Fiber-Optic Communications Based on Finite-Genus Solutions to the NLS with a Convolutional Neural Network Receiver

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Abstract *We present a novel fibre-optic transmission system based on the phase modulation via nonlinear Fourier transform for finite-genus signals with no periodicity constraint, which is achieved through designing a neural network-based receiver and demonstrate, as a proof of concept, signalling below 7% HD-FEC BER threshold.*

Introduction

The sustained growth of global data traffic and the quest for high-capacity communication networks have spurred significant interest in the development of advanced fiber optic communication systems. Central to the design and analysis of these systems is the Nonlinear Schrodinger (NLS) equation, which models the interplay between nonlinear and dispersive effects in optical signal propagation. This paper presents a pioneering method that combines the nonlinear Fourier transform (NFT) for finite-genus solutions[1] to the NLS equation and a neural network-based receiver, where we are able to get rid of the design deficiencies pertinent to the previously proposed communication systems of such a type.

NFT-based fiber-optic communication methods are based on the idea of transforming NLS equation for the signal propagation down the fiber into linear evolutionary equations inside the nonlinear Fourier domain. NFT-based transmission harnesses the linearizing transform for the NLS equation, allowing simultaneously compensating for dispersion and nonlinearity[1]–[3]. However, despite the promise that the "conventional" NFT methods have shown[4], there are several limitations that impact their performance. First, high computational complexity is a primary challenge of NFT-based methods[5]. Then, conventional NFT signals are assumed to be transmitted in the burst mode with substantial guarding intervals to avoid cross-talk[4]. Moreover, such systems provide very little control over signal duration and bandwidth. And last but not least, the presence of in-line noise can significantly affect the accuracy of the NFT technique, subsequently degrading the overall system performance[3].

To deal with these issues, the periodic NFT (PNFT) has been proposed as an alternative technique. PNFT approach provides control over the signal's duration and bandwidth as well as reduces the processing window at the receiver and noise impact[6]. The implementation of the PNFT-based transmission systems has been thoroughly investigated using the algebro-geometric approach[7],[8]. The algebro-geometric approach is linked to the computationally expensive Riemann theta function[9],[10], which makes this method rather impractical. As an alternative, the Riemann-Hilbert Problem (RHP) approach was proposed[11]. It is based on an analytic factorization problem in the nonlinear Fourier domain and has a computational complexity that is linearly proportional to the number of signal samples, which allows parallelization for efficient computing[6],[12],[13]. However, the methods mentioned above were constrained to use the exactly periodic signals, i.e. each supersymbol was appended with a cyclic prefix, similarly to coherent optical OFDM.

In this study, we introduce a fiber optic communication system based on the NFT for finite-genus NLS solutions, employing the RHP approach at Tx to modulate the phases and convolutional neural networks at Rx to demodulate the symbols. This methodology circumvents the limitations inherent in previous techniques[12],[13]: (i) we have devised a strategy for signal processing that does not require exact periodicity of a processed supersymbol, and (ii) we have surmounted the constraints on signal power and phases variation intervals from Refs.[12],[13]. This work addresses a proof-of-concept system, wherein we present our design proposal and provide estimates of its performance. We reserve an extensive array of

[1] In the mathematical and physical literature, finite-genus solutions are often referred to as finite-band or finite-gap solutions.

optimization-related inquiries for subsequent research.

Channel model and finite-genus solutions

The governing equation for signal propagation in an optical fiber is the NLS equation, written as:

$$iq_z - \frac{\beta_2}{2} q_{tt} + \gamma |q|^2 q = n(t, z), \quad (1)$$

where $q(t, z)$ denotes the signal envelope, with z being the coordinate along the optical fiber and t representing the temporal variable, β_2 is the chromatic dispersion, γ is the nonlinearity coefficient, $n(t, z)$ is the amplified spontaneous emission noise term.

Utilizing the RHP approach, we construct genus-N solutions associated with their main spectrum[13], consisting of points on the complex upper half-plane $\{\lambda_j \in \mathbb{C}\}_{j=0}^{N}$ and their complex conjugates. A particular genus-N solution is then specified by $N+1$ real-valued parameters, referred to as *phases* ψ_j, each lying in the interval $[0, 2\pi)$, see Refs.[13]–[15] and references therein.

We control the signal's duration and power by adjusting the main spectrum. The set of main spectrum points $\{\lambda_j\}_{j=0}^{N}$ determines the frequencies C^f of $N+1$ partial nonlinear modes for the genus-N solution: $\{C_j^f \in \mathbb{R}\}_{j=0}^{N}$ which are generally incommensurable. For our system, we process the largest period corresponding to the lowest C^f. To adjust the signal's power, we manipulate $\operatorname{Im} \lambda_j$.

As genus-N signals propagate through an optical fiber, the phases of partial nonlinear modes exhibit a trivial evolution:

$$\psi_j(z) = \psi_j(0) + (C_j^g - 2g_0)z, \quad (2)$$

where z represents the propagation distance and the constants g_0 and $\{C_j^g \in \mathbb{R}\}_{j=0}^{N}$ are determined by the main spectrum. This property is exploited to compensate for the signal's evolution at the receiver.

Convolutional neural network-based receiver

The core component of the proposed communication system is a receiver that relies on convolutional neural networks (CNNs)[14]. CNNs have demonstrated their effectiveness in processing signals within the context of NFT systems[16]–[18]. Our receiver processes the signals after their propagation through an optical fiber, and extracts the central portion of each supersymbol corresponding to the largest symbol's period, effectively removing the extension prefixes used to protect the supersymbols from overlapping/crosstalk along the propagation. The processed signal is then fed to the CNN's input layer, using 128 samples per signal. The goal of the CNN is then to retrieve the $N+1$ phases corresponding to the partially nonlinear modes embedded within the genus-N supersymbol.

The neural network architecture we use is similar to that from Ref.[14]; it comprises three convolutional layers and one fully-connected layer, Fig. 1. The CNN's hyperparameters have been optimized using Bayesian optimization[16],[19] for the specific symbols used in the transmission simulations. It turned out that the optimal distribution of hyperparameters is the same for all power levels studies and the resulting hyperparameter values are presented in Table 1. For further details regarding the implementation of the CNN see Ref.[14].

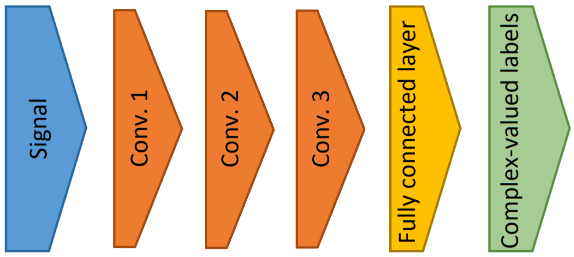

Fig. 1: Schematic of the CNN-based receiver used in our work for soft symbols.

The CNN produces complex-valued outputs that represent points on the unit circle. This choice is motivated by the need to ensure the periodicity of the labels corresponds to the phase periodicity of the solution. From the given points on the unit circle, the phases can be unambiguously retrieved.

	Filters	Kernel size	Activation
1 conv.	94	3	tanh
2 conv.	112	17	tanh
3 conv.	145	18	sigmoid
Fully-con.	128 neurons		sigmoid

Tab. 1: The hyperparemeters of the receiver CNN in Fig. 1.

Performance estimation

In the fiber-optic communication system illustrated in Fig. 2, we employ genus-4 solutions to the NLS equation (5 nonlinear modes per supersymbol). By manipulating the main spectrum of such solutions we can tune the signal duration and power, while the phases are used for data modulation. The following configuration of

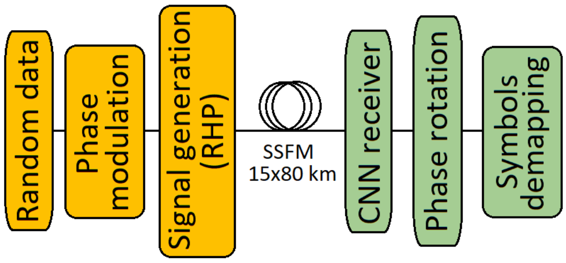

Fig. 2: Communication system principal scheme.

the main spectrum λ has been used for $N = 4$:

$$\lambda = \{-2 + ai, -1 + ai, ai, 1 + ai, 2 + ai\}, \quad (3)$$

where parameter a defines the power of the signal. The signal's duration (the longest period) was fixed to 1 ns.

Specifically, we implement 16-phase shift keying (16-PSK) modulation for each phase of the genus-4 solutions (5 phases), thus yielding 5×4 bits per supersymbol. To prevent signal overlap during propagation, extension prefixes are incorporated between supersymbols. To ensure that the central parts of adjacent signals do not overlap, we select extension prefixes substantially larger than required[2],[8]. Each supersymbol extends the duration of the central part by a factor of five. These supersymbols are then concatenated into lengthy sequences and input into the system as a single signal.

We investigate the propagation of optical signals in a system consisting of 15 spans (80 km each), resulting in an overall system length of 1200 km. The system employs standard single-mode fibers (SSMF) characterized by dispersion parameter $\beta_2 = -21.7\,\mathrm{ps}^2/\mathrm{km}$ and nonlinear coefficient $\gamma = 1.3\,\mathrm{W}^{-1}\mathrm{km}^{-1}$. An ideal distributed amplification model is utilized, with noise introduced at the end of each span. The noise power is expressed as $N_{ASE} = \alpha L \hbar \nu_s K_T NF$, where $\alpha = 0.2\,\mathrm{dBm/km}$ is fiber loss, $L = 80\,\mathrm{km}$ denotes the span length, $\hbar\nu_s$ is photon energy, $K_T = 1.13$, and $NF = 4.5\,\mathrm{dB}$ is the noise figure.

For each power level, we evaluate the BER as a metric for efficiency as a direct count of error bits within a signal comprising 2×10^4 symbols. The system performance, characterized by the relationship between BER and signal power, is depicted in Figure 3. Our results demonstrate successful data transmission at power levels of $\approx -6.3\,\mathrm{dBm}$ and $\approx -5\,\mathrm{dBm}$ below the FEC threshold, with a value of 3.8×10^{-3} and a 7% overhead[20].

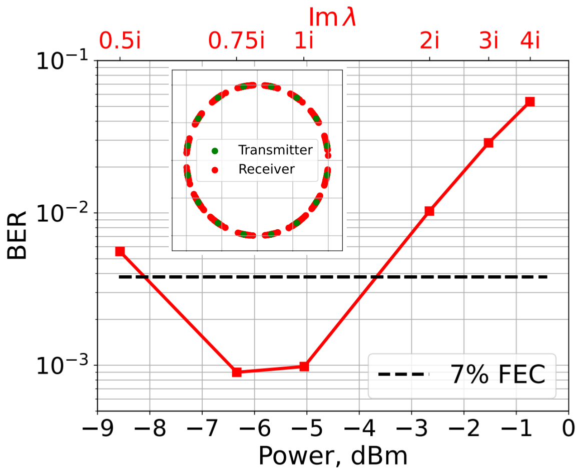

Fig. 3: BER versus signal power (lower axis) and corresponding values of $\mathrm{Im}\,\lambda$ (upper red axis). The black dashed line depicts 7% HD-FEC threshold[20]. The inset illustrates the symbol's phase distribution corresponding to the optimal power level $\approx -6.3\,\mathrm{dBm}$.

Conclusion

Although finite-genus solutions have been employed as data carriers in previous communications studies, those systems suffered from additional constraints imposed on the solution classes utilized and on phases allowed extent. Our novel approach, which utilizes CNN for phase retrieval at the Rx, relieves the finite-genus signals-based communications from the limitations of prior works. The general and adaptable nature of the proposed methodology makes this system both efficient and versatile for application in real-world systems.

Our proof-of-concept research aims to demonstrate the feasibility of data transmission below the 7% HD-FEC threshold and to lay the groundwork for future studies. The presented communication system can be significantly enhanced through the following approaches: (1) more extensive engagement of neural networks not only for phase retrieval of signals but also for addressing system imperfections, such as non-zero gain-loss profiles and noise, (2) comprehensive optimization of finite-genus solution parameters, including genus, main spectrum configuration, and cyclic extension length, and (3) developing theory for finite-genus solutions applied to the Manakov system, which describes data propagation in optical fibers with two polarizations. Addressing these crucial issues requires further investigation.

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