

# Application of SD-FEC to Optical Eigenvalue-modulated Signals Using a Neural Network-based Demodulator

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**Abstract** We propose the application of an SD-FEC technique to an eigenvalue-modulated signal using a multilabel neural-network demodulator. The experimental results indicate a successful operation with an error-free transmission through a 3000-km optical fiber line. ©2023 The Author(s)

## Introduction

Optical eigenvalue modulation<sup>[1]</sup> based on the inverse scattering transform (IST)<sup>[2]</sup> is a promising technology for overcoming the Kerr nonlinear limit in optical fiber communications. IST is recently well-known as nonlinear Fourier transform (NFT)<sup>[3]</sup>. Eigenvalues associated with the nonlinear Schrödinger equation (NLSE) are invariant during an optical fiber transmission even with the effects of dispersion and Kerr nonlinearity. Several studies have been made on multilevel modulation using multieigenvalue<sup>[4],[5]</sup> nonlinear spectrum<sup>[6]</sup>, scattering coefficient  $b$ <sup>[7],[8]</sup>, and joint modulation<sup>[9],[10]</sup> to achieve high-capacity transmission based on NFT.

Further, soft-decision forward error correction (SD-FEC) techniques have been applied to optical fiber communications to increase the transmission capacity and extend transmission distances<sup>[11],[12]</sup>. A logarithmic ratio of a *posteriori* probabilities (L-value) is calculated from received signals and generally utilized in SD-FEC. However, deriving the L-value from the received eigenvalue-modulated signal is complicated. This is because that the statics of the eigenvalue and scattering coefficients are not yet completely understood when white Gaussian noise is added to the time-domain signal of multieigenvalue transmission systems<sup>[13],[14]</sup>. Moreover, the applicability of SD-FEC to eigenvalue-modulated signals has not yet been investigated in detail.

In this paper, we propose a combination of a neural network (NN)-based demodulator and SD-FEC decoding. A multilabel NN-based demodulator is employed to compute the L-value from the eigenvalue input at the receiver. Through a simulation, an effective operation of SD-FEC with a clear waterfall BER curve is achieved. Furthermore, we experimentally demonstrate an error-free transmission through a 3000-km optical fiber.

## Eigenvalue Modulation

This study employed multieigenvalue transmission based on an on-off encoding<sup>[4],[5]</sup> of four eigenvalues. Fig. 1(a) presents an overview of the multieigenvalue transmission system. At the transmitter end, a 4-bit sequence is mapped onto an eigenvalue pattern. Note that the on-off encoding is based on a one-to-one mapping between a 4-bit input and the subsets of eigenvalues. For a bit value of 1 (or 0) at the  $j$ -th position  $b_j$ , the  $j$ -th eigenvalue  $\zeta^{(j)}$  is included (or excluded). Subsequently, the eigenvalue pattern is converted into a pulse and the pulse sequence is transmitted via the fiber as the resultant eigenvalue-modulated signal. During the fiber transmission, the eigenvalue  $\zeta$  is invariant even though the waveform and spectrum change owing to dispersion and nonlinearity.

At the receiver end, the eigenvalue pattern is detected using the IST from the complex envelope amplitude acquired by the coherent receiver. The total number of detected eigenvalues corresponds to the number of sampling points when using the Fourier collocation method<sup>[15]</sup>. Finally, the detected eigenvalue pattern is decoded into a bit sequence.

## SD-FEC Using NN-based Demodulator

In previous studies<sup>[16],[17]</sup>, an eigenvalue pattern was decoded into a bit sequence using an NN-based classifier on the assumption that hard-decision FEC was used. In this paper, we apply SD-FEC to the eigenvalue transmission system. Notably, the derivation of the L-value is difficult because the eigenvalues obtained via the IST do not exactly follow a Gaussian distribution<sup>[14]</sup>. Accordingly, this study proposes a multilabel NN-based demodulator to compute the L-value from the received eigenvalue pattern.

Fig. 1(b) presents an overview of the proposed

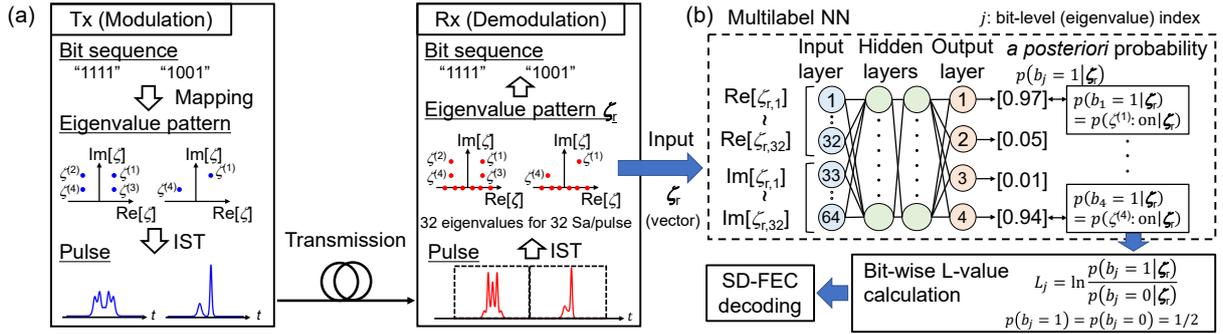


Fig. 1: Overview of (a) multieigenvalue transmission system and (b) multilabel NN-based demodulator for SD-FEC.

method. The real and imaginary parts of the detected eigenvalues  $\zeta_r$  (vector) were input to the NN. The number of output units was four corresponding to the number of the eigenvalues (bits) used for mapping. A logistic sigmoid function and a cross-entropy error function were employed as the output and loss functions, respectively. The input data were linked with the multilabel, that is, a *posteriori* probability of the on-state of  $j$ -th eigenvalue  $\zeta^{(j)}$  corresponding to  $p(b_j = 1 | \zeta_r)$  was the output<sup>[18]</sup>. The bit-wise L-value  $L_j$ , which was calculated from the ratio  $p(b_j = 1 | \zeta_r) / p(b_j = 0 | \zeta_r)$ , was input to the SD-FEC decoder.

## Simulations

Fig. 2 depicts the simulation model. The on-off states of the four eigenvalues  $\zeta = \{-0.25 + i0.25, 0.25 + i0.25, -0.25 + i0.5, 0.25 + i0.5\} \in \mathbb{C}$  were used for eigenvalue modulation. The modulation was performed at 10 GSa/s. The pulse duration was 1.6 ns and the bit rate was 2.5 Gb/s. The B-to-B operation was examined to demonstrate the feasibility of the proposed method.

At the receiver end, the eigenvalue patterns were detected from the received signal at 20 GSa/s. The bit-wise L-value was computed using the multilabel NN-based demodulator, as discussed in the previous section. Further, a four-layer perceptron configuration was employed while using a rectified linear unit (ReLU) activation function; the number of hidden units was set to 256. A total of 62 250 received pulses were divided into separate sequences of 10 000 and 52 250 pulses for training and BER test, respectively. Subsequently, the NN was trained using an Adam optimizer<sup>[19]</sup>. The following two training patterns were examined: The NN was trained (i) for each optical signal-to-noise ratio (OSNR) using each OSNR data and (ii) using the OSNR data in the vicinity of the SD-FEC limit (OSNR=3.4 dB). To present a comparison, condition (iii) without the NN demodulator was prepared as follows: The

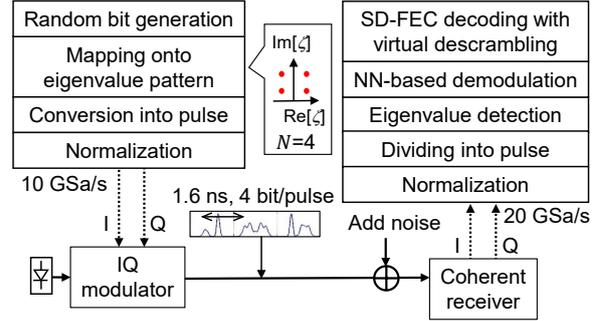


Fig. 2: Schematic of the simulation model.

L-value was calculated considering that the distribution of the eigenvalue pattern could be approximated by a Gaussian distribution following the 1-dimensional projection based on Fisher's linear discriminant<sup>[20]</sup>.

For SD-FEC, the simulation was performed using a random bit sequence, assuming the use of the scrambler and descrambler at the transmitter and receiver, respectively<sup>[21]</sup>. We employed the DVB-S2 low-density parity-check code (LDPC)<sup>[22]</sup> with the redundancy set to 16.7%. The number of decoding iterations for SD-FEC was optimized in the range from 1 to 10.

Fig. 3 shows the BER curves before and after SD-FEC. Evidently, the BER improved by SD-FEC for both training conditions (i) and (ii). However, certain residual errors were observed for the OSNR in the range of 8–12 dB for condition (i). In contrast, a clear waterfall curve was obtained without any residual errors for condition (ii). An error-free operation was achieved for OSNR values greater than 4.6 dB. In addition, the OSNR gain required to achieve an error-free operation was 1.3 dB compared with the case without the NN (condition (iii)).

Fig. 4 shows the Q-factor ( $Q_{\text{soft}}$ ) calculated from the soft information (asymmetric information (ASI))<sup>[23]</sup> and the number of decoding iterations for SD-FEC.  $Q_{\text{soft}}$  and the number of iterations were unstable when the residual errors were prevalent for condition (i). This implies an unsuccessful L-value computation because the

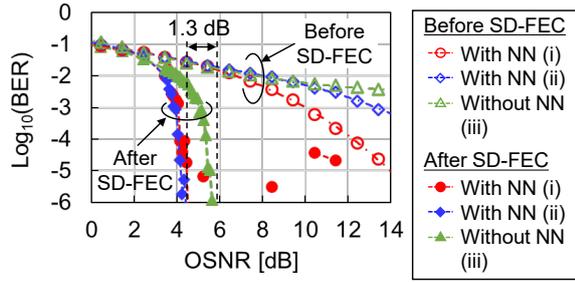


Fig. 3: BER results obtained through the simulation (B-to-B).

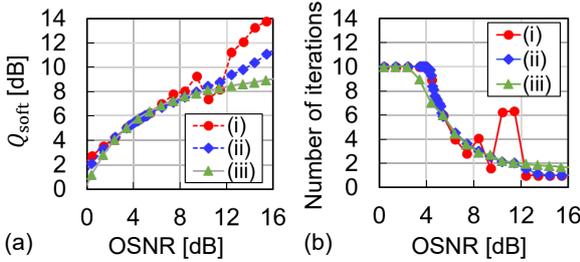


Fig. 4: (a)  $Q_{\text{soft}}$  and (b) number of iterations (B-to-B).

NN model was trained inadequately. Upon training the NN demodulator using the data of high OSNR for condition (i), wherein the data far from the ideal signal points is not included, processing noisy data and outliers was difficult. In contrast, under condition (ii), stable characteristics of  $Q_{\text{soft}}$  and iterations were obtained, as shown in Fig. 4.

## Experiments

Fig. 5 depicts the experimental setup with an offline NN-based receiver. For eigenvalue modulation, the same eigenvalue subsets and initial parameters used in the simulations were considered. An eigenvalue-modulated signal was generated using an offline digital signal processor (DSP) employing the same process as that in the simulation. An arbitrary waveform generator (AWG) and an IQ modulator were used to generate the optical signal. Subsequently, the optical signals were launched into a transmission loop that comprised a 50-km non-zero dispersion-shifted fiber (NZ-DSF). At the receiver end, the signals underwent analog-digital conversion using a digital storage oscilloscope (DSO). Digital signal processing for demodulation was performed offline at 20 GSa/s. The NN configuration, demodulation parameters, and SD-FEC parameters were maintained identical to the simulations. We employed condition (ii) for the NN training.

Fig. 6 shows the BER curves before and after SD-FEC. The BER was improved by SD-FEC and no residual errors were observed at OSNR value greater than 8 dB, even for the transmission distances of 2000 and 3000 km. At 2000 km, a better BER curve was observed because the spectrum

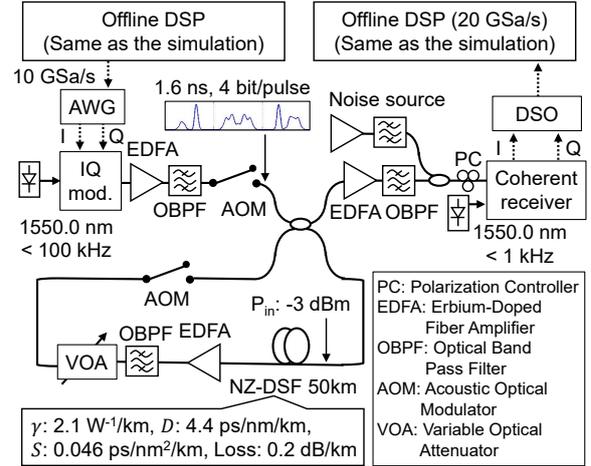


Fig. 5: Experimental setup.

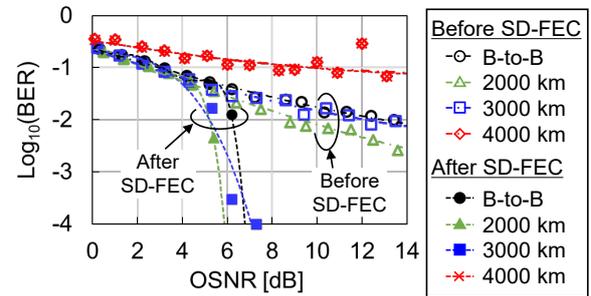


Fig. 6: BER results obtained through the experiment.

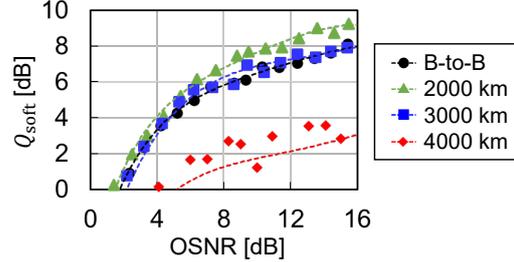


Fig. 7:  $Q_{\text{soft}}$  obtained through the experiment.

changed during the transmission and the noise effect was reduced. The trend of the BER characteristics after SD-FEC well correspond with that of  $Q_{\text{soft}}$  shown in Fig. 7. After a 4000-km transmission, the signal could not be demodulated due to the inter-symbol interference<sup>[16]</sup>.

## Conclusions

We proposed the application of SD-FEC to an eigenvalue-modulated signal using a multilabel NN-based demodulator. Successful error-free operation was experimentally performed at low OSNR even after a 3000-km transmission, where  $Q_{\text{soft}}$  well corresponds to BER after SD-FEC. The proposed method is expected to be useful for higher-order eigenvalue modulation systems.

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