Recursive Least Squares Based Low-Complexity Frequency-Domain MIMO Equalisation for MDL-Tolerant Long-Haul Space Division Multiplexed Transmission

Kohki Shibahara(1), Megumi Hoshi(1), Takemi Hasegawa(2),
Tetsuya Hayashi(2), and Yutaka Miyamoto(1)

(1) NTT Network Innovation Laboratories, NTT Corporation, kouki.shibahara@ntt.com
(2) Optical Communications Laboratory, Sumitomo Electric Industries

Abstract *We propose a low-complexity, fast-converging frequency-domain MIMO equaliser for mode-dependent-loss-impaired long-haul space-division-multiplexed transmission. An introduction of recursive-least-squares method performing computations only within signal bandwidth accelerates MIMO equaliser filter learning, achieving a world's-longest 10-spatial-mode transmission over 1560 km, while mitigating computational complexity requirement by 40%. ©2023 The Author(s)*

Introduction

Expanding per-link capacity over optical fibre is eagerly anticipated for realizing future optical transport systems. One prime candidate for it is space division multiplexing (SDM) technology allowing spatial channels to be parallelized within single fibre by means of core multiplexing and/or mode multiplexing. Known as a family of mode division multiplexing (MDM) approach, the latter utilizes transmission medium including coupled-core multicore fibres (CC-MCFs) or multi-mode fibres (MMFs), providing high throughput capacity exceeding 1 Pbs [1,2] and/or long transmission reach over 1000 km [3-6], while keeping fibre cladding diameter within 125 μm. This is greatly attributed to the help of multiple-input multiple-output equalisation (MIMO-EQ) for undoing intra- and inter mode mixing that occurs during propagation over MDM fibres.

A practical issue in applying MIMO-EQ for MDM is a slow convergence of filter coefficients affected by the presence of mode dependent loss (MDL) [7]. This is particularly prominent in an use of MIMO-EQ with the steepest descent algorithm including the least mean squares (LMS) method, because of a learning rate sensitivity w.r.t. eigenvalue spread of a correlation matrix of input signals into MIMO-EQ [8]. Furthermore, the convergence slowing is enhanced in high-mode-count MDM transmission over long distances due to accumulated MDL. A candidate solution to avoid this effect is an introduction of recursive least squares (RLS) based MIMO-EQ filter adaptation [7]. A convergence speed improvement by the frequency-domain RLS has been experimentally demonstrated using three spatial modes over up to 1000 km [9,10]. One remaining task for RLS-based filter adaptation is mitigating rather-enhanced computational complexity.

We have previously proposed a simplified scheme for LMS-adapted MIMO frequency-domain equalization (FDE) that avoids redundant computations outside of signal's bandwidth [11]. In this work, we extend its application into RLS-adapted one with the aim to simultaneously attaining fast convergence and low complexity. We reveal that complexity mitigation effect by the proposed FDE technique is enhanced in higher mode-count MDM scenarios. Based on experimental results over 4-core CC-MCF, we also show that convergence rate is improved by an order of magnitude at 5500 km. Also shown is a record-long-distance 10-mode WDM/MDM transmission over 1560 km with a decreased complexity by 40%, corresponding to 20% reach extension relative to our previous work [12].

Proposed: Out-of-band exclusive (OBE) FDE with RLS adaptation

This section briefly reviews the conventional FDE (C-FDE), followed by a description of the proposed low-complexity FDE technique. Throughout this section, we focus on an RLS-based filter adaptation instead of widely used

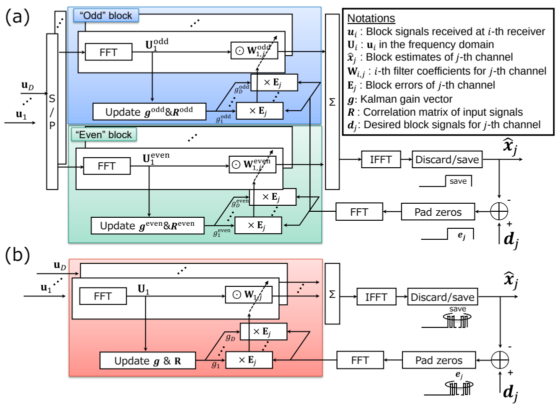


Fig. 1: Configuration comparison of RLS-adapted FDE schemes. (a): Conventional FDE (C-FDE). (b): Out-of-band exclusive FDE (OBE-FDE).

LMS one. We also assume an employment of the overlap-save MIMO-FDE algorithm with a block size of N working at the oversampling rate of 2 and the overlapping rate of 0.5. Accordingly, each block processing produces $N/4$ output symbols per block processing per spatial channel.

We consider an MDM system where D spatial channels including polarization ones are transmitted over SDM fibres. An objective of MIMO-EQ is to extract estimates of a data stream for j-th channel $\hat{\boldsymbol{x}}_j$ using a convolution of received signal block at i-th receiver $\mathbf{u}_i$ with filter coefficients $(1 \leq i, j \leq D)$. Figure 1(a) shows the conventional FDE (C-FDE) for j-th data stream output. The processing parts requiring complex multiplications/divisions include fast Fourier transform (FFT), output calculation, and updates for a filter coefficient $\mathbf{W}_{i,j}$, a Kalman gain vector $\boldsymbol{g}$, and a correlation matrix $\mathbf{R}$. It is unfortunate that we must omit a deeper description of C-FDE due to the limited space: readers can find detailed one in [9,10]. Resultant computational complexity for C-FDE, $C_{\mathrm{C-FDE}}$, that is measured with a required number of complex multiplications or divisions to obtain $ND/4$ symbols during one block processing period becomes $C_{\mathrm{C-FDE}} = 5ND^2 + ND + ND\log_2 N$. A drawback is that above computations are performed on the one-sample-per-symbol basis, because of the "odd/even" FDE configuration that individually processes odd/even blocks. This may result in ignoring the a priori knowledge of signal occupation spectrally shaped with a small roll-off factor α approaching 0, which is easily available in modern coherent transceivers.

Based on this motivation, we modify the algorithm to directly deal with twofold-oversampled received signals (Fig. 1(b)), which we refer to as out-of-band exclusive (OBE) FDE. The processing steps together with complexity requirement are summarized in Table 1. The main difference in comparison to C-FDE is that the proposed OBE-FDE selectively executes multiplications only for the in-band frequency entries of the signals with the size of $\sim N/2$ at the steps of output (step#2) and update (steps#4 through #6). The rest with the size of $\sim N/2$ defaults to zeros. Thus, with a sufficiently small α (~ 0), we can roughly halve a required number of complex multiplications or divisions, yielding total computational requirement for OBE-FDE as $C_{\mathrm{OBE-FDE}} = (5/2)ND^2 + ND + (3/2)ND\log_2 N$.

Then comparative computational complexity requirement for both FDE schemes is evaluated with a complexity reduction ratio $\eta \triangleq C_{\mathrm{OBE-FDE}}/C_{\mathrm{C-FDE}}$ in Figure 2, where we assume $N = \{256, 1024\}, D \in \{2, 6, 8, 12, 20, 24\}$, which should be achievable in standard-cladding single-mode fibre, MMF, or CC-MCF. We find that complexity reduction of η below 1 is achieved for almost all D scenarios, and more depressed in a larger D regime, indicating a suitable application to high-mode-count MDM system. Notably, η is less sensitive to a block size N because of the logarithmic dependence on η. In an extreme case where $D \gg \log_2 N$, η would approach to 0.5 (i.e., 50%), while in a practical case, the use of OBE-FDE reduces computational burdens by ~30% and ~40% in transmissions over 4-core CC-MCF $(D = 8)$ and 10-mode FMF $(D = 20)$, respectively, both of which will be experimentally evaluated in the next section.

Tab. 1: Algorithm of RLS-adapted OBE-FDE.

Step	Equation	$C_{\mathrm{OBE-FDE}}$ per step
1:	$\mathbf{U}_i = \mathrm{FFT}[\mathbf{u}_i]$	$ND/2\log_2 N$
2:	$\boldsymbol{p}_j \triangleq \mathrm{IFFT}[\Sigma_i \mathbf{U}_i \odot \mathbf{W}_{i,j}]$ $\hat{\boldsymbol{x}}_j = \text{odd entries at the last half of } \boldsymbol{p}_j$	$ND^2/2 + ND/2\log_2 N$
3:	$\mathbf{E}_j = \mathrm{FFT}[\mathbf{0}_{N/2}; \mathrm{upsample}(\boldsymbol{e}_j)]$	$ND/2\log_2 N$
4:	$\boldsymbol{g}[k] \leftarrow (\lambda + \mathbf{V}^H[k]\mathbf{R}[k]\mathbf{V}[k])^{-1}\,\mathbf{R}[k]\mathbf{V}[k]$	$ND^2 + ND$
5:	$\mathbf{R}[k] \leftarrow \lambda^{-1}\mathbf{R}[k] - \lambda^{-1}\boldsymbol{g}[k]\,\mathbf{V}^H[\mathrm{k}]\mathbf{R}[k]$	$ND^2/2$
6:	$\mathbf{W}_{i,j}[k] \leftarrow \mathbf{W}_{i,j}[k] + g_i^*[k]\mathbf{E}_j[k]$	$ND^2/2$

Notations: $\odot$: Hadamard product, k: frequency bin, λ: forgetting factor
$\mathbf{0}_M$: M-zeros vector, $\mathbf{V}$: input signal vector at each frequency

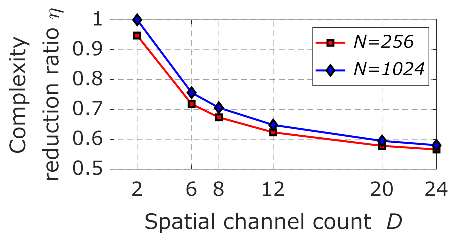
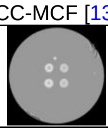


Fig. 2: Complexity reduction ratio η achieved by switching from C-FDE to OBE-FDE with RLS adaptation.

Experimental setup and results

We conducted MDM transmission experiments to evaluate the proposed OBE-FDE scheme by using almost the same setup of inline amplified 10-mode transmission experiments described in [12]. At the transmitter, dual 6-GBaud QPSK signals digitally shaped with α of 0.01 were combined with ASE-shaped 10-WDM 12.5-GHz-

Tab. 2: MDM fibre characteristics used in the experiments.

MDM fibre type	CC-MCF [13]	FMF [12]
Cross sectional view		
# of SDM channels	4	10
Length	53.9 km	52 km
Attenuation	0.157 dB/km	0.25 dB/km
Spatial mode dispersion	8 ps/(km)$^{1/2}$	157 ps/km

Note: parameters are evaluated at C band.

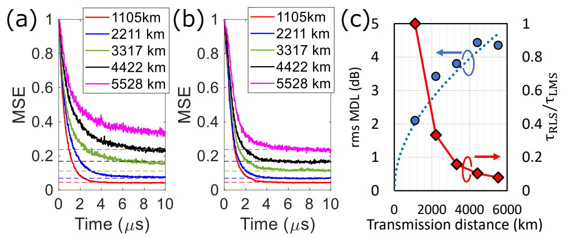


Fig. 3: Learning curves of MIMO-EQ coeffcients in 4-core CC-MCF transmission for (a) LMS adaptation and for (b) RLS adaptation. Dotted are MSE baselines achieved by the RLS after τ_{RLS}= 6 μs. (c): rms MDL evolution as a functoin of distance (left axis), and an accelerated MIMO-EQ learning rate brought by switching from the LMS to the RLS (right).

spaced signals locating from 1549.556 nm to 1550.558 nm, decorrelated, input into recirculating loop systems. We tested two kinds of MDM fibres listed in Table 2: one is a 4-core CC-MCFs [13], and another is a trench-assisted graded-index FMFs supporting 10 spatial modes [12]. Ten-fold recirculating loop system supported up to 10 spatial channel's parallel propagation, each containing EDFAs, OBPFs, AOMs, and mode multiplexer/demultiplexer (or, fan-in/fan-out) devices. After transmission, MDM signals were then processed by OBE-enabled reduced-complexity MIMO-FDE either with LMS or RLS adaptations. A FFT block size, a step-size parameter for LMS adaptation, and a forgetting factor for RLS adaptation were fixed to 800, 10^{-4}, and 0.99, respectively.

We start an experimental evaluation with transmission results over the 4-core CC-MCF. Figures 3(a)-(b) compare learning curves for MIMO-EQ filter coefficients obtained by LMS or RLS adaptations in terms of normalized mean squared error (MSE). Although the rate of convergence was almost identical in both adaptation schemes at 1105 km, it became slower for LMS adaptation at longer distances. This is because accumulated MDL directly affected on the eigenvalue spread of input signals into MIMO-EQ, which is seen in Fig. 3(c) with an estimation on root mean square (rms) MDL based on singular value decomposition of a transfer matrix. Also shown together is a theoretical curve [14], giving an rms MDL value per span of 0.45 dB. On the contrary, the learning curves by RLS adaptation was insensitive to increased transmission reach, fully converged by τ_{RLS}= 6 μs at all distances (Fig.3(b)). We also estimated a convergence time for the LMS-adapted cases, τ_{LMS}, by extrapolating the curves until reaching to baselines of MSE achieved by the RLS (dotted lines in Figs.3(a)-(b)). Fig.3(c) represents an accelerated learning rate defined by the ratio of τ_{RLS} to τ_{LMS} evaluated at each distance, showing that RLS adaptation provided a faster convergence at longer distance.

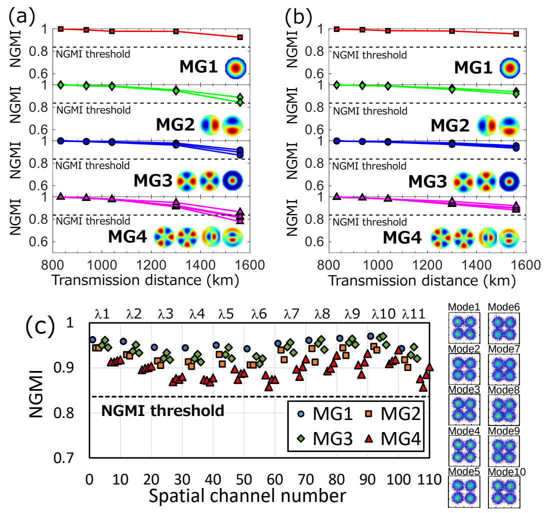


Fig. 4: NGMI for each MG in 10-mode FMF transmission at λ#7 obtained by (a): LMS-adapted MIMO-EQ or (b): RLS-adapted one. (c): NGMI for all wavelength/spatial channels at 1560 km with a representative constellations at λ#7. NGMI threshold of 0.836 assumes a combined use of LDPC and BCH codes [12,15].

Particularly, ten-fold faster learning of the filter coefficients was obtained at 5528 km. We also applied both FDE schemes to a long-haul 10-mode FMF transmission, whose normalized generalized mutual information (NGMI) results of each mode group (MG) are represented in Figures 4(a)-(b). Little NGMI difference between both FDE schemes was observed at distances of <1000 km, while decoupling for the mixed MDM signals was successfully performed at 1560 km only by RLS adaptation even in the presence of a peak-to-peak MDL exceeding 10 dB. From the NGMI results for all wavelength/spatial channels after 1560-km WDM/MDM transmission shown in Fig. 4(c), we confirmed that record-long-distance 10-mode FMF transmission was achieved by RLS-adapted OBE-FDE with a reach extension by 20% w.r.t. our previous work with 1300 km [12], while suppressing computational burden by 40% based on the discussion in the previous section.

Conclusions

We proposed a computationally efficient implementation technique for recursive least squares (RLS) adapted MIMO frequency-domain equalisation (FDE). Referred to as out-of-band exclusive FDE (OBE-FDE), the scheme achieves complexity reduction by omitting redundant computations on the outside bandwidth of signals. The advantages of RLS-adapted OBE-FDE were demonstrated in MDM transmissions, particularly achieving the world's longest MDL-tolerant 10-mode FMF transmission over 1560 km with a reduced computational complexity by 40%.

Acknowledgements

Part of this research is supported by NICT, Japan under the commissioned research of No. 01001.

References

[1] G. Rademacher, M. Hout, R.S. Luis, B.J. Puttnam, G.D Sciullo, T. Hayashi, A. Inoue, T. Nagashima, S. Gross, A.R Adams, M. Withford, J. Sakaguchi, C. Antonelli, C.M. Okonkwo, H. Furukawa, “Randomly Coupled 19-Core Multi-Core Fiber With Standard Cladding Diameter,” *in Optical Fiber Communication Conference 2023 (OFC2023)*, paper Th4A.4, 2023.

[2] G. Rademacher, R. S. Luís, B. J. Puttnam, N. K. Fontaine, M. Mazur, H. Chen, R. Ryf, D. T. Neilson, D. Dahl, J. Carpenter, P. Sillard, F. Achten, M. Bigot, J. Sakaguchi, and H. Furukawa, "1.53 Peta-bit/s C-Band Transmission in a 55-Mode Fiber," *in European Conference on Optical Communication 2022 (ECOC2022)*, paper Th3C.3, 2022.

[3] E. Ip, M. Li, K. Bennett, Y. Huang, A. Tanaka, A. Korolev, K. Koreshkov, W. Wood, E. Mateo, J. Hu, and Y. Yano, "146λ×6×19 Gbaud Wavelength and Mode Division Multiplexed Transmission over 10×50 km Spans of Few Mode Fiber with a Gain Equalized Few Mode EDFA," *in Optical Fiber Communication Conference 2013 (OFC2013)*, paper PDP5A.2, 2013.

[4] R. Ryf, J.C. Alvarado, B. Huang, J. Antonio-Lopez, S.H. Chang, N.K. Fontaine, H. Chen, R.J. Essiambre, E. Burrows, R. Amezcua-Correa, T. Hayashi, Y. Tamura, T. Hasegawa, and T. Taru, "Long-Distance Transmission over Coupled-Core Multicore Fiber," *in European Conference on Optical Communication 2016 (ECOC2016)*, paper Th.3.C.3, 2016.

[5] K. Shibahara, T. Mizuno, D. Lee, Y. Miyamoto, H. Ono, K. Nakajima, Y. Amma, K. Takenaga, and K. Saitoh., "DMD-Unmanaged Long-Haul SDM Transmission Over 2500-km 12-Core × 3-Mode MC-FMF and 6300-km 3-Mode FMF Employing Intermodal Interference Canceling Technique," *Journal of Lightwave Technology*, vol. 37, no. 1, pp. 138-147, 2019.

[6] D. Soma, S. Beppu, Y. Wakayama, S. Sumita, H. Takahashi, N. Yoshikane, I. Morita, T. Tsuritani, and M. Suzuki, “50.47-Tbit/s Standard Cladding Coupled 4-Core Fiber Transmission over 9,150 km,” J*ournal of Lightwave Technology*, vol. 39, no. 22, pp. 7099–7105, 2021.

[7] S.O. Arik, D. Askarov, and J. M. Kahn, “Adaptive frequency-domain equalization in mode-division multiplexing systems,” *Journal of Lightwave Technology*, vol. 32, pp. 1841–1852, 2014.

[8] S.S. Haykin, Adaptive filter theory. Pearson Education India, 2008.

[9] Z. Yang, J. Zhao, N. Bai, E. Ip, T. Wang, Z. Li, and G. Li, "Experimental demonstration of adaptive VFF-RLS-FDE for long-distance mode-division multiplexed transmission," *Optics Express,* vol. 26, no. 14, pp.18362-18367, 2018.

[10] Z. Pan, Y. Weng, X. He, and J. Wang, "Adaptive Frequency-Domain Equalization and MIMO Signal Processing in Mode Division Multiplexing Systems using Few-Mode Fibers," *in Advanced Photonics 2016*, paper SpW2G.1, 2016.

[11] K. Shibahara, T. Kobayashi, and Y. Miyamoto, "Out-of-band exclusive frequency-domain MIMO equalization with the reduced-overlap-and-save method for scalable mode multiplexed signal transmission," *Optics Express*, vol. 31, no. 2, pp. 2302-2315, 2023.

[12] K. Shibahara, M. Hoshi, and Y. Miyamoto, “10-spatial-mode 1300-km Transmission over 6-LP Graded Index Few-Mode Fiber with 36-ns Modal Dispersion,” *in Optical Fiber Communication Conference 2023 (OFC2023)*, paper M2B2, 2023.

[13] T. Hayashi, Y. Tamura, T. Hasegawa, and T. Taru, "Record-Low Spatial Mode Dispersion and Ultra-Low Loss Coupled Multi-Core Fiber for Ultra-Long-Haul Transmission," *Journal of Lightwave Technology*, vol. 35, no. 3, pp. 450-457, 2017.

[14] K.P. Ho and J.M. Kahn, "Mode-dependent loss and gain: statistics and effect on mode-division multiplexing," *Optics Express*, vol. 19, no. 17, pp. 16612-16635, 2011.

[15] K. Shibahara, T. Mizuno, H. Ono, K. Nakajima and Y. Miyamoto, "Long-Haul DMD-Unmanaged 6-Mode-Multiplexed Transmission Employing Cyclic Mode-Group Permutation," *in Optical Fiber Communications Conference and Exhibition 2020 (OFC2020)*, paper Tu3B.5, 2020.