Optimized Geometric Constellation Shaping for Wiener Phase Noise Channels with Viterbi-Viterbi Carrier Phase Estimation

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Abstract *The Viterbi & Viterbi (V&V) algorithm is well understood for QPSK and 16-QAM, but modifications are required for higher-order modulation formats. We present an approach to extend the standard V&V algorithm for higher-order modulation formats by modifying the transmit constellation with geometric constellation shaping. ©2023 The Author(s)*

Introduction

In recent years, the optimization of geometric constellation shaping for communication systems impaired by Wiener phase noise has been a topic of great interest [1]–[4]. The optimization is typically performed using a state-of-the-art auto-encoder approach in a symbol-wise or bit-wise manner [5]. To incorporate effects of the communication channel and subsequent carrier phase estimation into the optimization, either a residual phase noise process [1]–[3] or a Wiener phase noise process with a carrier phase estimation [4] has been included in the training. In the first case, the influence of the constellation on the operation of the carrier phase estimation is neglected and only the impairment of the signal at the output of the carrier phase estimation is considered in the training. Compared to the second approach, this simplifies the optimization and training. In the second case, to perform end-to-end (E2E) optimization with gradient descent the operations of the carrier phase estimation have to be implemented in a differentiable manner. In [4], we have introduced a differentiable blind phase search (BPS) to optimize constellations geometrically. In this work, we explore this approach for the V&V algorithm and introduce geometrically optimized 64-ary constellations for a Wiener phase noise channel with V&V carrier phase estimation (CPE). We modify the V&V algorithm to include a learnable and differentiable partitioning in the geometrical constellation shaping optimization process.

Feed-forward Carrier Phase Estimation

For high-rate coherent optical communication receivers, the choice of the CPE algorithm is driven by the symbol rate compared to the processing speed of digital signal processing (DSP). As the symbol rate is multiple orders of magnitude higher than the signal processing rate of DSP, the use of feedback CPE—common in communication receivers e.g. for wireless communications—is not possible. A popular feed-forward CPE implementation is the BPS [6], which is widely used in modern optical communication receivers. Another feed-forward CPE implementation is the V&V CPE [7], where computational complexity is traded off for performance.

To perform CPE with the BPS, the squared distance to each constellation point needs to be calculated for each received complex symbol. For square quadrature amplitude modulation (QAM), the distance all of the constellation points must be calculated for 1/4 of the test phases. For higher-order constellations, this increases the computational complexity significantly. For V&V-based CPE, the complexity does not scale with the number of constellation points, but with μ, since the μ-th power of each received complex symbol is computed followed by averaging and an estimation of the phase. For square QAM constellations, usually $\mu = 4$ is chosen due to the four-fold symmetry. Since we apply geometric shaping, the parameter μ can be chosen freely to increase or decrease the symmetry in the constellation with respect to square QAM. This is of particular interest for the robustness to cycle slips, as a lowered μ reduces the rotational symmetry of the constellation and the region of phase unambiguity increases. Additionally, a smaller μ leads to a less complex CPE.

CPE based on the V&V algorithm are not well-suited for higher order QAM, as not all constellation points are located on the symmetry lines, and therefore the inclusion of those points will result in reduced phase estimation performance. Previous approaches to improve V&V-based CPE use

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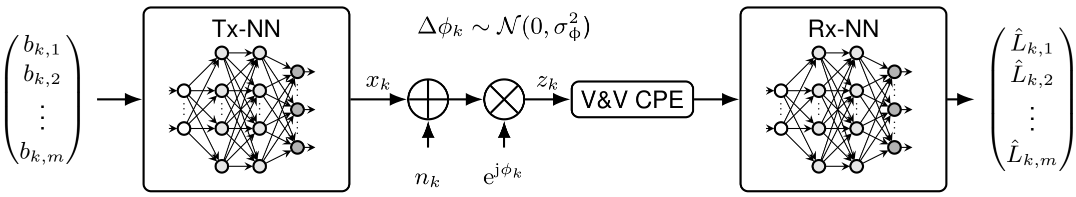

Fig. 1: System model to learn geometrically shaped constellations for V&V-based CPE.

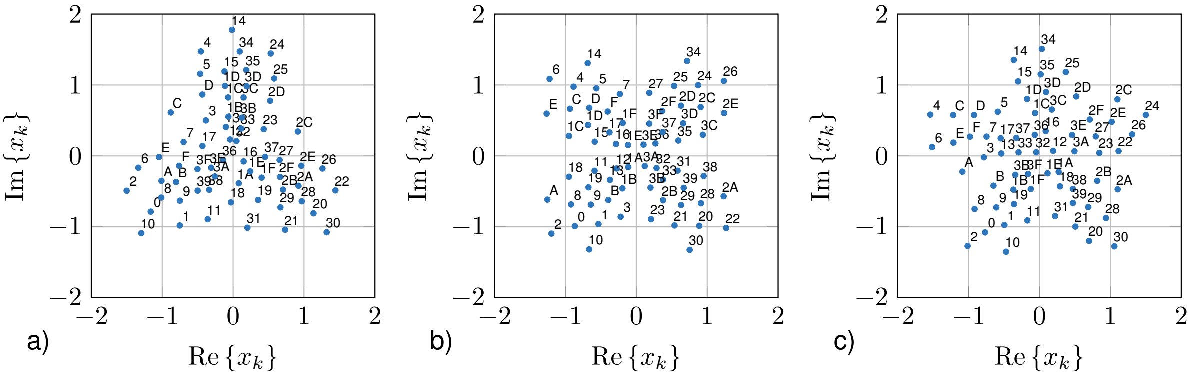

Fig. 2: Learned constellations for the V&V-based CPE for $m = 6\,\mathrm{bit/symbol}$ and a) $\mu = 3$, b) $\mu = 4$ and c) $\mu = 5$. Bit labels are obtained by converting the bit vectors to their hexadecimal representation, e.g., $(0, 1, 1, 1, 1, 1) \equiv 1F$

a system of partitioning the received constellation symbols based on their amplitude into multiple classes [8], [9]. In the second step, the V&V algorithm is only applied to symbols in a certain class. For the case of 16-QAM, only the outer- and innermost points are selected for the phase estimation. A solution that includes more points to improve the phase estimate of the V&V CPE rotates the received symbols to the symmetry lines [10]. Another approach to improve the CPE performance uses a multi-stage approach, where the first stage performs a coarse phase estimate at a lower complexity, while the following stages can refine the phase estimate using pre-compensated complex symbols [11]. Adding additional stages to the phase estimation increases latency and computational complexity. Therefore, we look at approaches that leave a single-stage V&V algorithm in place and instead adapt the transmit constellation to improve the performance.

Geometric Constellation Shaping with Modified V&V Algorithm

To improve the performance of the V&V algorithm for higher-order constellations, we apply geometric constellation shaping using the bitwise autoencoder approach. The V&V algorithm to obtain the phase estimate at time k for the received complex symbol z_k

$$\varphi_{k,\mathrm{est}} = \frac{1}{\mu}\mathrm{unwrap}\left(\arg\left(\sum_{k'=k-K}^{k+K} z_{k'}^{\mu}\right)\right), \quad (1)$$

is already a differentiable operation.Therefore we can insert the V&V in the E2E training with the bitwise auto-encoder as shown in Fig. 1. To perform geometric shaping, while still following the approach of partitioning, the previously presented partitioning approach [8] needs to be modified. Hard partitioning selecting certain symbols based on the amplitude will leave the optimization without any useful gradients and the partitioning cannot be directly included in the optimization. We introduce a novel, differentiable selection function that carries out the partition, with the goal to include many constellation points in the constellation optimization and phase estimate. The proposed selection function is

$$z'_{k,l} = \sigma\left(s_l(|z_k| - \theta_{0,l})\right)\sigma\left(s_l(\theta_{1,l} - |z_k|)\right)\frac{z_k}{|z_k|}, \quad (2)$$

which will apply a selection on each received complex symbol z_k to obtain $z'_{k,l}$. The function is parameterized by the trainable parameters s_l, $\theta_{0,l}$, and $\theta_{1,l}$ and we apply the non-linear Softplus activation function σ. To have multiple partitioning rings, we define multiple functions indexed by $l \in \{0, \ldots, L-1\}$ and calculate the phase estimate according to

$$\tilde{z}_{k,\mathrm{avg}} = \sum_{k'=k-K}^{k+K}\left(\sum_{l=0}^{L-1} z'_{k',l}\right)^{\mu} \quad (3)$$

$$\varphi_{k,\mathrm{est,mod}} = \frac{1}{\mu}\mathrm{unwrap}\left(\arg\left(\tilde{z}_{k,\mathrm{avg}}\right)\right). \quad (4)$$

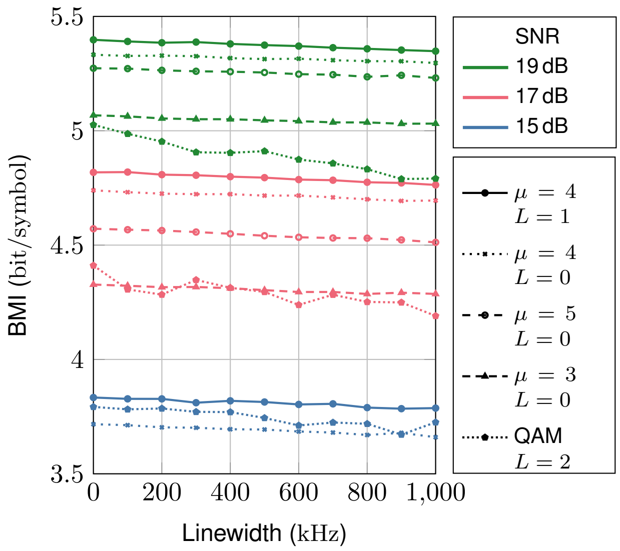

Fig. 3: Validation results for transmission with V&V-based CPE without partitioning and a genie-aided cycle slip compensation for $m = 6\,\mathrm{bit/symbol}$.

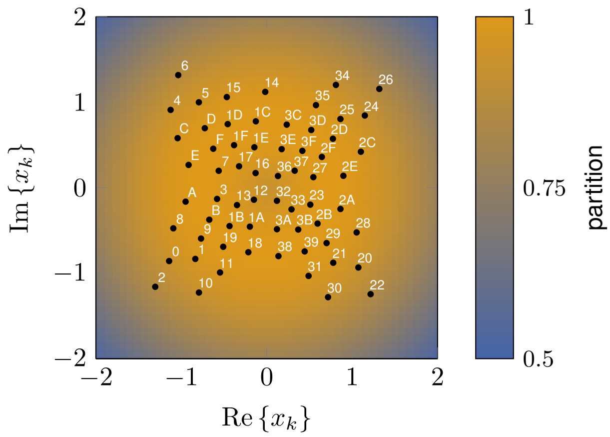

Fig. 4: Learned constellations for the V&V-based CPE with learned partitioning for $m = 6\,\mathrm{bit/symbol}$ and $\mu = 4$. Bit labels are obtained by converting the bit vectors to their hexadecimal representation, e.g., $(0, 1, 1, 1, 1, 1) \equiv 1F$

For $L = 0$ no partitioning is applied on the received symbols. In (3), a weighted average is calculated over $2K + 1$ neighboring phasors. The phase estimate $\varphi_{k,\mathrm{est,mod}}$ for the symbol at time step k is then obtained by taking the complex argument and performing phase unwrapping to have a continuous phase variation. In a subsequent step, we perform averaging across the neighboring phase estimates included in the partitioning to obtain phase estimates for constellation symbols that were not included in the partitioning step.

Results

Performing geometric constellation shaping (GCS) without partitioning for $\mu \in 2, 3, 4$, we obtain constellations with a significant amount of constellation points located close to the corresponding symmetry lines. We display the constellations for V&V CPE without partitioning in Fig. 2. With a bit-wise autoencoder approach, the bit labeling is also included in the optimization and we print the obtained bit labels for each constellation symbol in Fig. 2.

The training and validation were performed at a symbol rate of $R_\mathrm{S} = 32\,\mathrm{GBaud}$. For training, the signal to noise ratio (SNR) was fixed to $20\,\mathrm{dB}$, and the laser linewidth was fixed to $100\,\mathrm{kHz}$. For the validation, we use a genie-aided cycle slip compensation to simulate a system operating with a phase offset which can be corrected unambiguously with a V&V CPE. This can be achieved with a first stage CPE employing a pilot-based [9] or pilot-less [4] approach. Comparing the performance of the geometrically optimized constellations for varying μ in Fig. 3, we can observe that for lower SNRs the system with $\mu = 4$ shows the best performance and for high SNRs also the constellation and system with $\mu = 5$ approaches the performance of $\mu = 4$. From this, we conclude that the classical choice of $\mu = 4$ for square QAM also provides the best performance for a geometrically optimized constellation. The constellation trained with $L = 1$ for partitioning shown in Fig. 4 shows a consistently better performance compared to the constellation without partitioning. Since the partitioning is performed in a way, where most of the symbols are included in the V&V CPE, the increased performance might stem from the additional averaging step which is performed after the partitioning.

Conclusions

We present an approach to optimize the GCS of higher order modulations for V&V-based CPE. We firstly introduce our E2E optimization system based on the bitwise auto-encoder and then present a modified V&V CPE, which applies a differentiable partitioning on the constellation to further improve the performance. We compare the results of our approaches in terms of the bitwise mutual information (BMI) and find that optimization of the GCS allows for the application of the V&V CPE on higher order modulation formats without any partitioning. Application of partitioning further improves the performance, while adding some additional computational complexity to the DSP. Furthermore, an analysis of the exponent μ used in the V&V algorithm shows, that for 64-ary constellations and optimized GCS a value of $\mu = 4$ provides the best performance.

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