Analysis of the Scalar and Vector Random Coupling Models For a Four Coupled-Core Fiber

Ekaterina Deriushkina(1), Jochen Schröder(1), Magnus Karlsson(1)

(1) Photonics Laboratory, Chalmers University of Technology, SE-41296 Gothenburg, Sweden, ekader@chalmers.se

Abstract *We present an analytic comparative study of the scalar and vector random coupling models for a four coupled-core fiber. ©2023 The Author(s)*

Introduction

Coupled-core fibers (CCFs) have become of the most prominent among of all types of space division multiplexing (SDM) approaches due to the reduced accumulation of differential group delay (DGD) with the fiber length[1] and higher tolerance to nonlinearities because of larger effective areas. Application of CCFs in transmission links relies on accurate estimation of relevant link parameters using the modeling and simulations for analyzing not only an ideal, unperturbed fiber, but also how the random coupling regime impacts the studied characteristics.

Characterization of coupling effects in unperturbed CCFs is usually performed by coupled-mode theory[2]. One of the most studied concepts within this theory are the supermodes, the spatial eigenmodes of the superstructure that retain their shape during the propagation, which can be beneficial for transmission. Supermodes and their propagation constants were extensively studied for different symmetry structures of CCFs[3], and particularly, for a four coupled-core fiber (4CCF)[4]. However, polarization of the modes was neglected in these studies. Hereinafter, this case is referred to as a scalar model/case, while the case where the birefringence is considered will be referred to as the vector model/case.

Random coupling models in different types of CCFs were recently discussed in[5]. The fiber in these models is described by a concatenation of segments of constant length with given random bending curvatures. Polarization mixing can be taken into account by a rotation matrix characterizing the fiber twist, which is incorporated between each segment[6]. However, these studies lack the comparison between scalar and vector cases, which can be beneficial in analyzing conditions when the polarization effects have a strong impact on the investigated features.

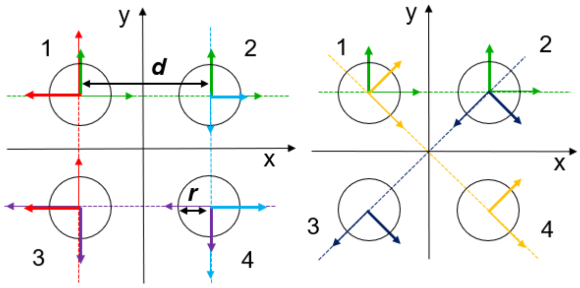

Fig. 1: Illustration of birefringence axes in a 4CCF, which manifests a combination of the 2CCF structure (green axes) with its $-90°$ (illustration on the left) and $-135°$ (illustration on the right) degree-rotated instances. The numbers $1, 2, 3, 4$ relate to the enumeration of the cores, d is the core pitch between the nearest cores, r is a core radii.

In this work we present a comparative analysis of the scalar and vector random coupling models for a 4CCF in the ideal case and random coupling regime. We show for the first time, to the best of our knowledge, analytically calculated supermodes and their propagation constants for a 4CCF in a polarization multiplexed case.

Supermodes and group delays in the unperturbed 4CCF

According to the coupled-mode analysis, the lossless interaction between the modes of the individual cores in a CCF can be described as $\frac{d}{dz}\vec{A} = -i\mathbf{M}\vec{A}$, where $\vec{A} = \left(A_1...A_D\right)^T$ is the complex amplitude of the electrical field with D components modeling the complex amplitude of the light in each core and $\mathbf{M}$ is the $D \times D$ coupling matrix. The solution then takes the form $\vec{A} = \mathbf{T}\vec{A}_0$, where $\mathbf{T}$ is the transfer matrix of a fiber and $\vec{A}_0$ is an input amplitude vector. In a CCF with no spatial randomness, the transfer matrix of a fiber of length L is given by $\mathbf{T} = \exp(j\mathbf{M}L)$.

The 4CCF supermodes and their propagation constants in the scalar case, where $D = 4$ and A_k refer to core k as numbered in Fig 1, were thoroughly discussed in Ref.[4]. The reader is referred to it to get more details. In the vector model, where $D = 8$, A_{2k-1}, A_{2k} refer to the x,y polariza-

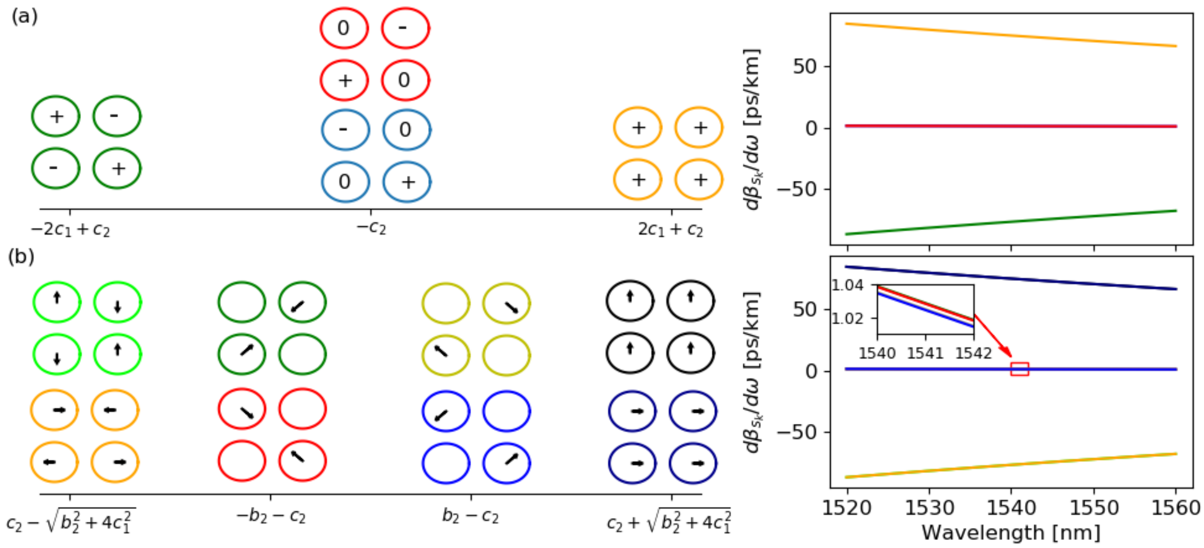

Fig. 2: Supermodes, their propagation constants $\beta_{s_k}(\omega)$ (as indicated on the axes) and derivatives of the propagation constants (right panels) of the 4CCF in case when (a) only the coupling effects take place (scalar model), (b) the coupling and birefringence are taken into account (vector model). Line colors of $d\beta_{s_k}/d\omega$ mark corresponding supermodes.

tion in core k and the coupling matrix $\mathbf{M}$ is then an 8×8 matrix that can be written in the form

$$\mathbf{M}(\omega)=\begin{pmatrix}\mathbf{B_1} & \mathbf{C_1} & \mathbf{C_1} & \mathbf{C_2}\\ \mathbf{C_1} & \mathbf{B_2} & \mathbf{C_2} & \mathbf{C_1}\\ \mathbf{C_1} & \mathbf{C_2} & \mathbf{B_3} & \mathbf{C_1}\\ \mathbf{C_2} & \mathbf{C_1} & \mathbf{C_1} & \mathbf{B_4}\end{pmatrix} \qquad (1)$$

where $\mathbf{B_1},\mathbf{B_2},\mathbf{B_3},\mathbf{B_4}$ are 2×2 birefringence matrices in every core describing the birefringence effects from the neighboring cores. This is the so-called *form birefringence* described in[7]. Matrices $\mathbf{C_1}=\begin{pmatrix}c_1 & 0\\ 0 & c_1\end{pmatrix}$ and $\mathbf{C_2}=\begin{pmatrix}c_2 & 0\\ 0 & c_2\end{pmatrix}$ characterize the coupling between adjacent and diagonal cores accordingly, and $c_1 \gg c_2$. We assume that orthogonal polarizations between modes do not couple. The relation of the coupling coefficient c to the physical parameters of step-index cores can be found in[8].

The birefringence matrix of two parallel cores is $\mathbf{B}=\begin{pmatrix}b_x & 0\\ 0 & b_y\end{pmatrix}$[8] and we can assume that $b_y=-b_x=b$. The expression for b and its dependence on fiber parameters and frequency can be found in[7],[8]. The birefringence for the full 4CCF can be described using a superposition of birefringences from pairwise linear-array structures as shown in Fig. 1. It should be noted that in this case there will be two types of birefringence matrices, representing the adjacent cores $\mathbf{B}=\begin{pmatrix}-b_1 & 0\\ 0 & b_1\end{pmatrix}$ and the diagonal cores $\mathbf{B}'=\begin{pmatrix}-b_2 & 0\\ 0 & b_2\end{pmatrix}$. The contribution from the adjacent cores $\mathbf{B}$ will cancel out in the 4CCF case, and the final birefringence matrices becomes $\mathbf{B_1}=\mathbf{B_4}=-\mathbf{B_2}=-\mathbf{B_3}=\begin{pmatrix}0 & b_2\\ b_2 & 0\end{pmatrix}$.

Combining all the calculated birefringence and coupling matrices in Eq. 1 it is possible to construct the coupling matrix $\mathbf{M}(\omega)$ and find the supermodes and their propagation constants by solving the eigenvalue problem for this matrix.

The supermodes and their associated propagation constants $\beta_{s_k}(\omega)$ for vector and scalar cases are shown in Fig. 2. The parameters used for this calculation are $r=4.75\ \mu m$, $d=22.5\ \mu m$, index difference $\Delta=0.44\%$, the refractive index is found by using Sellmeier formulas for SiO_2[9]. We find that in case of the scalar model there are four supermodes, two of which are degenerate (have the same propagation constant). In case of the vector model, there will be eight supermodes, four of which are degenerate.

The group delays (GDs) scale with the derivative of the propagation constant over the frequency as $\tau_k=\frac{d\beta_{s_k}(\omega)}{d\omega}L$. As can be seen from the plots for $d\beta_{s_k}(\omega)/d\omega$ in Fig. 2, the vector and scalar models give very similar results. The reason for this is that the birefringence is usually much weaker in CCFs than the coupling effects and does not impact the calculation result from the scalar case significantly.

Random coupling model

A random propagation in a realistic CCF can be modeled by applying a concatenation rule originated from the PMD calculus, which allowed the determination of the PMD vector of an assembly of concatenated fiber segments when the PMD

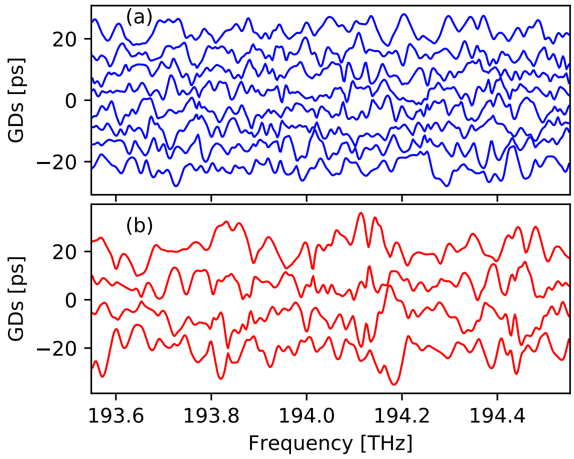

Fig. 3: The GDs of the 4CCF calculated for (a) vector and (b) scalar random coupling models, $N = 200$.

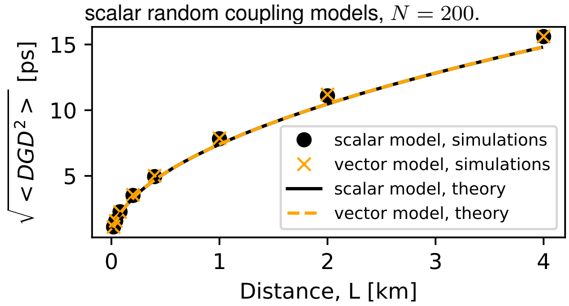

Fig. 4: Calculated dependence of averaged DGD on propagation distance.

vectors of the individual segments are known[10]. A piece of length of a CCF is modeled with the delay matrix $\mathbf{T_d} = \exp(j\mathbf{M}(\omega)L)$. The randomness in real CCFs is likely a variation in core size and separations along the fiber due to manufacturing imperfections and it is modeled as a random unitary matrix $\mathbf{U}$ that has the Haar distribution and can be realized using QR-factorization[11],[12]. The random coupling in a CCF can thus be modeled as a concatenated sequence of N such matrices giving a total transfer matrix $\mathbf{T_{tot}} = \mathbf{T_{d,N}} \cdot \mathbf{U_N} \cdot ... \cdot \mathbf{T_{d,1}} \cdot \mathbf{U_1}$, which has to be unitary, as it is a product of unitary matrices.

Group delays, DGD and impulse response of the 4CCF in the random coupling regime

The GDs calculated as eigenvalues λ_i of the delay operator $\mathbf{D} = -j\mathbf{T}^{\dagger}_{\mathbf{tot}} \frac{d\mathbf{T_{tot}}}{d\omega}$ in the range 1542-1550 nm are shown in Fig. 3. These GDs were estimated for one realization and for $N = 200$ concatenations, while having a constant element (concatenation) length of $l = 20$ m. They show random, but periodic behavior and notably, do not overlap. The reason for that can be that the delay operator is a random Gaussian matrix, and since the probability for such a matrix to have two identical eigenvalues is negligibly small, the GDs does not coincide[13].

The average DGD can be calculated analytically as $\langle DGD^2 \rangle = N \sum_{i=1}^{D} \lambda_i^2 / D$, where λ_i are the eigenvalues of the individual delay elements which are taken from the deterministic model given previously. The agreement with simulations is shown in Fig. 4. It is clearly seen that the curves manifest a square root behavior on propagation distance, as expected for CFFs.

The total power intensity impulse response (IIR) of the 4CCF calculated using the vector model is shown in Fig. 5. A linearly polarized input pulse in core 1 is used, with blue (orange) representing x (y)-polarization. As can be seen, the shape of the IIR depends strongly on the input state of polarization, which is surprising since the form birefringence is much smaller - even negligible - than the core coupling in the deterministic model. However, in the random model the polarization effects still manifest. On average though, polarization does not impact the IIR and the IIR shape becomes Gaussian, in agreement with[14]. These results agree well with measured impulse responses of the 4CCF[15], and with eigenvalue analysis of these measurements[16].

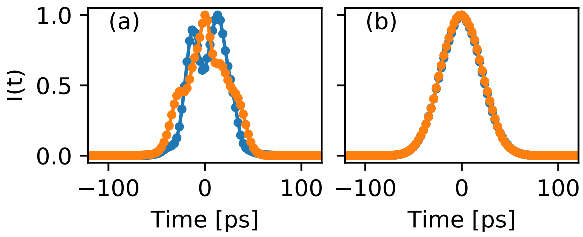

Fig. 5: Normalized total power IIR calculated for $N = 20$ for the vector model in case of (a) 1 realization and (b) averaged over 200 realizations. Orange and blue lines are related to the instances where a Gaussian pulse is injected to x- and y-polarizations respectively.

Conclusions

In conclusion, we presented a comparative analysis of the scalar and vector random coupling models for a 4CCF. Vector supermodes for an unperturbed 4CCF were presented analytically for the first time.

Simulation files within this work are accessible in Ref.[17].

Acknowledgements

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