Approximate Maximum a Posteriori Carrier Phase Estimator for Wiener Phase Noise Channels using Belief Propagation

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Abstract *The blind phase search (BPS) algorithm for carrier phase estimation is known to have sub-optimal performance for probabilistically shaped constellations. We present a belief propagation based approximate maximum a posteriori carrier phase estimator and compare its performance with the standard and an improved BPS algorithm. ©2023 The Author(s)*

Introduction

Probabilistic amplitude shaping (PAS) is being investigated as a way to improve performance of fiber-optic communication systems [1]–[4]. PAS allows to finely adapt the information rate and has also been demonstrated to have improved tolerance to the fiber nonlinearity [3], [5]–[7]. In recent works [8]–[11], it was shown that the nonlinear shaping gain from probabilistic shaping is reduced in the presence of a carrier phase estimation (CPE) system. A CPE system is essential in practical systems as it is required to correct the phase noise arising from non-ideal lasers. CPE also appears to aid in nonlinearity compensation as the fiber nonlinearity manifests partly as a phase noise on the received symbols [12]. Hence, it is beneficial to design new CPE systems or modify existing CPE systems to aid in nonlinearity mitigation [13].

The blind phase search (BPS) algorithm [14] is a standard feed-forward CPE algorithm used in fiber-optic communication systems [8]. By design, the BPS algorithm does not take into account the channel parameters and transmitted symbol probabilities. While this makes the BPS algorithm versatile and easy to implement, its performance is sub-optimal for probabilistically shaped constellations [15]. The maximum a posteriori (MAP) CPE for the Wiener phase noise channel was investigated in [16]. The BPS algorithm is essentially a simplification of the MAP CPE. It was shown that the MAP CPE has a superior residual phase noise performance compared to the BPS algorithm. However, a Monte Carlo integration method was used in [16] to implement the MAP CPE which has an enormous computational cost, rendering it impractical.

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In this paper, we demonstrate that an approximate MAP CPE for the Wiener phase noise channel can be implemented using belief propagation (BP) at a reasonable cost. We show that it has improved bit-wise mutual information (BMI) performance compared to the BPS algorithm. The improvement however cannot always be justified by the increased computational cost. Hence as a low-complexity alternative we explore machine learning based improvements to the BPS algorithm and show that the performance of the BPS algorithm can be improved considerably with minimal changes.

MAP CPE

We work with the Wiener phase noise channel which has the following discrete model

$$y_k = x_k \mathrm{e}^{\mathrm{j}\varphi_k} + n_k, \tag{1}$$

where $x_k \in \mathcal{X} \subset \mathbb{C}$ are transmitted symbols with probability distribution $P(x)$ and $y_k \in \mathbb{C}$ are the received symbols. The symbol x_k at time instant k is affected by phase noise modeled as $\varphi_k = \varphi_{k-1} + \theta_k,\ \theta_k \sim \mathcal{N}(0, \sigma_\theta^2)$ and circular additive-white Gaussian noise (AWGN) $n_k \sim \mathcal{CN}(0, \sigma_n^2)$. Given $2N+1$ received symbols $\boldsymbol{y} = [y_{k-N}, \dots, y_k, \dots, y_{k+N}]$, the MAP phase estimate [16] is given by $\hat{\varphi}_k = \arg\max_{\varphi_k} P(\varphi_k|\boldsymbol{y})$.

$$P(\varphi_k|\boldsymbol{y}) = \int \cdots \int \prod_{i=k-N}^{k+N} R(y_i, \varphi_i) Q(\varphi_i|\varphi_{i-1}) \, \mathrm{d}\varphi_{k-N} \cdots \mathrm{d}\varphi_{k-1} \mathrm{d}\varphi_{k+1} \cdots \mathrm{d}\varphi_{k+N} \tag{2}$$

where $R(y_i, \varphi_i) = \sum_{x \in \mathcal{X}} P(y_i|x, \varphi_i) P(x)$ with $P(y_i|x, \varphi_i) \propto \exp\left(-\frac{|y_i - x\mathrm{e}^{\mathrm{j}\varphi_i}|^2}{2\sigma_n^2}\right)$ due to the AWGN and $Q(\varphi_i|\varphi_{i-1}) \propto \exp\left(\frac{-(\varphi_i - \varphi_{i-1})^2}{2\sigma_\theta^2}\right)$ due to the phase noise.

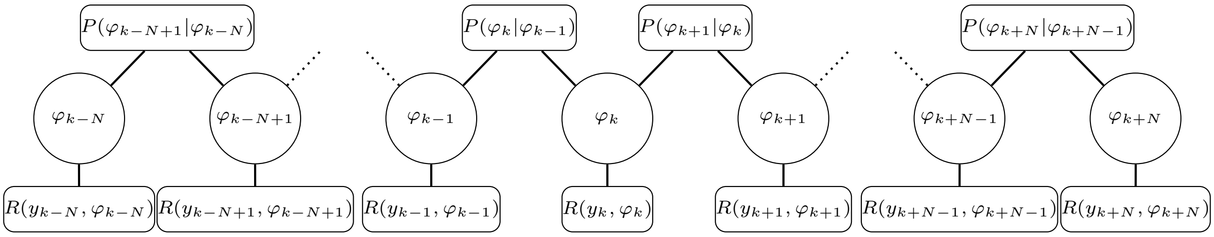

Fig. 1: Factor graph of the product term in Eq. 3

For numerical implementation, we assume that the φ_k can only take on a finite set of phase values in $\boldsymbol{\phi} = \{\phi_1, \phi_2, \ldots, \phi_M\}$. Then the approximate MAP estimator is given by

$$\hat{\varphi}_k = \arg\max_{\varphi_k \in \boldsymbol{\phi}} \sum_{\varphi_{k-N}\in\boldsymbol{\phi}} \cdots \sum_{\varphi_{k-1}\in\boldsymbol{\phi}} \sum_{\varphi_{k+1}\in\boldsymbol{\phi}} \cdots \sum_{\varphi_{k+N}\in\boldsymbol{\phi}} \prod_{i=k-N}^{k+N} R(y_i, \varphi_i) P(\varphi_i|\varphi_{i-1}). \tag{3}$$

Assuming phase noise remains constant in the window of $2N+1$ symbols and equiprobable transmit symbols, considering only the constellation symbol that provides the largest contribution in $P(y_i|x,\varphi_i)$ yields the BPS algorithm:

$$\begin{aligned}\hat{\varphi}_k &= \arg\max_{\varphi\in\boldsymbol{\phi}} \prod_{i=k-N}^{k+N} \mathrm{e}^{-\frac{|y_i - \hat{x}_n \mathrm{e}^{\mathrm{j}\varphi}|^2}{2\sigma_n^2}} \\ &= \arg\min_{\varphi\in\boldsymbol{\phi}} \sum_{i=k-N}^{k+N} |y_i - \hat{x}_n \mathrm{e}^{\mathrm{j}\varphi}|^2, \end{aligned} \tag{4}$$

where $\hat{x}_n = \arg\min_{x\in\mathcal{X}} |y_n - x\mathrm{e}^{\mathrm{j}\varphi}|^2$.

Implementation of MAP CPE using BP

The MAP phase estimation problem in (3) is a marginalization problem which can be implemented efficiently using belief propagation. We can represent the product term in (3) as the factor graph in Fig. 1. As the graph in Fig. 1 is a tree, the marginalization problem can be solved exactly by applying the sum-product algorithm on the respective graph [17]. The specific algorithm for obtaining the phase estimate $\hat{\varphi}_k$ in (3) is given in Alg. 1.

Algorithm 1 Approximate MAP phase estimation using BP

Require: $\boldsymbol{y}$, $\boldsymbol{\phi}$, $R(y_i, \varphi_i)$, $Q(\varphi_i|\varphi_{i-1})$
  $\boldsymbol{Q} \leftarrow Q(\boldsymbol{\phi}|\boldsymbol{\phi}) \in \mathbb{R}^{M\times M}$
  $\boldsymbol{m}_- \leftarrow (1,1,\ldots,1)^\intercal \in \mathbb{R}^M$
  $\boldsymbol{m}_+ \leftarrow (1,1,\ldots,1)^\intercal \in \mathbb{R}^M$
  for $i = k-N$ to $k-1$ **do**
    $\boldsymbol{m}_- \leftarrow \boldsymbol{Q}\,(\boldsymbol{m}_- \odot R(y_i, \boldsymbol{\phi}))$
    ▷ $\odot$ Hadamard product
  end for
  for $i = k+N$ to $k+1$ **do**
    $\boldsymbol{m}_+ \leftarrow \boldsymbol{Q}\,(\boldsymbol{m}_+ \odot R(y_i, \boldsymbol{\phi}))$
  end for
  return $\hat{\varphi}_k \leftarrow \arg\max_{\varphi\in\boldsymbol{\phi}} \boldsymbol{m}_- \odot R(y_k, \boldsymbol{\phi}) \odot \boldsymbol{m}_+$

Simulation results

We perform numerical simulations on the Wiener phase noise channel (1) with Maxwell-Boltzmann shaped 64-QAM constellations for varying signal-to-noise ratios (SNRs) and σ_θ^2 values. We set the test angles as $\phi_i = -\pi/n + \frac{(i-1)2\pi}{nM}$ for $i = 1,\ldots,M$ with n equal to the degree of rotational symmetry of the constellation. For QAM constellations, we have $n=4$. To account for phase wrapping, we use the wrapped normal distribution for the phase noise

$$P(\varphi_i|\varphi_{i-1}) \propto \sum_{r=-\infty}^{r=\infty} \exp\left(\frac{-(\varphi_i - \varphi_{i-1} + 2\pi r/n)^2}{2\sigma_\theta^2}\right).$$

A mismatched circular Gaussian demapper is used with optimized noise variance. We use the BMI calculated using the method from [5] to compare the performance of the new estimator (MAP) with the BPS algorithm and the algorithm with constant phase noise assumption from [16] (CPN). Phase unwrapping is applied to the estimated phase sequences and a fully data-aided cycle-slip compensation is used. We simulate sequences of 2^{15} symbols and report the median BMI value over 100 realizations. In Fig. 2, we can see the results for half-window size $N=32$ and $M=60$ test phases. Of the three estimators in Fig. 2, the MAP estimator has the best performance. The improvement at higher σ_θ^2 values can be attributed to the use of a model for the phase noise. While the gains of the new estimator are clear in Fig. 2, they are diminished when a more practical number of $M=15$ test phases is used, as shown in Fig. 3. For lower number of test phases, the approximation (3) deviates significantly from the MAP estimator (2) which explains the reduced improvement in Fig. 3. In this scenario the additional cost of the MAP estimator cannot be justified by the improvement in BMI. We

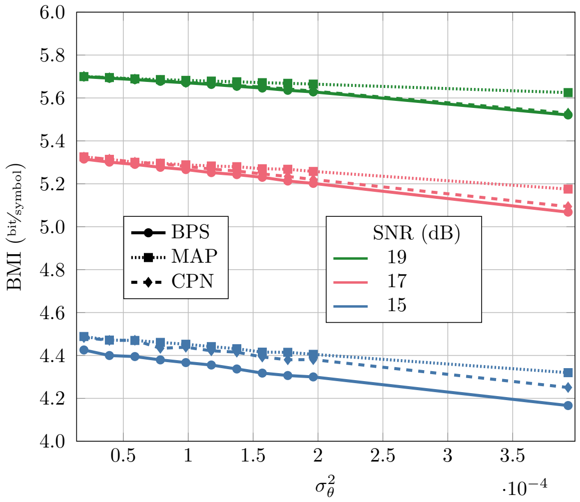

Fig. 2: Performance comparison between CPE algorithms with $N = 32$ and $M = 60$ for varying channel parameters.

hence explore machine learning based improvements to the BPS algorithm.

Improved BPS

We start by modifying the differentiable BPS algorithm from [18] as follows:

$$d_{i,m} = \min_{x \in \mathcal{X}} |y_l - x e^{j\phi_m}|^2$$

$$\forall\, i \in \{k - N, \ldots, k + N\},\ \forall\, m \in \{1, \ldots, M\}$$

$$D_m = \sum_{i=k-N}^{k+N} w_i d_{i,m}$$

$$\hat{\varphi}_k = \frac{\arg\left(e^{j\phi n} \cdot \mathrm{softmin}_t(\boldsymbol{D})\right)}{n}, \tag{5}$$

An arbitrary normalization $\sum_{i=k-N}^{k+N} w_i = 1$ is chosen and the softmin with temperature t is defined as

$$\mathrm{softmin}_t(x_i) = (\mathrm{softmin}_t(\boldsymbol{x}))_i = \frac{e^{-\frac{x_i}{t}}}{\sum_j e^{-\frac{x_j}{t}}} \tag{6}$$

The first difference to [18] is the use of weights w_i and the second difference is that the dot-product is taken between $\mathrm{softmin}_t(\boldsymbol{D})$ and $e^{j\phi n}$ rather than ϕn. The second difference solves the performance degradation problem due to the phase discontinuity reported in [18, Sec. VI.C]. For uniform weights $w_i = 1/(2N + 1)$ and temperature $t \to 0$, we recover the standard BPS algorithm. Setting $N = 32$ and $M = 15$, for each value of SNR and of σ_θ^2, we learn the weights $\boldsymbol{w}$ and temperature t using an end-to-end optimization approach similar to the one used in [18]. Training is performed over 100 epochs using the Adam optimizer in PyTorch with a learning rate of 10^{-3}. The number of batches is increased from 10 to 100 and the batch size is increased from 2^{12} to 2^{17} symbols over the 100 epochs. We report the performance of the improved BPS algorithm (BPS-optimized) in Fig. 3. The performance of the BPS-optimized algorithm with $M = 15$ matches the performance of the MAP algorithm with $M = 60$, at a significantly lower computational cost. In Fig. 4 we show the weights w_i learned by the BPS-optimized algorithm for $\sigma_\theta^2 = 1.18 \times 10^{-4}$.

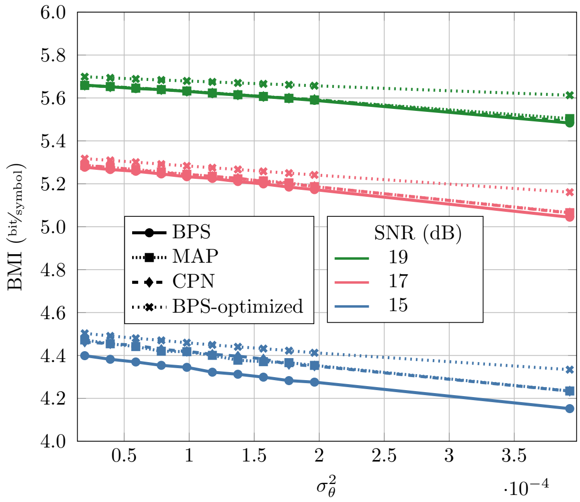

Fig. 3: Performance comparison between CPE algorithms with $N = 32$ and $M = 15$ for varying channel parameters.

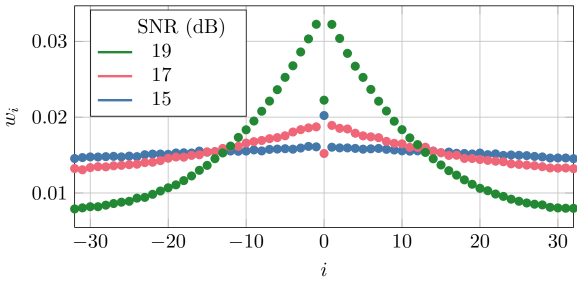

Fig. 4: Weights w_i learned by the BPS-optimized algorithm with $N = 32$ and $M = 15$ for $\sigma_\theta^2 = 1.18 \times 10^{-4}$.

Conclusions

We firstly presented a BP based approximate MAP CPE and demonstrated its superior performance compared to the standard BPS algorithm, especially for higher values of phase noise variance σ_θ^2. The performance gain is diminished when a small number of test angles are used. We then proposed improvements to the BPS algorithm using end-to-end learning. The improved BPS algorithm has performance similar to the MAP CPE at a lower computational cost. End-to-end learning may be used similarly over a differentiable model of the fiber-optic channel for optimization of the BPS algorithm to aid in nonlinearity mitigation.

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