

# Generalized OSNR Penalty induced by SDM Amplifiers' Differential Spatial-Lane Gain

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**Abstract** The maximum allowed differential spatial-lane gain (DSG) of SDM amplifiers is key information for amplifier designers. We have demonstrated a simplified method based on relative calculations to estimate the impact of DSG under different conditions.

## Introduction

Space-division multiplexing (SDM) erbium doped fiber amplifiers (EDFAs) provide gains to multiple spatial lanes (SLs) simultaneously. The physical entities of the SLs could be parallel single-mode fibers, cores in a multi-core fiber (MCF) or spatial modes in a multi-mode fiber (MMF). Reducing the gain difference among the SLs, *i.e.* the differential spatial-lane gain (DSG), has been one of the major research topics of SDM in the past decade [1]–[3]. The motivation to reduce DSGs in amplifiers is similar to that of the gain equalization technique in the wavelength division multiplexing (WDM) systems [4]. In the future SDM optical transport network, the non-equalized SL gains will make the signal-to-noise ratio (SNR) of some SLs worse than the others, leading to a degradation of the overall system performance.

There already are, on the table, various technical solutions to reduce DSGs. For example, in multi-fiber EDFAs, more pump diodes and attenuators can be used to control separately the SL gains; in a multicore EDFA, the core signals can be fan-out, equalized and fan-in again [5]; in multimode EDFAs, the refractive index profile as well as the  $\text{Er}^{3+}$  ions' doping profile can be engineered [3] so as to reduce the EDF's intrinsic DSGs. However, they all add up to the amplifier's

complexity and cost. This is contradictory to the major objective of SDM: to achieve lower cost per bit and lower pump consumption per bit. SDM devices should be as integrated as possible: less number of control parameters, more shared components [6], smaller foot print [7], *etc.* After all, designing an SDM amplifier would always be about the trade-off. To that extent, how much DSG of the amplifier a system can tolerate is very useful information: it would notably impact the complexity of the SDM amplifier.

In this paper we have developed a simplified method for the estimation of the generalized OSNR (gOSNR) penalty induced by DSG. This method takes the ideal-case gOSNR as the reference and calculate the relative deviation with the presence of DSG. No information about the absolute values of the SDM link is needed for this calculation.

## SDM Link Architecture and Noise Model

Fig. 1(a) depicts a general SDM link with  $N$  SLs. It consists of multiple cascaded optical multiplexing sections (OMS), which is defined between adjacent SDM reconfigurable optical add/drop multiplexers (ROADM). The SDM ROADM [8]–[10] performs SL-level multiplexing and grooming. It has the capability of re-adjusting the signal powers that are launched into multiple

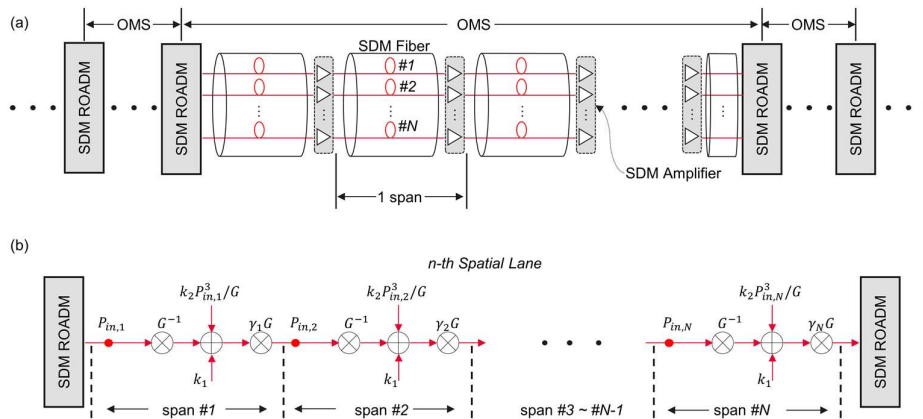


Fig. 1. (a) General architecture of an SDM link; (b) signal-noise model of the  $n$ -th spatial lane.

SLs of the SDM fiber. A long section of SDM fiber and an SDM amplifier compose a “span”. A chain of spans connects the adjacent SDM ROADMs.

In the ideal case with uniform spans, the SDM fiber in each span should induce identical losses  $\alpha$  to all the SLs; in the mean time, there should be no DSG, the amplifiers then provide the same gain with the value of  $G = \alpha^{-1}$  to all the SLs so that the signal power can be recovered at the output of each span. However, due to the presence of DSG, in  $n$ -th SL, the amplifier gain of the  $i$ -th span becomes  $\gamma_i G$  rather than  $G$ , where  $\gamma_i$  is the deviation factor. The value of  $\gamma_i$  is not necessarily the same for all the spans due to the design of the SDM amplifiers and the fabrication tolerance. For example, in SDM amplifiers based on multi-fibers or MCFs,  $\gamma_i$  would most likely follow a random distribution because their DSGs are mostly caused by the random non-uniformity of the  $\text{Er}^{3+}$  doping levels in fibers/cores and component variations, while for EDFAs based on MMFs, some spatial modes may have a fixed gain offset compared to the others since the overlap factors between spatial mode fields' distributions and the  $\text{Er}^{3+}$  doping profile may have an intrinsic discrepancies between each other.

In each span, there are two noise sources (see Fig. 1 (b)). The first one is the EDFA's equivalent input noise  $k_1$ .  $k_1$  is proportional to the noise figure of the EDFA, which is assumed to be the same for all the SDM EDFAs in the span. The second noise source considered here is the nonlinear noise which is proportional to the 3<sup>rd</sup> power of the signal power at the input of the  $i$ -th SPAN, being written as  $k_2 P_{in,i}^3 / G$  [11], where  $k_2$  is dependent on the features of the SLs, such as the length of the SDM transmission fiber, mode field diameters, etc.. It will be shown in the following that the absolute values  $k_1$  and  $k_2$  are not needed for the calculation of gOSNR penalty.

After  $N$  spans, the signal power becomes:

$$P_s = P_{in} \times C \quad (1)$$

where  $P_{in}$  is the signal power launched into the 1<sup>st</sup> span, it equals  $P_{in,1}$  shown in Fig. 1 (b). The accumulated ASE noise  $P_{ASE}$  and nonlinear noise  $P_{NL}$  can also be calculated after doing similar algebra:

$$P_{ASE} = k_1 \Gamma_1 \times G \times C \quad (2)$$

where  $\Gamma_1 = 1 + \frac{1}{\gamma_1} + \frac{1}{\gamma_1 \gamma_2} + \dots + \frac{1}{\gamma_1 \gamma_2 \dots \gamma_{N-1}}$ , and

$$P_{NL} = k_2 \Gamma_2 \times P_{in}^3 \times C \quad (3)$$

where  $\Gamma_2 = 1 + \gamma_1^2 + (\gamma_1 \gamma_2)^2 + (\gamma_1 \gamma_2 \gamma_3)^2 + \dots + (\gamma_1 \gamma_2 \dots \gamma_{N-1})^2$  and  $C = \prod_{i=1}^N \gamma_i$  is a common term for Eq. (1), (2) and (3). The gOSNR can then be calculated as:

$$gOSNR^{-1} = \frac{k_1 \Gamma_1 G}{P_{in}} + k_2 \Gamma_2 P_{in}^2 \quad (4)$$

#### Optimal gOSNR w/o DSG

When there is no DSG,  $\Gamma_1$  and  $\Gamma_2$  converge to the value of  $N$  (the number of spans). The gOSNR w/o the effect DSG can be calculated as:

$$(gOSNR_{wo})^{-1} = N \times \left( \frac{G k_1}{P_{in}} + k_2 P_{in}^2 \right) \quad (5)$$

Eq. (5) indicates that there exists an optimal value of the input signal power,  $P_{in,wo}^{opt}$ . It can be found out by taking the first derivative of  $gOSNR^{-1}$  w.r.t.  $P_{in}$ :

$$\begin{cases} P_{in,wo}^{opt} = \left( \frac{k_1}{2k_2} G \right)^{1/3} \\ (gOSNR_{wo}^{opt})^{-1} = \frac{3}{2} G^{\frac{2}{3}} N k_1^{\frac{2}{3}} (2k_2)^{\frac{1}{3}} \end{cases} \quad (6)$$

Fig. 2 shows the gOSNR degradation of one span in function of the deviation of the input signal power  $P_{in}$  from the optimal value  $P_{in,wo}^{opt}$ . When  $P_{in} = P_{in,wo}^{opt}$ , the ASE and nonlinear noises come to a balance point, the gOSNR is maximized. As long as  $P_{in} \neq P_{in,wo}^{opt}$ , the gOSNR will degrade.

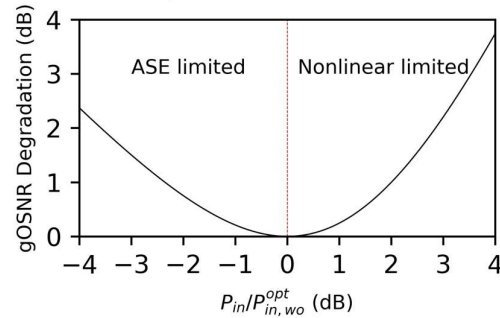


Fig. 2. gOSNR degradation of one span when  $P_{in}$  deviates from  $P_{in,wo}^{opt}$  (in the case of w/o DSG).

#### gOSNR Penalty w/ DSG

$gOSNR_{wo}^{opt}$  will then be set as the reference. The gOSNR penalty is calculated by taking the ratio between the gOSNR obtained from Eq. (4) and that from Eq. (6).  $k_1$  and  $k_2$  will be eliminated during the calculation, therefore it is not necessary to have their exact values. This is in fact the key advantage of the method proposed in this paper.

The gOSNR penalty is highly dependent on how  $P_{in}$  is chosen. Two cases are considered: 1)  $P_{in} = P_{in,wo}^{opt}$ , which means that the power is fixed to be the optimal value for the case w/o DSG, no power adjustment is done at the ROADM node; 2)  $P_{in}$  is optimized to obtain of the best gOSNR at the end of the OMS. In a real application scenario, 2) can be done for each OMS in an iterative way.

Following the same methodology as that in the case w/o DSG, we can find the optimal launching power and the optimal gOSNR w/ DSG:

$$\begin{cases} P_{in,w}^{opt} = \left( \frac{k_1 \Gamma_1}{2k_2 \Gamma_2} G \right)^{1/3} \\ (gOSNR_w^{opt})^{-1} = \frac{3}{2} G^{\frac{2}{3}} (k_1 \Gamma_1)^{\frac{2}{3}} (2k_2 \Gamma_2)^{\frac{1}{3}} \end{cases} \quad (7)$$

The gOSNR penalties ( $R$ ) of one OMS are:

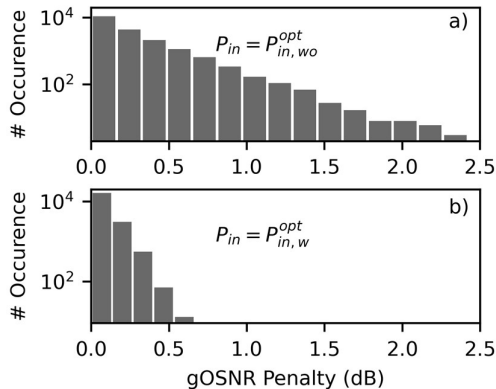
$$R = \begin{cases} \frac{\Gamma_1^{2/3} \Gamma_2^{1/3}}{N}, & P_{in} = P_{in,wo}^{opt} \\ \frac{2(\Gamma_1 + \Gamma_2/2)}{3N}, & P_{in} = P_{in,w}^{opt} \end{cases} \quad (8)$$

Because the signal power will be re-adjusted to the optimal value for the next OMS at the ROADM node, one OMS's  $R$  will be independent from the other OMSs'  $R$ . For  $K$  cascaded OMSs, the overall gOSNR penalty  $R_{tot}$  will be a weighted average over all the OMSs:

$$R_{tot} = \frac{\sum_{i=1}^K \{[gOSNR_{wo,i}^{opt}]^{-1} \times R_i\}}{\sum_{i=1}^K \{[gOSNR_{wo,i}^{opt}]^{-1}\}} \quad (9)$$

## Results

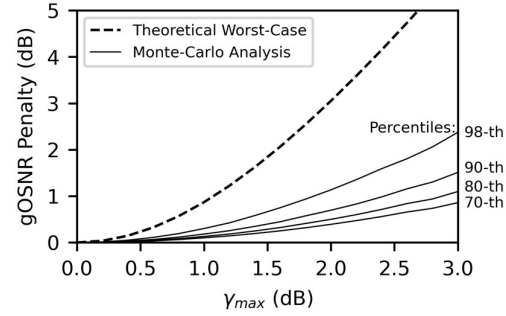
Since  $R$  will not accumulate along the chain of OMSs (see Eq. (9)) as long as the procedure of power adjustment is done at each ROADM node, our investigation is restricted within only one OMS. Firstly, a Monte-Carlo analysis is done for 7 spans. The DSG (a set of  $\{\gamma_1, \gamma_2, \dots, \gamma_7\}$ ) is assumed to have a random, uniform distribution within a boundary of  $[-\gamma_{max}, +\gamma_{max}]$ . This would most likely be the case for the SDM EDFAs based on multi-fibers or MCFs, as already discussed above.  $\gamma_{max}$  is swept from 0 to 3 dB. Eq. (8) is used for calculating the gOSNR penalties under the two conditions:  $P_{in}$  equals 1)  $P_{in,wo}^{opt}$  and 2)  $P_{in,w}^{opt}$ . For each  $\gamma_{max}$ , 20,000 calculations are done. Fig. 3 plots the histograms of the gOSNR penalties when  $\gamma_{max}$  is 1 dB. It is clearly shown that in the case of  $P_{in} = P_{in,w}^{opt}$ , the gOSNR penalty is significantly improved compared to the case of  $P_{in} = P_{in,wo}^{opt}$ . This result confirms the necessity to adjust the signal launching power into each SL along the link.



**Fig. 3. Histograms of gOSNR penalties of one OMS section which contains 7 spans and  $\gamma_{max} = 1$  dB.**

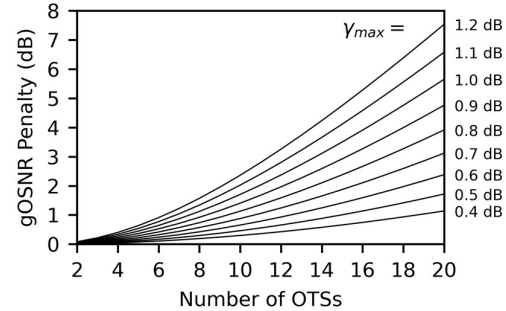
The solid curves of Fig. 4 are the gOSNR penalties corresponding to different percentiles taken from the results of Monte-Carlo analysis. For comparison, the theoretical worst-case ( $P_{in} = P_{in,w}^{opt}$ ) is also depicted (the dashed line), which is

calculated by Eq.(8) with all the amplifiers having the maximum DSG ( $\gamma_i = \gamma_{max}$ ). The theoretical worst-case would most-likely happen in amplifiers whose SL gains have fixed offsets, such as those based on MMFs. It can be seen that there is a significant gap between Monte-Carlo curves and the theoretical worst-case. This indicates that the fixed offset of the SL gains in the amplifier is more detrimental and should be avoid as much as possible.



**Fig. 4. gOSNR penalties v.s.  $\gamma_{max}$  of 7 spans.**

The gOSNR penalty is also a function of the number of the spans, as shown in (5). For an OMS which contains more spans, the requirement to the amplifier's DSG would be stricter. One solution is to insert one or more SL gain equalizers in-between the spans, within one OMS section. Of course, this would again be a trade-off between the cost and performance.



**Fig. 5. Worst-case gOSNR penalty in function of the number of spans, in the case of  $P_{in} = P_{in,w}^{opt}$ .**

## Conclusions

In this paper we have investigated the relation between the amplifier's DSG and the gOSNR penalty in an SDM link. Adjusting the SL's signal launching power at the ROADM node is very important in order to reduce the gOSNR penalty of the whole SDM link. Amplifiers having randomly-distributed DSGs are more preferred than those having fixed DSG offset since the latter would lead to a much worse gOSNR penalty. Finally, the number of spans between SDM ROADMs (or equivalently the SL gain equalizers) plays a non-negligible role and it should be carefully chosen w.r.t. the amplifiers' DSG level in order to achieve the best system trade-off.

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