Statistical Characterization of Modal Dispersion in Field-Deployed Multi-Core Fiber

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Abstract We present a comprehensive study of a field-deployed SDM system based on a coupled-core four-core fiber, providing the first detailed comparison between SDM theory and experimental data. The study includes validation of the model's accuracy and examination of the intensity impulse response (IIR) duration. ©2023 The Author(s)

Introduction

The fiber-optic communications industry is facing an imminent capacity crunch^[1], prompting the need for new and scalable technologies. One promising solution is SDM transmission^[2], which combines multi-core fibers (MCF)^[3] and multimode (MMF) fibers^[4]. SDM utilizes multiple-input multiple-output (MIMO) techniques that are implemented through electronic digital signal processing (DSP). To design efficient MIMO DSP algorithms, it is necessary to have an understanding of the statistics related to the signal propagation in SDM systems. Two key propagation effects in SDM transmission systems are modal dispersion (MD) and mode-dependent loss (MDL), and several models have been proposed to evaluate their impact on system performance^{[5],[6]}. A recent contribution is a unified model proposed in^[7], which considers the effects of both MD and MDL on the system's intensity impulse response (IIR). That work focused on a scenario commonly found in coupled-core multi-core fibers, where all modes are randomly coupled, and introduced a complex MD vector $\vec{\tau}$ that accounts for the presence of MDL. Using a mathematical representation of multiple-mode propagation, the model examines how MD and MDL accumulate along the link and describes their impact on the IIR^[8] of the system. The duration of the IIR determines the memory requirements for a MIMO-SDM receiver, and the model shows that it is related to the mean square value of $\vec{\tau}$. Nonetheless, to date, theoretical models for SDM fiber systems have only been validated through numerical simulations or experiments on spooled fibers. This paper presents a novel contribution: the first statistical analysis of experimental data measured in a field-deployed SDM fiber installation, located in the Italian city of L'Aquila^[9]. Our results show a strong agreement with the theory, providing a solid foundation for the random coupling model of SDM fiber links.

Theoretical model

In this section, we brief the theoretical model^[7] for the propagation of light in a fiber that supports 2Nmodes, where N is the number of spatial modes and the factor two accounts for polarization degeneracy. This model builds on the concept of the field vector $\vec{E}(z,t)$, which can be constructed by stacking the 2N complex envelopes, describing the excitations of the individual modes, on top of each other and denoted its Fourier transform by $\vec{E}(z,\omega)$. Linear propagation of light along the fiber can be described by the relation $\vec{E}(L,\omega) =$ $\mathbf{T}(L,\omega)\vec{E}(0,\omega)$, where $\mathbf{T}(L,\omega)$ is the transfer matrix of the fiber link and L is its length. In order to simplify the notation, in what follows we drop the dependence of T on L. Neglecting scalar factors common to all modes, the frequency dependence of **T** is given by^[7]:

$$\frac{\partial \mathbf{T}(\omega)}{\partial \omega} = i \mathbf{Q}(\omega) \mathbf{T}(\omega) = i \frac{\vec{\tau} \cdot \vec{\Lambda}}{2N} \mathbf{T}(\omega).$$
(1)

Here, Q is a matrix with eigenvectors representing the principal modes of propagation (PMPs), and the eigenvalues contain information about the delays associated with the PMPs^[10]. Specifically, the real part of the eigenvalues gives the corresponding modal group delay (GD), while the imaginary part provides information about loss. The quantity $\vec{\Lambda}$ that appears in Eq. (1) is a vector whose elements Λ_n are generalized Pauli matrices and $\vec{\tau}$ is the complex-valued generalized Stokes vector introduced in^[7], whose components are given by

$$\tau_n = -i \operatorname{Trace}\{\Lambda_n \mathbf{Q}(\omega)\}.$$
 (2)

The complex MD vector $\vec{\tau}$ extends the complex polarization-mode dispersion (PMD) vector^[11] to the multi-mode case in the presence of MDL, providing a complete characterization of both MD and MDL. In the single-mode case, the complex PMD vector is related to the distortion of the propagated signal^[12]. To proceed, it is important to characterize the statistical properties of the vector $\vec{\tau}$. This can be achieved by examining the theoretical expressions for two-frequency correlation functions, which have been derived in^[7].

$$f_{\vec{\tau},\vec{\tau}}(\omega) = \langle \vec{\tau}(\omega) \cdot \vec{\tau}(0) \rangle$$
$$= \frac{D}{\omega^2} \left\{ 1 - \exp\left[-\frac{\tau^2 \omega^2}{D}\right] \right\}$$
(3)

$$f_{\vec{\tau},\vec{\tau}^*}(\omega) = \langle \vec{\tau}(\omega) \cdot \vec{\tau}^*(0) \rangle$$
$$= \frac{D\tau^2}{\omega^2 \tau^2 - \alpha^2} \left\{ 1 - \exp\left[-\frac{\omega^2 \tau^2 - \alpha^2}{D}\right] \right\} \quad (4)$$

where $D=4N^2-1$ is the dimensionality of the generalized Stokes space. These expressions depend on the scalar parameters τ^2 and α^2 which can be extracted from experimental data. They account for mode coupling and the local effects of MDL, respectively. It is worth noting that only Eq. (4) is impacted by the presence of MDL, as seen through the parameter α^2 , whereas Eq. (3) retains the same expression as it would in the absence of MDL. The matrix Q, or equivalently the vector $\vec{\tau}$, provides a complete description of MDL. However, due to their complex nature and difficulty in measurement and interpretation, summary parameters are often employed to capture the phenomenological aspects of the link. For instance, the IIR^[8] is a useful parameter that provides valuable information about the link's behavior. The relationship between the MD vector and the IIR is discussed in the following. By analyzing the IIR, it is possible to determine the effects of mode mixing on the propagation of light in the fiber and to optimize the fiber's performance for specific applications. The IIR of a fiber is defined as follows: a frequency-flat broad-band signal is launched into the fiber exciting only one mode, and the total output power is measured as the sum of the output powers in all the modes. By exciting the various fiber modes one by one and by averaging the received power signals, we obtained the quantity I(t) in which we are interested. Mathematically, this quantity can be expressed as:

$$I(t) = \frac{1}{2N} \operatorname{Trace}[\mathbf{H}(t)\mathbf{H}^{\dagger}(t)]$$
 (5)

where $\mathbf{H}(t)$ is the inverse Fourier transform of the transfer matrix $\mathbf{T}(\omega)$. In the regime of strong mode mixing, the analytical expression of the average IIR is given by^[7]:

$$\tilde{I}(t) = I_0 \, \exp\left(-\frac{2N^2 t^2}{\tau^2}\right) \tag{6}$$

where I_0 is a normalization constant accounting for gain and loss. From Eq. (6), we would like to highlight the physical meaning of the parameter τ^2 , which represents the duration of the IIR.

Experimental results

In this section we validate the proposed model through a comparison with experimental data taken on a field-deployed cabled coupled-core four-core fiber installed in the city of L'Aquila^[13], Italy. The cable is 6.3 km long, contains 18 MCFs, and is installed in an underground tunnel. Specifically, it comprises 12 strands of coupled-core fourcore fiber and the link is formed by splicing together 11 of these strands, resulting in a total length of 69 km. The average MDL of the link is around 2.5 dB. A detailed description of the experimental setup as well as of the technique that was used to acquire the data can be found in^[14]. To obtain an accurate transfer matrix T for the fiber, it is essential to make coherent measurements of all the outputs from a given input. Once the transfer matrix \mathbf{T} has been obtained, we can characterize the GD of the fiber. To this end, we compute the matrix Q from Eq. (1) as:

$$\mathbf{Q} = -i \frac{\log \left[\mathbf{T}(\omega_n) \mathbf{T}^{-1}(\omega_{n-k}) \right]}{\omega_n - \omega_{n-k}}$$
(7)

where $\log m$ denotes the matrix logarithm, and the value of k determines the frequency step used to compute the derivative of **T**. Using the matrix **Q**, we can calculate the eigenvalues and determine the distribution of their real parts. This corresponds to evaluating the probability density

function (pdf) of the GDs. Figure 1 illustrates the marginal pdf of the GDs for the fiber under test, which supports eight modes of propagation (2N=8). The marginal pdf exhibits eight peaks,



Fig. 1: Marginal pdf of the GDs for a eight-mode fiber.

corresponding to the GDs of eight different principal modes, and it is consistent with the findings of^{[15],[16]}. Given the matrix Q and using Eq. (2), we can determine the frequency-dependent vector $\vec{\tau}$. Figure 2 showcases the first component among the D=63 components of $\vec{\tau}$ to provide an example. The remaining components exhibit analogous statistical properties. From $\vec{\tau}$ we can ex-



Fig. 2: Real and imaginary part of the first component of $\vec{\tau}$.

tract the two scalar parameters τ^2 and α^2 from $\langle \vec{\tau}\cdot\vec{\tau}\rangle=\tau^2$ and

$$\langle \vec{\tau} \cdot \vec{\tau}^* \rangle = \frac{D\tau^2}{\alpha^2} \left[\exp\left(\frac{\alpha^2}{D}\right) - 1 \right].$$
 (8)

Then, we compute the two autocorrelation functions and fit them with Eqs. (3) and (4), substituting the computed values of τ^2 and α^2 . The results of this procedure are presented in Figure 3. The two expressions are in fairly good agreement. In both figures, the dashed yellow curve corresponds to the result obtained by substituting the two scalar parameters, retrieved from the vector $\vec{\tau}$, into equations (3) and (4). The lower frequency correlation in the experimental data may be attributed to some noise in the measurement procedure or in the derivation of the matrix Q. Nevertheless, the experimental results confirm the theoretical prediction that the imaginary part of the correlation is largely negligible. The final step of our analysis involves calculating the IIR through the formula outlined in Eq. (5) and verifying its coherence with the theoretical expression described



Fig. 3: Real and imaginary part of the autocorrelation functions (a) $\langle \vec{\tau} \cdot \vec{\tau} \rangle$ and (b) $\langle \vec{\tau} \cdot \vec{\tau}^* \rangle$ versus frequency. The dashed yellow lines refer to the theoretical expressions in Eqs. (3) and (4)

in Eq. (6). In particular, using for τ^2 the value of 0.035 ns^2 extracted from the data, we can fit the computed IIR with the theoretical expression. The results of the fitting process are presented in Figure 4, which demonstrates an excellent agreement between data and theory. Additionally, the



Fig. 4: The solid line is a plot of the computed IIR, while the dashed line refers to the theoretical expression for the IIR from Eq. (6)

model proves that modifications to the system's MDL do not impact the IIR duration, despite the fact that both MD and MDL affect the accumulation of $\vec{\tau}$.

Conclusions

In summary, our work provides insights for the continued development of SDM fiber systems, enabling a deeper understanding of the effects of MD and MDL and validating theoretical findings in a real scenario. To the best of our knowledge, this study represents the first comprehensive comparison between SDM theory and experimental data measured in a field-deployed coupled-core multicore fiber for SDM transmission.

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