

# Experimental Comparison of Probabilistic and Geometric Shaping in Transmission

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**Abstract** Probabilistic- and geometric-shaped (PS and GS) constellations were compared in four-channel transmission experiments. For 64-ary constellations over 3022.5 km of SMF, PS outperformed GS by 0.17 bit/4D. For short distance (232.5 km) and 1024 points, this advantage diminishes due to increased transceiver noise, and both schemes perform similarly.

## Introduction

Constellation shaping is a technique to assign a non-uniform distribution to the input symbols, such that the mutual information (MI) of the channel under consideration is maximised. This is achieved by changing the distribution of the transmitted symbols to resemble a Gaussian distribution and can reduce the gap to capacity for optical communication systems. There are two approaches, namely, probabilistic and geometric shaping. While the former adjusts the occurrence rates of individual square-QAM symbols, the latter shifts the target symbol positions in the complex plane. Both methods result in an increased achievable information rate when compared to square QAM constellations and have been demonstrated in transmission experiments but have not been directly compared over a wide range of parameters.

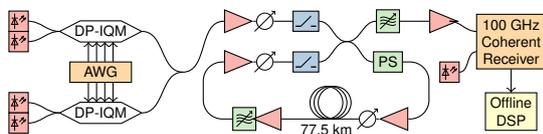
The choice of the technique is usually based on its compatibility and complexity of implementation in deployed optical fibre systems<sup>[1]</sup>. Yet to understand the range of operation and quantify benefits it is important to compare both approaches over the same system. A previous comparison of PS and GS was reported in<sup>[2]</sup> but this was carried out in terms of the mutual information (MI) performance and for low cardinality constellations only, i.e, up to 32 constellation points.

In this work, using the approach described in<sup>[3]</sup>, high cardinality PS and GS constellations, (up to 1024 points) were designed and experimentally compared in terms of the more appropriate metric

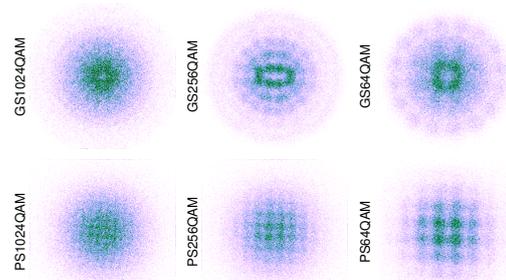
of generalised mutual information (GMI)<sup>[4]</sup>. The transmission experiment was carried out for distances of 232.5, 1007.5 and 3022.5 km, using a recirculating loop with 4 channels. The experimental results were validated using the Enhanced Gaussian noise (EGN) model<sup>[5]</sup>.

## Experimental Setup

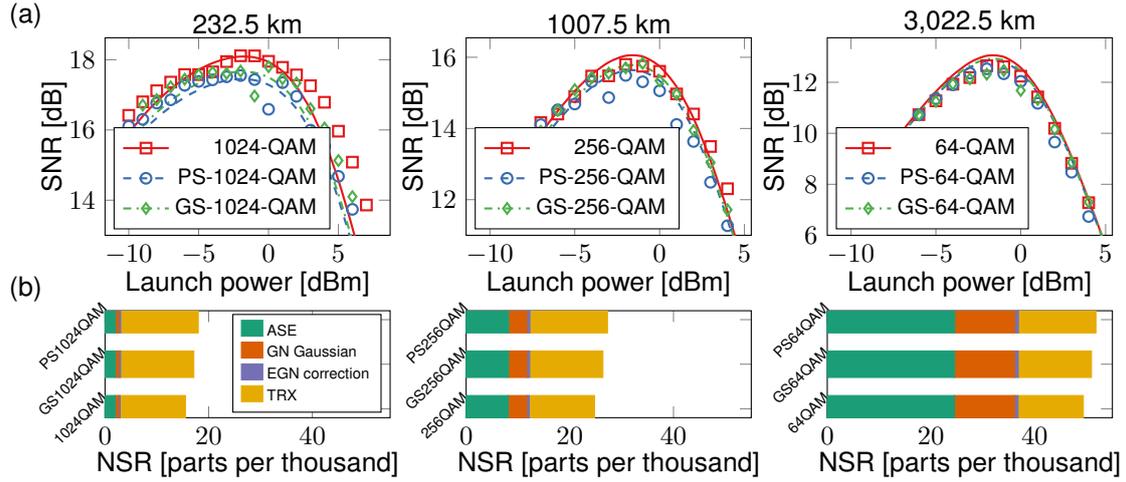
The experimental setup is shown in Fig 1. A 4-carrier wavelength division multiplexed (WDM) signal was transmitted, with the transmitter comprising two independently-modulated dual-polarisation inphase-quadrature modulators (DP-IQMs) with two external cavity lasers (ECLs), each of them with <100 kHz linewidth. The channel symbol rate was 49.5 GBd, and, together with 500 MHz guard bands between channels, the four channels occupied 200 GHz of the optical spectrum. The root-raised cosine filter had a 1 % roll-off. Pilot-based DSP<sup>[6]</sup>, a 1024-symbol header for channel equalisation, and 1 in 32 symbols for carrier phase estimation were used. For modulation formats, square, GS and PS constellations were chosen. The transmission was carried out using a recirculating loop with the distance varied by adjusting the number of recirculations. The loop consisted of 77.5 km of Corning<sup>TM</sup> ultra-low loss single-mode fibre (ULL-SMF), with a wavelength-



**Fig. 1:** Experimental transmission setup with 4 channel transmitter, recirculating loop of 77.5 km and 100 GHz coherent receiver.



**Fig. 2:** Experimentally received constellation heatmap for the modulation formats used in this work after 232.5, 1007.5 and 3022.5 km for 64, 256 and 1024 QAM respectively. The results are obtained for -2 dBm launch power.



**Fig. 3:** (a) The SNR vs launch power for square QAM and the designed PS and GS constellations. The SNR for 1024, 256 and 64-ary constellations are shown for 232.5, 1007.5 and 3022.5 km transmission respectively. (Markers - experiments, Lines - simulations) (b) The NSR at optimum launch power for the designed PS and GS constellations with the contributions from ASE, GN model, EGN modulation corrections and transceiver noise separated out.

selective switch (WSS) for gain flattening and a loop-synchronous polarisation scrambler (LSPS). The receiver consisted of a 100-GHz coherent receiver connected to a 256 GSa/s scope. The full 200-GHz transmitted spectrum was received. The carrier demultiplexing and digital signal processing (DSP) were implemented offline.

The shaped constellations were optimised for SNR values of 12, 15 and 18 dB for 64, 256 and 1024-ary QAM, respectively. The constellations were optimised assuming an additive white Gaussian noise (AWGN) channel. The GS constellations were designed using a gradient descent algorithm as described in<sup>[3]</sup>. For the PS constellations, a Maxwell-Boltzmann distribution was optimised to maximise GMI for the target SNR value. The distribution was quantised for use with a constant composition distribution matcher (CCDM)<sup>[7]</sup> with distributions {10503, 7021, 3138, 938}, {5454, 4527, 3118, 1783, 846, 333, 109, 30} and {3122, 2844, 2360, 1784, 1228, 770, 440, 229, 108, 47, 18, 7, 2, 1, 0, 0} for 64, 256 and 1024-ary PS, respectively. In Fig. 1(b), the received constellations are shown for -2 dBm launch power after 232.5, 1007.5 and 3022.5 km for 64, 256 and 1024-ary constellations respectively. The resulting constellations are shown in Fig. 2.

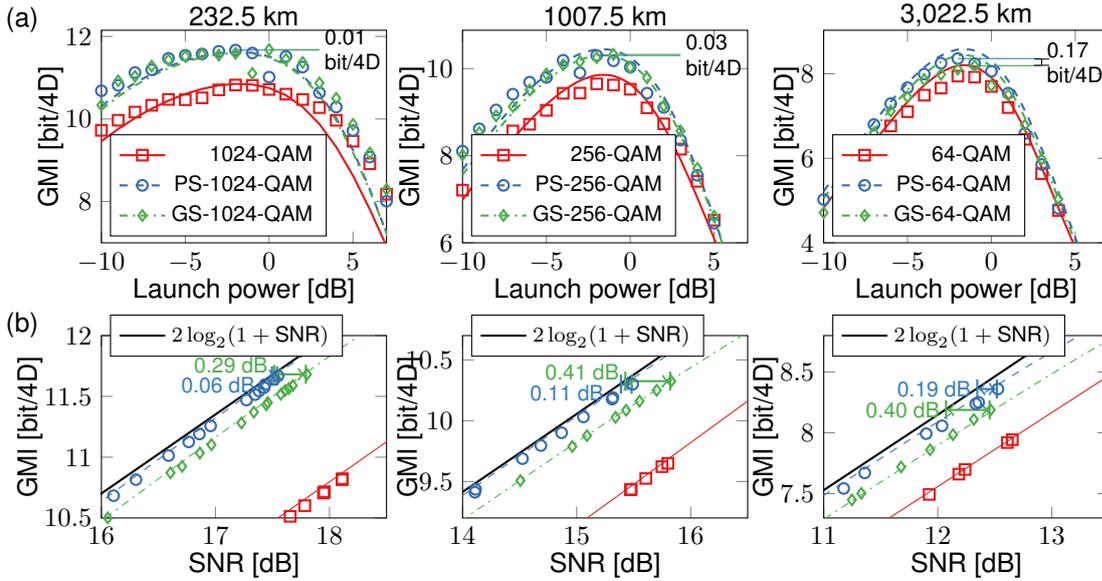
## Results

To validate the experimental investigation, the measured results were compared with the EGN model<sup>[5]</sup>. This model includes the dependence of the modulation format on the nonlinearity generated by the fibre propagation, accurately estimating the SNR for different constellations de-

signs. This is carried out using both fourth- and sixth-order moments of the transmitted constellations. The transceiver (TRX) SNR was calculated by measuring the back-to-back performance. For QAM, GS and PS, these values were 19.0, 18.5 and 18.25 dB, respectively, independent of the constellation cardinality.

Fig. 3(a) shows the received SNR as a function of launch power for the different modulation formats. For ease of understanding, Fig. 3(b) separates out the different sources of noise contributing to the SNR calculation. The contributions of the noise sources are shown as noise-to-signal ratios (NSRs) in parts per thousand and are computed using the optimum launch power profile. Note that, for each transmission distance, the contributions from amplified spontaneous emission (ASE) and the Gaussian Noise (GN) model with Gaussian signal assumption are similar. This means that for each distance, the SNR variation between the constellations is mainly due to the TRX noise and the modulation format correction contribution from the EGN model. These different contributions are reflected in the SNR results shown in Fig. 3(a).

For the shortest distance transmission, greater differences between the modulation formats in the SNR results were observed; this is because the TRX noise is the main limiting source of noise for this case and its variation for each modulation is greater when compared to the correction contribution from the EGN model. Additionally, shaped constellations have higher transceiver noise when compared to square ones, worsening their performance in terms of SNR. How-



**Fig. 4:** GMI vs (a) launch power and (b) SNR for square QAM and the designed PS and GS constellations. The GMI for 1024, 256 and 64-ary constellations are shown for 232.5, 1007.5 and 3022.5 km transmission respectively.

ever, perhaps counter-intuitively, this does not mean shaped constellations ultimately perform worse than square ones. In fact, the shaped constellations achieve a higher GMI because the GMI calculation depends on the input symbols distribution<sup>[4]</sup>. Also, note that PS constellations have more TRX noise when compared to GS ones; this may mean that GS has the potential to outperform PS for short-distance transmission, where the system performance is limited by TRX.

In Fig. 4(a), the GMI for one of the centre channels is shown as a function of launch power; as mentioned, in comparison to square QAM constellations, PS and GS achieve better performance in terms of GMI. Additionally, this plot shows higher shaping gains for shorter-distance transmission, because higher values of shaping gain are expected for the high-SNR regime<sup>[8]</sup>.

Fig. 4(b) presents the GMI as a function of SNR, showing that PS outperforms GS for all the cardinalities designed in this work in terms of gap to capacity. Indeed, for the peak values of measured SNR shown in Fig. 3(a) for each of the distances, Fig. 4 shows that by using PS rather than GS, the gap to capacity is decreased from 0.29, 0.41 and 0.40 dB, to 0.06, 0.11 and 0.19 dB, for 64, 256, and 1024 constellations designs, respectively. These values are indicated with arrows in this figure; these same values are also indicated in terms of GMI as 0.01, 0.03 and 0.17 bit/4D.

For shorter distances, there is no discernible difference in the performance of GS and PS (within the achievable experimental accuracy). The choice of shaping method should be made

based on other considerations, such as implementation complexity. Conversely, for longer distance (e.g., 3022.5 km), with the reduced impact of transceiver SNR equalising their SNR performance, PS outperforms GS by 0.17 bit/4D due to the 0.21 dB reduction in gap to capacity.

## Conclusion

An experimental investigation of the performance of probabilistic- and geometric-shaped constellations using high-order QAM (64, 256 and 1024) was carried out for the transmission distances of 232.5, 1007.5 and 3022.5 km. It is shown that PS outperforms GS for long-distance transmission (3022.5 km), where low-cardinality constellations have better performance; the improvement is 0.17 bit/4D due to 0.21 dB reduction in gap to capacity. For the case of short distances (232.5 and 1007.5 km), where high-cardinality constellations are preferable, both PS and GS strategies perform similarly due to the increased impact of transceiver noise, within the margins of measurement errors. The choice of constellation would depend on the trade-off between the performance impact of the transceiver and the implementation complexity of the shaping method. In conclusion, whilst PS achieves a smaller gap to capacity resulting in a performance benefit over GS for long-haul and trans-oceanic transmission, at a shorter distance the increased transceiver noise diminishes this advantage over GS.

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