# Low-Complexity PDLUT Based on Symmetry and Unification for High-Order Modulation Format

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**Abstract** *We propose a low-complexity PDLUT approach employing symmetry property and unification for 64QAM. Compared to the conventional PDLUT, the proposed approach reduces LUT size by up to 89% with negligible performance penalty.*

## Introduction

Nonlinear distortions induced by transmitter components, including digital-to-analogue converter (DAC), radio-frequency (RF) amplifiers (modulator driver) and Mach-Zender modulator, have been recognized as one of the major impairments that limit the transceiver performance, especially for high-order modulation formats at high baud rates. The nonlinear effects are coupled with the linear response of individual components, which results in a strong pattern dependency with a considerable memory length.

Pattern-dependent look-up table (PDLUT) approach has shown good performance in compensating the transmitter nonlinearities [2]. For a given memory length, the PDLUT size increases exponentially with the modulation levels. For example, for a memory size of 3, the PDLUT size for 16QAM is 64, while for 64QAM, the size increases to 512. Therefore, it requires enormous resources to implement PDLUT for 64QAM or higher order QAM, which in turn increases dramatically the complexity of ASIC implementation. Several approaches have been proposed to reduce the LUT size [3-5]. However, the performance penalty is relatively large compared to the conventional PDLUT. Reducing the PDLUT size while maintaining the compensation performance for high-order QAM signals is highly desired. In this paper, we propose a low-complexity PDLUT method based on symmetry and unification (PDLUT_SU) to reduce LUT size with negligible performance penalty.

## Principle of PDLUT based on symmetry and unification

We take 64QAM with a memory length of 3 as an example to illustrate the principle of the proposed PDLUT_SU approach. Fig.1 shows the conventional PDLUT and its corresponding patterns in a specific order. The PDLUT is obtained by calibrating a coherent transmitter. The PDLUT size is $8^3$=512 points. We can see from the figure that the LUT is roughly symmetric about the centre point. That is because the sign of the patterns are opposite about the centre point. For example, the 1st pattern [-7 -7 -7] is opposite to the last pattern [7 7 7], the 2nd pattern [-7 -5 -7] is opposite to the 2nd last pattern [7 5 7], and etc.

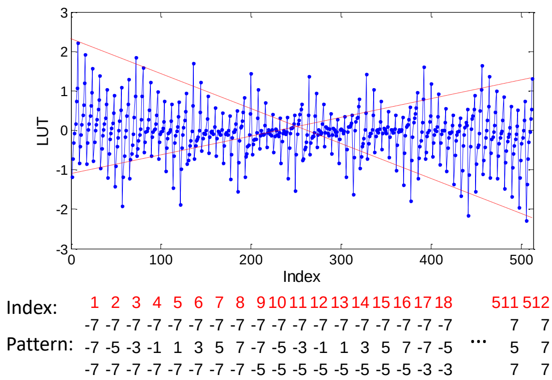


**Fig. 1: A typical PDLUT of 64QAM with memory size of 3.**

After folding and taking the opposite, the two half-LUTs overlap with each other, as shown in Fig. 2. That means we can use one half-LUT to represent the other half-LUT. Taking advantage of the symmetry, the LUT size could be reduced by half.

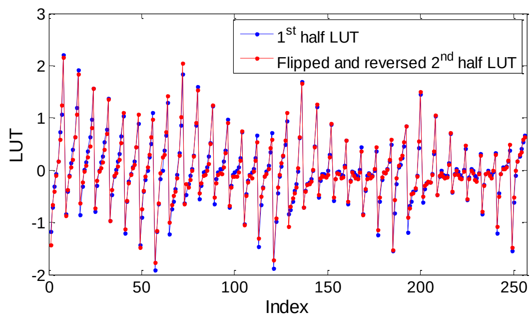


**Fig. 2: PDLUT after folding and doing reverse.**

The folded half-size LUT can be further divided into 4 groups (with corresponding index: 1-64, 65-128, 129-192, and 193-256) and each group is symmetric about its centre point. This observation is also from the amplitude symmetric characteristics of the nonlinear pattern distortion: after fixing the left end symbol, the centre and right symbols of the 1st pattern [-7 -7 -7] and the last pattern [-7 7 7] of group 1 are opposite. Therefore, their distortions are opposite and this observation is the 2nd symmetry. After the 2nd folding and taking the opposite, the two half-sub-LUTs overlap with each other, as shown in Fig. 3. By using the 2nd symmetry, the LUT size could be reduced to 1/4 of its original, from 512 to 128.

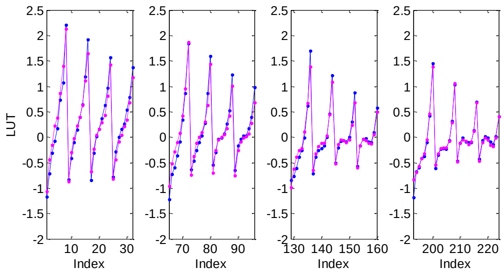

**Fig. 3: PDLUT after the second folding and reverse.**

**Tab. 1: Mapping table.**

| | Group1 | Group2 | Group3 | Group 4 |
|---|---|---|---|---|
| Curve Index | 4 | 0 | 7 | 14 |
| a | 0.85 | 1.21 | 0.84 | 1.02 |
| b | -0.08 | -0.20 | -0.03 | 0.03 |
| Curve Index | 8 | 2 | 9 | 15 |
| a | 0.81 | 0.84 | 1.09 | 0.71 |
| b | -0.19 | 0.01 | -0.17 | -0.01 |
| Curve Index | | 3 | 10 | |
| a | | 0.74 | 0.74 | |
| b | | -0.02 | -0.08 | |
| Curve Index | | 5 | 13 | |
| a | | 0.83 | 0.92 | |
| b | | -0.14 | -0.18 | |

Next, we propose to further reduce the LUT size by unification. The simplified LUT can be treated as 16 curves, as shown in Fig. 4. Each curve includes 8 points which corresponds to the under-correct symbol varying from -7 to 7. From the figure we can see that some curves share similar patterns, therefore, the curves can be classified into several groups based on their similarity. For each group, one curve is selected as a base curve while the others are represented by the base curve with certain stretching and bias,

$$S = aS_{base} + \mathrm{b}$$

where $a$ is a scaling factor which stretches the base curve while b being the bias. Assuming 8 points in one curve, $a$ and b can be learned using linear regression in a supervised learning way by mapping the 8 points and that of a base curve in the same group.

To divide the curves without a symmetric property into several groups (e.g. 16 curves in Fig. 4), we proposed an unsupervised learning approach with two steps. Firstly, point magnitudes in each curve are considered as features. A dimension reduction algorithm, principal component analysis (PCA), was used to reduce multi features (e.g. 8 features in each curve in Fig. 4) into two features as shown in Fig. 5(a). Secondly, an unsupervised learning method, K-means, was used to group all curves represented by features into several clusters. In this case, the 16 curves can be classified into four groups, as shown in Fig. 5 (b). If we use more groups for clustering, the performance will be slightly better but LUT size will increase. The number of groups could be determined by a trade-off between performance and complexity.

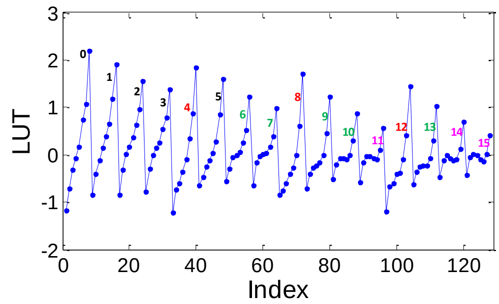

**Fig. 4: 128-point LUT is denoted by 16 curves.**

In each group of curves, the base-curve selection is achieved by blind search. We choose the curve which has the least fitting error with other curves as a base curve. The base curves and their corresponding mapping table (a, b) are shown in Fig. 6 and Tab. 1. The 4 base curves contain 32 points, and the mapping table (a, b) contain (16-4)×2=24 values. Thus, the 128-point simplified LUT is further compressed to a 56-point (32+24=56) LUT.

**Tab. 2: LUT size comparison.**

| | Size | Size Reduction |
|---|---|---|
| PDLUT(3) | 512 | Baseline |
| PDLUT(3)_SU | 56 | 89% |
| PDLUT(3+2) | 576 | Baseline |
| PDLUT(3+2)_SU | 126 | 78% |

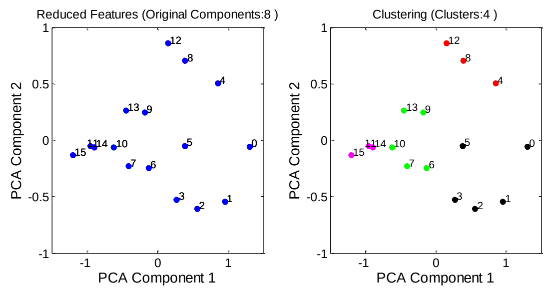


**Fig. 5: (a) Reduced features of 16 curves in Fig. 4 after PCA (b) Four groups of curves after K-means clustering.**

## Principle of Cascaded PDLUT_SU

The symmetry and unification method can also be used for cascaded PDLUT [6]. The cascaded PDLUT uses two smaller LUTs instead of single large one. One LUT only contains the nonlinear effect from the left-side symbols (or precedent symbols), and the other LUT only contains the nonlinear effect from the right-side symbols (or latter symbols). By applying the two LUTs successively, the distortion affected by both sides is compensated. In this way, a long-memory LUT is replaced by two short-memory LUTs, which results in a reduced total size. The size or memory of the two LUTs could be different, depending on the device nonlinearity property. PDLUT(m+n) denotes a cascaded PDLUT approach that has one LUT with a memory of m and another LUT with a memory of n.

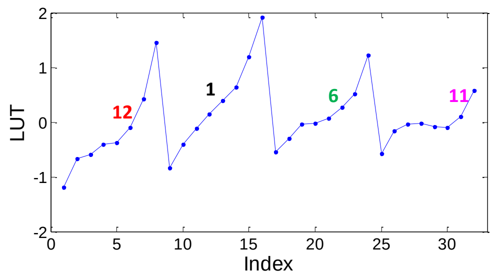


**Fig. 6: Four base curves for the 4 groups.**

It should be noted that the PDLUT(m+n) and PDLUT(n+m) has the same LUT size but the performance is different due to inter-symbol interference (ISI) asymmetry caused by the nonlinearity. The one with better performance will be selected.

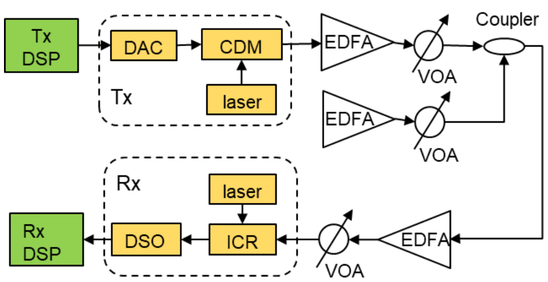


**Fig. 7: Experimental setup**

Taking PDLUT(3+2) as an example, the total LUT size is 512+64=576. The two cascaded LUTs are both symmetric about each centre point. By applying this symmetry, the LUT size could be reduced to half of its original (512+64 to 256+32). Unfortunately, there's no second symmetry for cascaded LUTs. But the larger table with a size of 256 can be further simplified using the same unification method. Five groups are used in K-means clustering. The 256-point LUT is compressed to 94-point LUT after the unification step.

The complexity comparison is summarized in Tab. 2. Compared to the original PDLUT, the proposed PDLUT_SU reduces size by 89% for a single LUT with a memory of 3 and by 78% for a cascaded LUT with a memory of (3+2).

## Experimental Verification

Fig. 7 shows the experimental setup of 64GBd 64QAM. The digital signals are up-sampled and pre-compensated in transmitter DSP, and then converted to analogue signal by DAC. A combined driver modulator (CDM) is used to modulate the data signal at a wavelength of 1548.5nm. In the experiment, to generate different optical signal-to-noise ratio (OSNR) values, an erbium-doped fibre amplifiers (EDFA) is used as ASE noise source and a variable optical attenuators (VOA) is used to adjust the OSNR. After being detected by integrated coherent receiver (ICR), the signals are sampled digitally by a digital storage real-time oscilloscope (DSO). An offline DSP platform is used to process the digital signals to characterize PDLUT compensation performance. The PDLUT is calibrated by comparing the received and transmitted ideal patterns. Multiple data traces with different data loading are used in the calibration to improve the LUT accuracy and avoid over fitting. Fig. 8 shows the performance of the proposed PDLUT_SU method based on offline test bed. Compared to original PDLUT (3) and PDLUT(3+2), the required OSNR (at a BER of 4e-2) penalty of the proposed PDLUT(3)_SU and PDLUT(3+2)_SU is negligible (less than 0.05dB). The results confirmed that the PDLUT size for 64 QAM can be reduced significantly (up to 90%) with a negligible compensation performance penalty. It worth noting that the PDLUT based on symmetry and unification can be implemented any high-order modulation formats like 128 QAM and 256QAM.

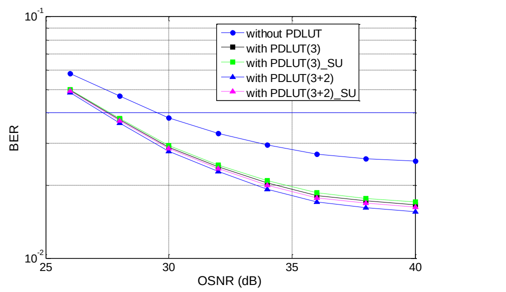


**Fig. 8: Performance comparison for different PDLUT implementations.**

## Conclusion

The proposed low-complexity PDLUT method based on symmetry and unification greatly reduces PDLUT size and provides an efficient transmitter nonlinearity compensation solution for 64QAM or higher modulation format. Compared to the conventional PDLUT, the proposed PDLUT_SU reduces size by 89% for single LUT with a memory of 3 and by 78% for cascaded LUT with a memory of (3+2), with less than 0.05dB required OSNR penalty.

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