Experimental Demonstration of Physical-Layer Network Coding Based on Residual Field of Algebraic Integers in OFDM-VLC Two-Way Relay Networks

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Abstract *We propose a decode-and-forward (DF) physical-layer network coding (PNC) scheme based on residual field of algebraic integers for OFDM-VLC two-way relay networks. Experiments with up to 1.47-Gbit/s throughput show that the proposed scheme enables DF-PNC of high-order QAM and significantly outperforms the conventional amplify-and-forward scheme.*

Introduction

Visible light communication (VLC) is a promising solution for the future 6G, yet it is sensitive to occlusion in the link due to its inherent line-of-sight dependence. Relay-assisted physical-layer network coding (PNC) can effectively solve this issue and simultaneously improve the throughput [1-6]. For example, in a typical two-way relay network (TWRN) where two users under blockage exchange information via a relay, PNC can reduce the required time slots from 4 to 2.

Current PNC works mainly adopt the amplify-and-forward (AF) strategy: the relay detects the signals from the two users simultaneously that are superposed in the power domain, and then broadcasts the superposed signal back to users. However, the superposed signal has a larger constellation size and thus is susceptible to noise. Decode-and-forward PNC (DF-PNC), on the other hand, re-encodes the superposed signal to reduce the constellation size so that the performance at the broadcast stage can be improved [3],[4]. Conventional DF-PNC uses XOR for re-encoding and this method is only applicable to BPSK or QPSK. PNC based on Gaussian integers was proposed in [5],[6] for higher-order QAM formats, however, the performance is sensitive to the quality of uplinks.

In this paper, we propose a novel DF-PNC scheme based on the residual field of algebraic integers. The proposed scheme can be applied to any-order QAM and re-encode the superposed signal at the relay with a small constellation size, whilst guaranteeing that the symbols of one user are solely determined by those of the other user and the re-encoded signal. OFDM-VLC TWRN experiments using uniform 8QAM/16QAM and adaptively-loaded QAMs show that the proposed scheme outperforms the conventional AF-PNC with one order of magnitude reduction in the BER.

Principle

Fig. 1(a) shows a TWR network, wherein users A and B exchange information with each other. Because the LOS link is blocked, node R is used as the relay. PNC reduces the required time slots from 4 to 2 compared to traditional solutions: At the 1st time slot, users A and B send signals x_A and x_B simultaneously, which are superposed at node R, resulting in a large constellation size, as shown in Figs. 1(b)-(c); At the 2nd time slot, node R broadcasts signal x_R back to users A and B, based on which each user can then extract the targeted information using its own information. As aforementioned, the superposed signal has a large constellation size and is vulnerable to noise in conventional AF-PNC. As such, we propose a PNC scheme to group the constellation points of the superposed signal and re-encode them to a smaller constellation, as depicted in Figs. 1(c)-(d).

The key problem is how to re-encode the superposed signal, which should meet: 1) x_A (or x_B) should be solely determined by x_R and x_B (or x_A); 2) the minimal Euclidean distance between groups of the superposed constellation, $u(x_A, x_B)$, should be maximized.

The proposed PNC is based on the residual field of algebraic integers. The algebraic integer is defined as $Z(\xi)=a+b\cdot\xi$, where a and b are integers and ξ is:

$$\xi=\begin{cases}\sqrt{D} & D\equiv 2 \text{ or } 3 \pmod 4\\ \left(-1+\sqrt{D}\right)/2 & D\equiv 1 \pmod 4\end{cases} \tag{1}$$

where D<0. When D=-1, $Z(\xi)$ degenerates to the Gaussian integer $a+b\cdot(-1)^{1/2}= a+b\cdot i$. The residual field of $Z(\xi)$ is denoted as $Z(\xi)/p$, where p is a prime number in $Z(\xi)$. It represents the remainder of $Z(\xi)$ divided by p and has $|p|$ elements. Fig. 1(e) shows the representative elements of

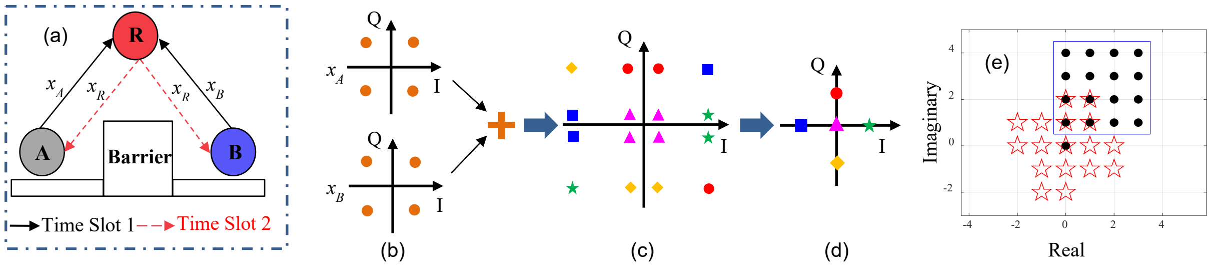

Fig. 1: (a) PNC scheduling in TWRN; (b) 4QAM constellations of an OFDM subcarrier at nodes A and B; (c) The superposed signal at the relay R. 16 superposed points are categorized into 5 groups $u(x_A, x_B)$; (d) Constellation of re-encoded x_R; (e) Two sets (stars and dots) of representative elements for Z(ξ)/(4+i) when D=-1. The dots in the blue square are selected as 16QAM.

Z(ξ)/(4+i). The total number is $|4+i|=17$. Note that the representative elements can be selected in different ways, as shown by the dots set_{dot} and the stars set_{star} in Fig. 1(e).

Tab. 1: p for 8QAM and 16QAM under different D. $Z(\xi)$ is not a Euclidean ring for D=-5, -6, which are not considered here.

	D=-1	D=-2	D=-3	D=-7
8QAM	3	3+ξ	4+3ξ	3+2ξ
16QAM	4+ξ	3+2ξ	5+2ξ	5+2ξ

The constellations of the transmitted signals at users, x_A and x_B, are selected from Z(ξ)/p, under the condition that the constellation size M should be no more than $|p|$. For example, p=4+i and the points circled by the blue square in Fig. 1(e) can be used for 16QAM when D=-1. Note that there is an offset whose effect can be eliminated by adding a bias. Tab. 1 shows the value of p used for 8QAM and 16QAM under different D. In OFDM, Hermitian extension and IFFT are applied to generate a real time-domain signal. At the relay, after transformed back to the frequency domain, the superposed signal at a OFDM subcarrier, $y_R(x_A, x_B)$, is:

$$y_R = h_A x_A + h_B x_B + n_R \tag{2}$$

where h_A (or h_B) is the channel response between user A (or B) and relay R. n_R is the noise. y_R has M^2 constellation points. The relay would re-encode these constellation points to $|p|$ points by:

$$x_R = \alpha \cdot x_A + \beta \cdot x_B \pmod{p} \tag{3}$$

where α and $\beta \in$ Z(ξ)/p\{0}. Accordingly, the constellation of y_R is categorized into $|p|$ groups $u(x_A, x_B)$: if (x_{A1}, x_{B1}) and (x_{A2}, x_{B2}) are in the same u, $x_R(x_{A1}, x_{B1}) = x_R(x_A, x_B) \pmod{p}$, otherwise $x_R(x_{A1}, x_{B1}) \neq x_R(x_A, x_B) \pmod{p}$. Because Z($\xi$)/$p$ is a field, the inverses of α and β are in Z(ξ)/p\{0}. Therefore, user A can solely recover x_B using x_R and its own information:

$$x_B = \beta^{-1}(x_R - \alpha x_A) \pmod{p} \tag{4}$$

Similarly, user B can recover x_A from x_R and its own information x_B. In theory, any α and β can realize Eq. (4). However, given the channel h_A and h_B, there are optimal α and β that maximize the minimal distance between $u(x_A, x_B)$, $d_{min}^{(\alpha,\beta)}$:

$$\left(\alpha_{opt}, \beta_{opt}\right) = \underset{(\alpha,\beta)\in Z(\xi)/p\setminus\{0\}}{\arg\max}\; d_{\min}^{(\alpha,\beta)} \tag{5}$$

Fig. 2 shows an example of (α, β) and its corresponding $d_{min}^{(\alpha,\beta)}$.The optimal (α, β) can be calculated beforehand and set according to the channel in practice. In addition to (α, β), d_{min} also depends on the phase of the channel coefficient ratio h_A/h_B, which can be pre-aligned at the transmitters of users A and B.

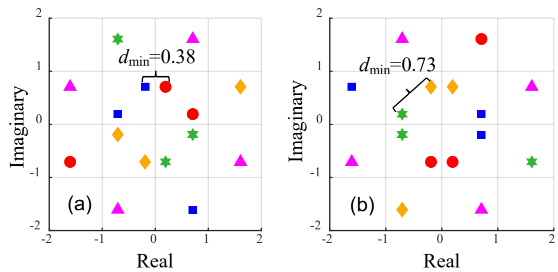

Fig. 2: (a) Classification results and d_{min} when (α, β) is (a) (1,1+i) and (b) the optimal (1, i). D=-1, h_A = 0.9·exp(jπ/4), and h_B = 1. The formats at users A&B are 4QAM. The superposed signal y_R has 16 points, which are classified into five groups.

Experimental setup and results

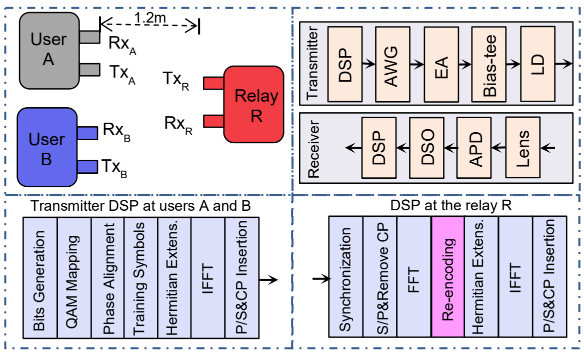

Fig. 3: Experimental setup. AWG: arbitrary waveform generator; EA: electrical amplifier; LD: laser-diode; APD: avalanche photo-diode; DSO: digital storage oscilloscope.

Fig. 3 shows the experimental setup. The bits of two users were mapped into 8QAM, 16QAM or adaptively-loaded QAMs. The phase of each subcarrier was aligned to maximize d_{min}. Hermitian extension and a 256-point IFFT were employed to generate a real signal. The payload was carried at subcarriers #2 to #m, where m was varied to obtain different data rates. The length of cyclic prefix was 16. Offline-generated OFDM signals for users A and B were uploaded into a 200-MHz AWG at 1 GSa/s. The signals passed EAs before been used to drive blue LDs. At the relay, the received signal was detected by an APD and captured by a DSO with a 200-MHz bandwidth. After re-encoding, the signal was broadcast back to users A and B. Each user extracted the information of the other user using the encoded signal x_R and its own information.

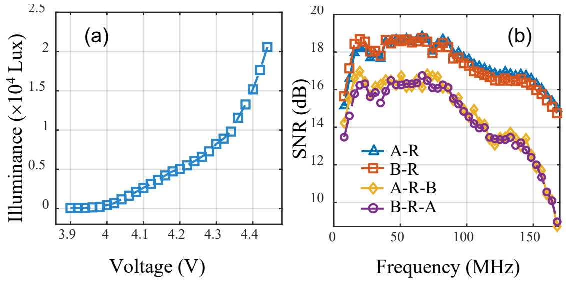

Fig. 4: (a) The illuminance versus voltage for the LD. (b) the measured SNRs for the links A-R, B-R, A-R-B, and B-R-A.

Fig. 4(a) shows the illuminance of the LD versus the voltage. In the experiment, we set the DC bias and the V_{pp} of the AC signal as 4.2 V and ~140 mV respectively at both users A&B and the relay R. The measured signal-to-noise ratios (SNRs) for different links are depicted in Fig. 4(b).

We first investigated the performance of the proposed scheme with uniform 8QAM/16QAM across the subcarriers. Fig. 5(a) shows the performance under different D. It is seen that the optimal performance was achieved when D=-3. Fig. 5(b) compares the proposed method with D=-3 to conventional AF-PNC. By reducing the constellation size at the relay, the proposed scheme enhances the tolerance to the noise and device nonlinearity, which results in significant improvement in throughput. The improvement is more prominent at high-frequency subcarriers, as shown in Fig. 6(a), where the SNRs are lower and the performance of the conventional AF-PNC is degraded more significantly.

We then investigated adaptively-loaded OFDM. Fig. 6(b) depicts the allocation results based on the SNR curve, whose value at each subcarrier is the lower one of the SNRs in links A-R-B and B-R-A. Figs. 7(a) and 7(b) show the BER versus the subcarrier index and the throughput, respectively. It is seen that the system using D=-3 exhibits a slightly lower BER than that using D=-1. Compared with the conventional AF-PNC, the proposed scheme using both D=-1 and D=-3 offers around one order of magnitude reduction in the BER.

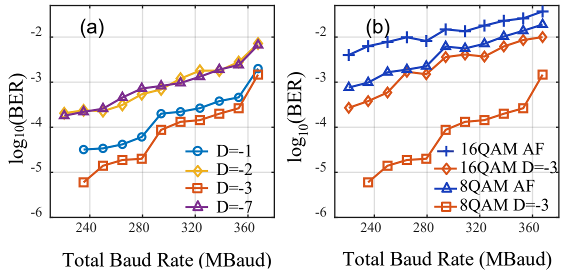

Fig. 5: BER versus the total baud rate of the two users for (a) the proposed method using 8QAM under different D, (b) the proposed method under D=-3 and conventional AF-PNC.

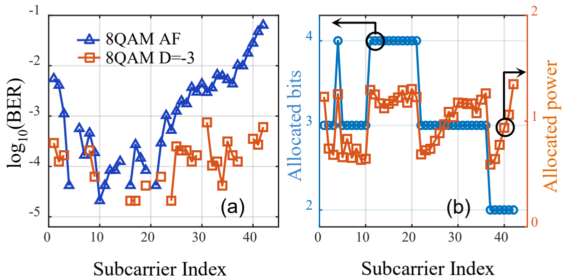

Fig. 6: (a) BER versus the subcarrier index when the total baud rate is 309 MBaud using 8QAM under D=-3. (b) bit-and-power loading results using Chow's algorithm at 0.97 Gbit/s.

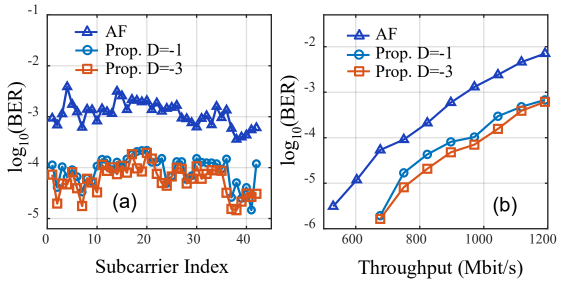

Fig. 7: BER versus (a) the subcarrier index at 1.014 Gbit/s, and (b) the throughput for adaptively-loaded OFDM.

Conclusions

We propose a novel scheme using the residual field theory of algebraic integers to achieve DF-PNC of any-order QAMs in OFDM-VLC systems. The constellation size of the superposed signal at the relay can be effectively reduced while the uniqueness to extract the targeted information for users is guaranteed. Through OFDM-VLC TWRN experiments with 0.53-1.47Gbit/s throughput, we show that the proposed PNC scheme offers significant BER reduction (around one order of magnitude) relative to the conventional AF-PNC in both cases of uniform 8QAM/16QAM and adaptively-loaded QAMs.

Acknowledgements

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