Enhanced Achievable Information Rate for NFDM Systems Using FBMC Wave-carriers

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Abstract *We demonstrate, for the first time, a high-capacity NFT-based transmission system, based on PHYDYAS wave-carriers, which is a variant of the FBMC technique, to obtain a significantly improved achievable information rate of 7.2 bits/s for NFDM systems.*

Introduction

The nonlinear frequency division multiplexing (NFDM) scheme, which is based on the nonlinear Fourier transforms (NFT) concept, has been shown to offer immunity against Kerr nonlinearity and dispersion in optical fiber transmission systems. However, some issues such as low achievable information rate (AIR) and the interaction of the optical signal with the inline amplifier noise still constitute a major setback for NFT-based schemes[1]. In this work, we examine for the first time, the use of the PHYsical layer for Dynamic Spectrum Access (PHYDYAS), which is a type of filter bank multi-carrier (FBMC) method, as a wave-carrier in NFT-based systems. The performance of the PHYDYAS wave-carrier is then compared with the Hermite-Gaussian (HG) based NFT schemes, which has been demonstrated to offer better performance than the traditional sinc-based methods. We demonstrate that the proposed PHYDYAS-based NFT system offers a high AIR of up to 7.2 bits/symbol and shows a highly desirable resilience to inline amplifier noise when compared to HG-based methods.

Channel Model of the Proposed PHYDYAS-based NFT Transmission

NFT-based systems entail the transmission of optical signals in the nonlinear Fourier domain and propagated independently based on the nonlinear Schrodinger equation (NLSE), which can be described as[1],[2]

$$\frac{\partial E}{\partial Z} + j\frac{\beta_2}{2}\frac{\partial^2 E}{\partial T^2} - j\,\gamma\,|E|^2 E + \alpha E = n(Z,T), \quad (1)$$

for a single polarization, noiseless channel, where Z represents the coordinate along the fiber, β_2 denotes the chromatic dispersion (CD) coefficient, which is negative for a standard single mode fiber (SSMF), while γ represents the nonlinear Kerr coefficient. Also, α denotes the fiber loss while T describes the time in the frame co-moving with the envelope of the signal E. Also, $n(Z,T)$ represents the amplifier spontaneous emission (ASE) noise, which is assumed to be a complex white Gaussian process. The lumped amplification scheme is employed, where N_a amplifiers are positioned at the tail end of lossy spans of length L_a for signal regeneration. A path-averaged model is employed for the proposed system[3],[4], where the field distribution is averaged over short amplifier spans, leading to effectively lossless NLSE, described as $\gamma_{eff} = \gamma(G-1)/(G\ln G)$, $G = \exp(\alpha L_a)$[3]. In NFT-based systems, it is customary to describe the model in the normalized soliton units, such that the time is normalized to a characteristic pulse duration T_n, where $T = t/T_n$, while the envelope is normalized to a nonlinear power $q = E\,(\gamma L_D)^{1/2}$. Also, Z is normalized by a dispersion length, $L_D = T_n^2/|\beta_2|$.

For a typical NFDM scheme, all the steps involved in the transmission process are well detailed in[1],[5]. Now considering an NFT-based transmission scheme, where b-modulation is employed[5],[6], the modulated data at the transmitter can be expressed as

$$u(0,\xi) = \sqrt{S}\sum_{n=1}^{N} c_n\,\mathrm{sinc}(\xi - n). \quad (2)$$

The expression in (2) can be termed as the loading spectrum, where c_k is a set of N symbols modulating N shifted orthogonal nonlinear subcarriers. The loading spectrum is then mapped to the b-coefficient as detailed in[5],[6] to ensure the constraint $|b| < 1$. Also, S represents the effective parameter that defines the average power per carrier.

The sinc-based method as implemented in[1] offers low spectral efficiency due to the unique characteristics of the sinc carriers. In[7],[8], the HG-based NFT transmission system was shown to outperform sinc-based methods and this is due to the excellent time-bandwidth-product efficiency of HG pulses. In this work, we therefore, seek to implement a more efficient wavecarrier, based on the FBMC technique and compare the results with the HG pulses.

Like the HG wavecarriers, the FBMC pulses are well localized in the time and (nonlinear) frequency domain. The FBMC technique possesses low out-of-band (OOB) emission and is also robust against intersymbol interference (ISI), even without cyclic prefix, as seen in the wireless domain[9]–[11]. In order to maximize the gains of the FBMC technique, the PHYDYAS project was proposed in[9],[10]. The main idea behind the implementation of PHYDYAS is the replacement of the rectangular pulse shaping filter with a carefully designed filter with excellent localization properties and low spectral leakage[10]. The PHYDYAS filter is realized by determining the frequency coefficients that agrees with the Nyquist criteria[9],[12]. The obtained coefficients are then used to determine the frequency response by interpolation. The frequency coefficients obtained are dependent on the overlapping factor $K = L/N$, where N represents the number of subcarriers, while L denotes the number of coefficients of the impulse response of the filter. Therefore, considering an NFT-based transmission scheme, where the PHYDYAS filter is employed, the expression in (2) becomes

$$u(0,\xi) = \sqrt{S} \sum_{n=0}^{N-1} c_n \, v(\xi). \quad (3)$$

and $v(\xi)$ is given as[10],[12]

$$v(\xi) = \sum_{k=-(K-1)}^{K-1} R_k \frac{sin(\pi(\xi - \frac{k}{NK})NK)}{NKsin(\pi(\xi - \frac{k}{NK}))}. \quad (4)$$

where R_k represents the frequency coefficients. The values of the frequency coefficients for overlapping factor $K = 2, 3, 4$ are shown in Table 1.

Tab. 1: Frequency coefficients for different values of K

K	R_0	R_1	R_2	R_3
2	1	0.707106	-	-
3	1	0.911438	0.411438	-
4	1	0.971960	0.707106	0.235147

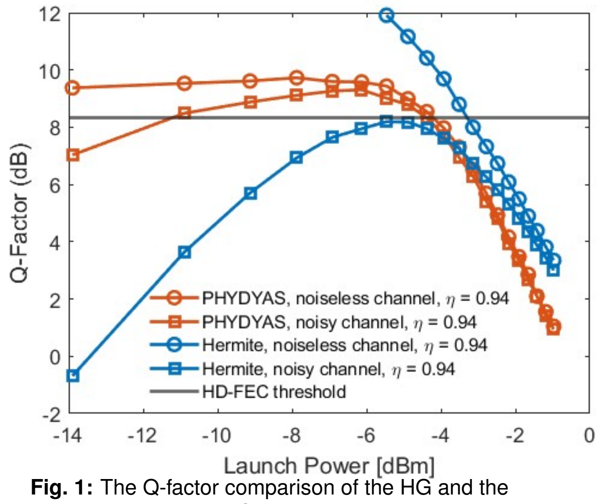

Fig. 1: The Q-factor comparison of the HG and the PHYDYAS wavecarriers.

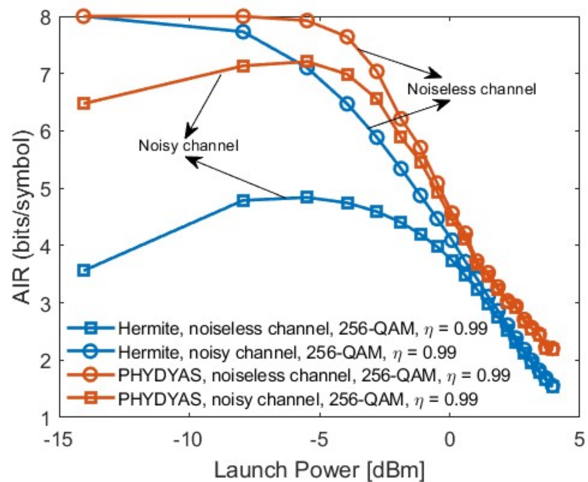

Fig. 2: The comparison of the AIR performance of PHYDYAS and HG wavecarriers for a 256-QAM system.

For this work, we have used an overlapping factor of $K = 4$, so as to reduce interference from adjacent side lobes and to significantly minimize OOB emissions[9],[10],[12]. Also, employing the PHYDYAS filter requires that orthogonality needs to be maintained only for neighbouring subcarriers[9], thereby, ensuring a simple demodulation in a similar way as the Hermite-Gaussian pulses.

Simulation and Discussion

For the simulations, we have considered high order QAM formats, using 32 and 256-QAM, with total propagation length of 960 km. We used practical values for channel parameters $\beta_2 = -21$ ps^2/km, $\gamma = 1.27$ km^{-1}W^{-1}, $\alpha = 0.2$ dB/km, EDFA noise figure $NF = 5$ dB and $N = 64$ subcarriers. Also, to account for the practical interactions between neighboring bursts, multiple bursts were transmitted simultaneously. The Q-factor and AIR (as detailed in[1]) were used

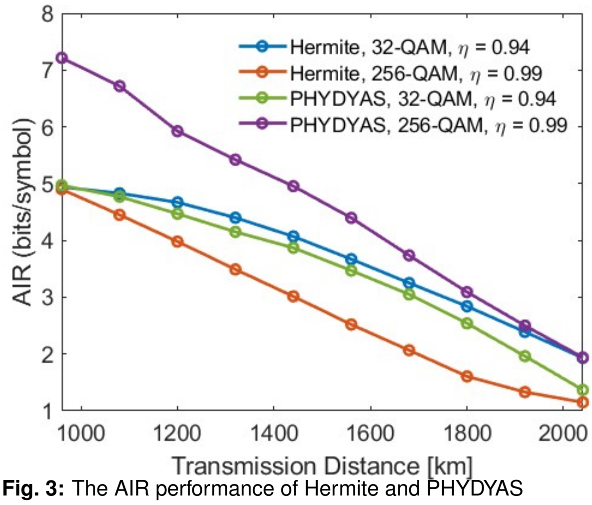

Fig. 3: The AIR performance of Hermite and PHYDYAS wavecarriers versus the transmission distance.

as figures of merit. The Q-factor was calculated via the bit error rate (BER), using $Q = 20\lg(\sqrt{2}\mathrm{erfc}^{-1}(2\,BER))$), where the BER was obtained by direct counting.

Since it has been shown in[7] that the performance of sinc carriers even get worse with $\eta < 1$ (η being the spectral efficiency penalty[1]), this work therefore, focuses on the comparison of PHYDYAS and Hermite carriers, as we seek to achieve high AIR, while using a strict constraint of $\eta < 1$. Also, the considered wavecarriers are implemented without the use of any pilot scheme or equalization algorithm at the receiver.

In Fig. 1, we compare the performance of HG pulses with PHYDYAS wavecarriers. As reported in[5], HG pulses achieved the most desirable BER/SE performance at $\eta = 0.94$. Hence, we have used this value of η to fairly compare the performance of the two wavecarriers for a 32-QAM system with a baudrate of 17 Gbaud. The plot shows that HG pulses outperforms PHYDYAS for the entire range of launch power in a noiseless channel, impacted only by the processing noise. However, in the presence of the ASE noise, the performance of HG pulses drops significantly, while PHYDYAS shows greater resilience. The results clearly shows an important attribute of the PHYDYAS filter design, which is very robust against interference and noise. These much desired attributes have therefore been carried over to the nonlinear optical domain.

In Fig. 2, the corresponding AIR performance for HG and PHYDYAS wavecarriers is shown for a 256-QAM system, with a baudrate of 26 Gbaud, in both noiseless and noisy channel. For a high modulation order of 256-QAM, where the effect of interference is more conspicuous, PHYDYAS attained an AIR of 7.2 bits/symbol and effectively outperforms HG carriers, which only achieved an AIR of 5.0 bits/symbol. If we go by the definition of the spectral efficiency used in[1],[13], the 256-QAM PHYDYAS-based system will therefore, give a spectral efficiency of $SE = \frac{AIR}{\eta}$ = 7.27 bits/s/Hz. Thus, PHYDYAS wavecarriers show better performance, especially when compared to[14], where the maximum SE obtained is 3.6 bit-s/s/Hz and more recently in[14], where the maximum SE obtained is 5.5 bits/s/Hz. Therefore, the results show the robustness of PHYDYAS, especially at high powers, where the performance of HG pulses are significantly degraded. Although both wavecarriers are known to possess excellent localization properties, the special attributes of PHYDYAS, stemming from the careful filter design have ensured better performance.

The plot of the AIR versus the transmission distance is shown in Fig. 3. It can be seen from the plot that the Hermite and the PHYDYAS wavecarriers have a relatively similar performance for a 32-QAM system, over the transmission distance considered. However, for a higher-order 256-QAM system, where the impact of interference and ASE noise is more significant, the AIR performance of PHYDYAS is conspicously better than that of Hermite, over the entire haul. This further confirms the innate robustness of PHYDYAS wavecarriers against interference as well as ASE noise and also shows their superiority, especially for long-haul optical communication systems.

Conclusion

In this work, employing PHYDYAS filters has enabled us to achieve a considerably high AIR of 7.2 bits/symbol for a single polarization NFT-based transmission system. The results in this work are of great interest as no pilot or equalization algorithm was employed. Therefore, the reported AIR can be improved upon, if efficient training and equalization techniques are implemented at the receiver. Also, this work shows that employing carefully designed filters like PHYDYAS, significantly enhances the immense potential of NFT-based systems, as we seek viable methods for future nonlinearity-resistant optical communication systems.

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