
[20] A. Mecozzi et al., "Coupled Manakov equations in multimode fibers with strongly coupled groups of modes, " Opt. Express, vol. 20, no. 21, pp. 23436-23441, (2012).
[21] A. Mecozzi et al., "Nonlinear propagation in multi-mode fibers in the strong coupling regime, " Opt. Express, vol. 20, no. 11, pp. 11673-11678, (2012).
[22] A. Carena, et al, ``EGN model of non-linear fiber propagation, " Optics. Express, vol. 22, no. 13 pp.16335-16362, (2014).
[23] P. Serena and A. Bononi, ``A Time-Domain Extended Gaussian Noise Model, " JLT. vol. 33, no. 7, pp. 1459-1472, (2015).
[24] P. Johannisson and M. Karlsson, ``Perturbation Analysis of Nonlinear Propagation in a Strongly Dispersive Optical Communication Systems, " JLT. vol. 31, no. 8, pp. 1273-1282, (2013).
[25] M. Secondini and E. Forestieri, "Analytical Fiber-Optic Channel Model in the Presence of Cross-Phase Modulation, " PTL, vol. 24, no. 22, pp. 2016-2019 (2012).
[26] P. Serena, ``Nonlinear Signal Noise Interaction in Optical Links With Nonlinear Equalization, " JLT, vol. 34, no. 6, pp. 1476-1483, (2016).
[27] A. Ghazisaeidi, ``A theory of nonlinear interactions between signal and amplified spontaneous emission noise in coherent wavelength division multiplexed systems, JLT, vol. 35, no. 23, pp. 5150-5175, (2017).
[28] E. Ip and J. Kahn, ``Compensation of Dispersion and Nonlinear Impairments Using Digital Backpropagation, " JLT, vol. 26, no. 20, pp. 3416-3425 (2008).
[29] G. Kramer, et al., ``Upper bound on the capacity of a cascade of nonlinear and noisy channels, " IEEE Information Theory Workshop (ITW) (2015).
[30] M. Secondini, et al., ``Achievable Information Rate in Nonlinear WDM Fiber-Optic Systems With Arbitrary Modulation Formats and Dispersion Maps, " JLT, vol. 31, no. 23, pp. 3839-3852, (2013).
[31] K. Keykhosravi, et al., ``A Tighter Upper Bound on the Capacity of the Nondispersive Optical Fiber Channel, " Proc. ECOC, Goteborg, Sweden, 2017.
[32] A. Ghazisaeidi, "Noise Analysis of Zero-Forcing Nonlinear Equalizers for Coherent WDM Systems, " JLT, vol. 37, no. 6, pp. 1552-1559, (2019).
[33] J. Renaudier and A. Ghazisaeidi, "Scaling Capacity Growth of Fiber-Optic Transmission Systems Using 100+nm Ultra-Wideband Semiconductor Optical Amplifiers, " JLT, vol. 37, no. 8, pp. 1831-1838 (2019).
[34] J. Renaudier et al., "First 100-nm continuous-band WDM transmission system with 115Tb/s transport over 100km using novel ultra-wideband semiconductor optical amplifiers, " Proc ECOC, Goteborg, Sweden, 2017.
[35] D. Semrau et al., "The Gaussian noise model in the presence of inter-channel stimulated Raman scattering, " JLT, vol. 36, no. 14, pp. 3046-3055, (2018).
[36] A. Ghazisaeidi, "Theory of Coherent WDM Systems Using in-Line Semiconductor Optical Amplifiers, " JLT, vol. 37, no. 17, (2019)