On the Performance of Multidimensional Constellation Shaping for Linear and Nonlinear Optical Fiber Channel

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Abstract *Multidimensional constellation shaping of up to 32 dimensions with different spectral efficiencies are compared through AWGN and fiber-optic simulations. The results show that no constellation is universal and the balance of required and effective SNRs should be jointly considered for the specific optical transmission scenario. ©2023 The Author(s)*

Introduction

The application of multidimensional (MD) modulation in optical fiber communications has been intensively investigated since the fiber-optic channel offers different physical dimensions of light waves (quadratures, polarizations, wavelengths, modes and cores) simultaneously[1],[2]. Compared to the typical paradigm of transmitting independent data on all of these dimensions, improved performance can be obtained by optimizing the distribution of the constellation points or encoding data jointly on several dimensions, i.e., by *multidimensional constellation shaping*[3],[4].

To harvest performance gains in optical transmission systems, the optimization of novel MD modulation formats has attracted considerable attention for reducing nonlinear interference (NLI) distortions in 4D[5]–[11], 8D[12]–[15], and higher dimensions (up to 32D[16]). The key idea relies on the fact that MD shaping can reduce the NLI generated by the variations of the transmitted signal energy in addition to the usual (linear) shaping gain, and it could potentially be higher than the ultimate shaping gain (1.53 dB) in additive white Gaussian noise (AWGN) channel[17],[18]. However, the problem of designing optimal MD constellations with a finite nuumber of dimensions for arbitrary fiber-optic channels is still open.

In this paper, we analyze the performance of various MD modulation formats with geometric shaping (GS) for both the AWGN channel and the optical fiber channel. By evaluating the dependency of the effective signal-to-noise ratio (SNR) on the MD modulation format in multi-span and single-span transmission systems with the help of a recently proposed 4D NLI model[19],[20], we show that the MD formats should be evaluated by combing the increasing shaping gain in AWGN channel and the enhanced effective SNR in optical fiber channels.

Performance Metrics: MI, GMI and SNRs

As the most popular information-theoretical performance metrics, mutual information (MI) and generalized MI (GMI) can quantify the maximum number of information bits per transmit symbol in symbol-wise and bit-wise coded modulation systems, respectively[21]. For arbitrary M-ary N-dimensional modulation formats with equal probabilities that can carry at most $m = \frac{4}{N} \log_2 M$ bit/4D-symbol as spectral efficiency (SE), the maximum number of information bits per transmit bit can be obtained by normalizing the MI or GMI as NMI = MI/m or NGMI = GMI/m, where $0 \leq$ NGMI $\leq$ NMI ≤ 1. The values of NMI and NGMI indicate the largest ideal code rate that is able to produce error-free post-FEC results.

MD constellation shaping aims at reducing the required SNR over the AWGN channel but in practice often leads to a smaller received (effective) SNR due to hardware distortions and fiber nonlinearities. Thus, true performance improvement of MD formats in optical channel is achieved if and only if the reduction in the required SNR is larger than the loss in terms of the effective SNR. The required SNR is defined as the minimum SNR to achieve a target value of a performance metrics (here NMI and NGMI).

The analysis in this paper is based on a set of modulation formats, which combines the MI or GMI-optimized 2D and 4D formats in our online database[22] and MD Voronoi constellations (VCs) in[16],[23]. In the choice of MD modulation format, there is an inherent threefold trade-off among the SE, the noise tolerance, and the complexity of the format[24]. Thus, the performance of a modulation format is heavily dependent on which channel and performance metric is considered.

Results: Shaping Gap to AWGN Capacity

In this section, we perform extensive MI and GMI evaluations for the considered formats [1] in the AWGN channel. For the AWGN channel, the performance of an ideally shaped QAM system matches that of an ideal system using Gaussian modulation, which represents the case of a Gaussian input distribution (equivalent to the case of $N \to \infty$)[17].

To quantify the gains offered by different modulations, we plot the "SNR gap to AWGN capacity" (ΔSNR) in Fig. 1. The value of ΔSNR in dB measures how far a given symbol-wise or bit-wise coded modulation scheme is operating from the Gaussian modulation, which are defined as

$$\Delta\text{SNR}_{\text{req}}^{R} = \text{SNR}_{\text{req}}^{R} - \text{SNR}_{\text{req}}^{C}, \qquad (1)$$

where the R is MI or GMI and C is Shannon capacity. Assuming a coded modulation system with an ideal soft-decision FEC with $R_c = 0.8$ code rate into account, the required SNR is obtained for $R = 0.8m$ and $C = 0.8m$, where the m is the number of bits of the constellation.

Fig. 1 (a) and (b) collectively indicate that (i) MD geometric shaping can provide significant gains compared to uniform QAM, (ii) by increasing the dimension or SE, the SNR gap of constellations can be brought closer to the Gaussian capacity as shown in Table. III[3], (iii) the 4D-GS constellation (red circle ●) exhibits a larger shaping gain than the 2D-GS constellation (green circle ●), (iv) for the SE of a 4D constellation with odd bits, a single 2D constellation does not exist, i.e., 4D formats achieve a finer granularity than 2D formats.

For MD-VCs (8D-VC ◆, 16D-VC ▲ and 32D-VC *), the $\Delta\text{SNR}_{\text{req}}^{\text{MI}}$ decreases as the dimension increases, as shown in Fig. 1 (a). As the SE increases, the $\Delta\text{SNR}_{\text{req}}^{\text{MI}}$ converges to the asymptotic gap $1.53 - \gamma_s$ [dB] (dotted lines), where γ_s is the asymptotic shaping gains of the shaping lattices listed in[23]. Compared to 2D-GS and 4D-GS formats, 16D- and 32D-VCs realize lower $\Delta\text{SNR}_{\text{req}}^{\text{MI}}$ between 4 and 14 bit/4D-sym, and provide a good trade-off at high SEs due to their low-complexity encoding and decoding algorithms[23]. However, for MD-VCs finding efficient labelings which lead to a better performance than QAM at NGMI= 0.8 is challenging, thus the $\Delta\text{SNR}_{\text{req}}^{\text{GMI}}$ results for MD-VCs is not shown in Fig. 1 (b).

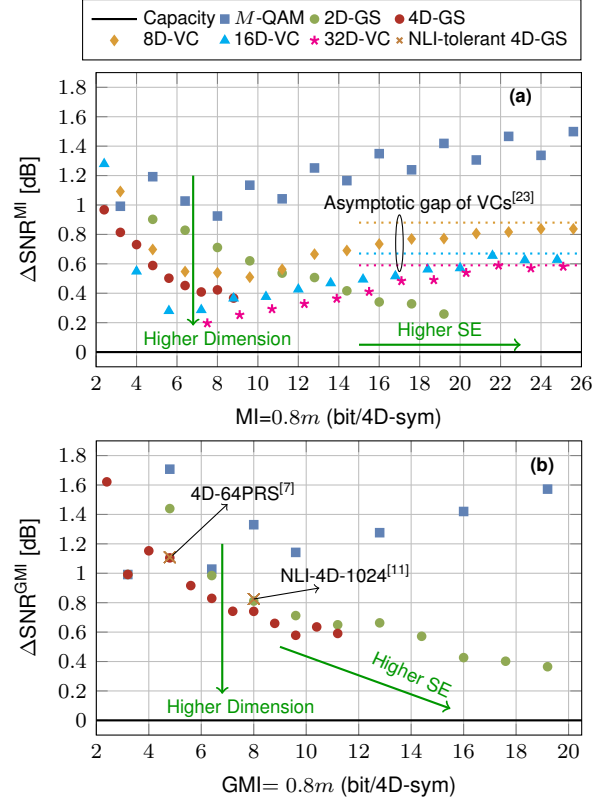Lastly, two nonlinear-tolerant 4D formats (4D-64PRS[7] and NLI-4D-1024[11]) are highlighted as

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[1]The numerical results in this paper are based on a set of modulation formats which combines the MI or GMI-optimal 2D and 4D formats in[22] and MD Voronoi constellations (VCs) in[16],[23].

Fig. 1: Comparison of the gap to AWGN capacity for different MD formats at the target rates: (a) NMI=0.8 or (b) NGMI=0.8.

cross marks × for GMI of 4.8 bit/4D-sym and 8 bit/4D-sym in Fig. 1 (b). These two constellations lead to a small but negligible loss compared to the AWGN-optimized 4D-GS formats. However, more gains are expected in the nonlinear optical fiber channel (see Fig. 2 in next section).
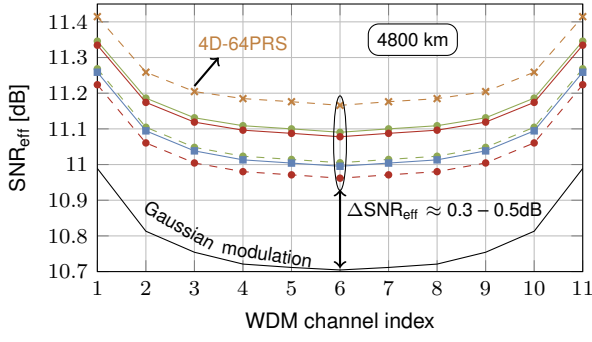
Results: Shaping Gains in Nonlinear Optical Fiber Channel

The optical system under consideration is a 11-channel WDM system with 96 GBaud symbol rate and 100 GHz channel spacing. For getting NMI or NGMI approximately at $0.8m$, two transmission cases with standard single-mode fiber, 60×80 km multi-span system and a 205 km single-span system, are considered. The fiber parameters are a loss coefficient of 0.2 dB/km, a chromatic dispersion of 17 ps/nm/km, and a nonlinear coefficient of 1.3 /W/km. Fiber losses are compensated after each span by erbium-doped fiber amplifiers, with a noise figure of 5 dB.

To evaluate the performance in the nonlinear optical fiber channel, we estimate the optimum effective SNR at a given distance for each channel using the 4D NLI model proposed in[20]. The 4D NLI model with considering modulation-dependent interference enables a quick computation of the NLI power σ_{NLI}^2 as a function of the input constellation for any dual-polarization 4D format.

In Fig. 2 (a) and (b), the optimal effective SNR

(a) Multi-span system at 60×80 km with 11 WDM channels.
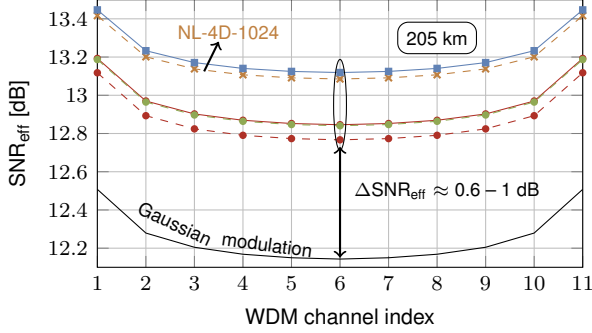
(b) Single-span system at 205 km with 11 WDM channels.

Fig. 2: The effective SNR performance comparison over two different transmission cases via 4D NLI model[20]. The considered constellations are shown in Table. 1.

Tab. 1: The $\mathrm{SNR}_{\mathrm{req}}^{R}$ and $\mathrm{SNR}_{\mathrm{eff}}$ of the constellations in Fig. 2.

SE (m)	Constellation	$\mathrm{SNR}_{\mathrm{req}}^{\mathrm{MI}}$	$\mathrm{SNR}_{\mathrm{req}}^{\mathrm{GMI}}$	$\mathrm{SNR}_{\mathrm{eff}}$
6	① 8QAM (–■–)	7.502	8.117	10.995
	② hepta2-8 (–●–)	7.215	7.830	11.090
	③ C4-64 (–●–)	6.901	8.835	11.078
	④ DSQ2-8 (-●-)	7.332	7.752	11.005
	⑤ GS-AWGN-4D-64 (-●-)	7.242	7.417	10.962
	⑥ 4D-64PRS (-✗-)	7.257	7.421	11.166
	Gaussian distribution	$\mathrm{SNR}_{\mathrm{req}}^{C} = 6.312$		10.705
10	⑦ 32QAM (–■–)	12.686	13.091	13.119
	⑧ GS-AWGN-2D-32 (-●-)	12.472	12.572	12.841
	⑨ C4-1024 (–●–)	12.184	14.614	12.846
	⑩ 4D-OS1024 (-●-)	12.291	12.502	12.767
	⑪ NL-4D-1024 (-✗-)	12.417	12.586	13.085
	Gaussian distribution	$\mathrm{SNR}_{\mathrm{req}}^{C} = 11.761$		12.143

$\mathrm{SNR}_{\mathrm{eff}}$ of each channel are shown for modulation formats with an SE of 6 bit/4D-sym and 8 bit/4D-sym, which listed in Table. 1, over multi-span and single-span system, respectively. In order to compare the required and effective SNR differences between the selected MI or GMI-optimal modulation formats in Fig. 1, the $\mathrm{SNR}_{\mathrm{req}}^{\mathrm{MI}}$, $\mathrm{SNR}_{\mathrm{req}}^{\mathrm{GMI}}$ for AWGN channel and the $\mathrm{SNR}_{\mathrm{eff}}$ of center optical channel are also shown in Table. 1. In addition, we can also observe that the 4D-64PRS and NL-4D-1024 exhibit excellent nonlinear tolerance. Despite the 2D-GS and 4D-GS designed for the AWGN channel have remarkable required SNR gains, they result in the degradation of the effective SNR due to the impact of nonlinear impairments caused during the propagation.

To further quantify the gains offered by 2D and 4D shaped constellations in nonlinear opti-
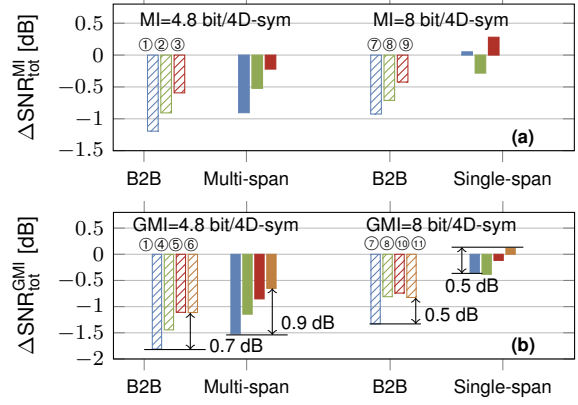

Fig. 3: The relative shaping gap or gain of the considered constellations in Table 1 (marks as ① – ⑪) with respect to Gaussian distribution.

cal fiber channel, the relative shaping gains in terms of the SNR ($\Delta\mathrm{SNR}_{\mathrm{tot}}^{\mathrm{MI}}$ and $\Delta\mathrm{SNR}_{\mathrm{tot}}^{\mathrm{GMI}}$) for the considered 2D and 4D modulation formats with respect to Gaussian distribution for both multi-span and single span scenarios are shown in Fig. 3. The total SNR gap or gains considering both linear and nonlinear effects are defined as $\Delta\mathrm{SNR}_{\mathrm{tot}}^{R} = -\Delta\mathrm{SNR}_{\mathrm{req}}^{R} + \Delta\mathrm{SNR}_{\mathrm{eff}}$, where $\Delta\mathrm{SNR}_{\mathrm{eff}}$ represent the modulation-dependent effective SNR ($\mathrm{SNR}_{\mathrm{eff}}$) difference between a given constellation and Gaussian constellation itself in Table 1, resp.

In Fig. 3, for the back-to-back (B2B) case (pure AWGN channel), the NLI power equals zero, i.e., $\Delta\mathrm{SNR}_{\mathrm{eff}} = 0$, thus $\Delta\mathrm{SNR}_{\mathrm{tot}}^{R} = -\Delta\mathrm{SNR}_{\mathrm{req}}^{R}$ (bar with lines). Note that all the formats reduce the gap to the theoretical AWGN-optimal Gaussian modulation in optical channel due to the largest NLI and lowest effective SNR of Gaussian modulation. In addition, 4D-GS formats always provide positive gains over uniform QAM, but 2D-GS may even have negative penalty (see the single-span case in Fig. 3 (a)) with respect to QAM. In Fig. 3 (b), the nonlinear-tolerant 4D formats (brown bars) can maintain or even provide larger SNR gains in the optical fiber channel than the gains in B2B cases. Thus, MD shaping is an efficient way to search for constellations which could tolerant the linear noise and nonlinear noise.

Conclusion

In this work, we evaluated the performance of various MD formats for both the AWGN channel and nonlinear optical fiber channel with two types of transmission systems. The results suggest that more gains could be harvested with higher dimensional space for the AWGN channel and more nonlinear-tolerance could be found with four-dimensional space for the nonlinear optical channel. Therefore, higher dimensional space design may be the key to unlock the full potential of nonlinear constellation shaping.

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