European Conference on Optical Communication (ECOC) 2022 © Optica Publishing Group 2022

Fourier Neural Operator Based Fibre Channel Modelling for Optical Transmission

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Abstract *We propose a Fourier Neural Operator based fibre channel modelling method with both time-domain and frequency-domain operators. The proposed method performs a high accuracy in the WDM long-haul transmission system. ©2022 The Author(s)*

Introduction

Optical fibre channel modelling is essential for network design, optimization and estimation of quality of transmission (QoT). To realize that, many analytical methods such as Gaussian noise (GN) model [1] are proposed, and a fast evaluation of the signal to noise ratio (SNR) can be achieved. However, the rich information in the waveform cannot be provided by those methods. On the other hand, although such information can be obtained by solving the nonlinear Schrödinger equation (NLSE) with the split-step Fourier method (SSFM) [2], the complexity can be quite high, which may be not proper for a timely control of the optical network.

To solve the problems above, machine-learning (ML)-based methods are widely studied recently. A bidirectional long short-term memory model-based method in [3] and a generative-adversarial network-based in [4] are proposed for waveform modelling. However, the estimation accuracy of these works needs further improvement. Some works such as the physics-informed neural network (PINN) [5] are proposed, which incorporate theoretical knowledge into the design of ML algorithm to improve interpretability [6]. However, their applications are typically limited in pulse modelling.

In this paper, we propose a waveform modelling method based on Fourier neural operator (FNO) [7], which contains both time-domain and frequency-domain operators. The structure is designed by theoretically analysing the Manakov equation. To demonstrate the accuracy, we perform extensive simulations. The results show that the modelling error of the proposed method can be less than 0.1 dB, demonstrating the effectiveness of the method.

Principle

The Manakov equation can be written as [8]

$$\frac{\partial \boldsymbol{E}(z,t)}{\partial z} = (\mathcal{L}+\mathcal{N})\boldsymbol{E}, \quad (1)$$

where $\mathcal{L}$ and $\mathcal{N}$ represent the linear and nonlinear operator, respectively. $\boldsymbol{E}=\left[E_x, E_y\right]^T$ represents the two polarizations of the optical signal. The two operators can be written as

$$\mathcal{L}(\boldsymbol{E}) \triangleq \frac{j\beta_2}{2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \frac{j\beta_2}{2}\mathcal{F}^{-1}[\omega^2\mathcal{F}(\boldsymbol{E})], \quad (2)$$

$$\mathcal{N}(\boldsymbol{E}) \triangleq -j\frac{8}{9}\gamma e^{-\alpha z}\left|E_x^2+E_y^2\right|\boldsymbol{E}, \quad (3)$$

where z and t represent the propagation distance and time, respectively. β_2, α and γ are the chromatic dispersion (CD), attenuation and

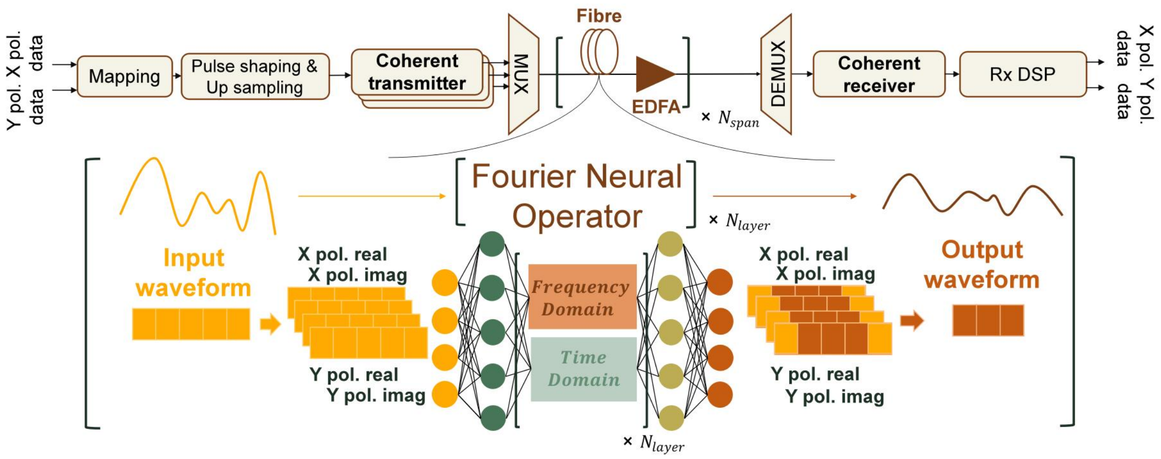

Fig. 1: The structure of the SSFM-inspired optical fibre channel modelling using Fourier neural operator.

nonlinear coefficient of the optical fibre. $\mathcal{F}$ and $\mathcal{F}^{-1}$ represent Fourier transform and inverse Fourier transform, respectively.

Eq. (2) and (3) show that the linear effect and nonlinear effect can be solved in different domains. However, most of the data-driven methods only focus on time-domain processing. In this work, we propose to apply FNO in waveform modelling, which consists of a frequency-domain operator and a time-domain operator as shown in Fig. 1. The calculation of it can be written as

$$\boldsymbol{E}(z + \Delta z, t) = \sigma[(\mathcal{W} + \mathcal{R})\boldsymbol{E}], \qquad (4)$$

where $\mathcal{W}$ and $\mathcal{R}$ represent the time-domain operator and the frequency-domain operator as

$$\mathcal{W}(\boldsymbol{E}) = W * \boldsymbol{E}, \qquad (5)$$

$$\mathcal{R}(\boldsymbol{E}) = IFFT[R \cdot FFT(\boldsymbol{E})], \qquad (6)$$

For the input waveform, $\mathcal{W}$ performs a convolution on it in time-domain. For the frequency-domain operator $\mathcal{R}$, fast Fourier transform (FFT), a fully connected layer, inverse fast Fourier transform (IFFT) are performed successively. The final output of FNO is obtained by performing an activation operation σ on the summation.

The workflow of the proposed method is detailed as below. The input waveform is first mapped to a higher dimension channel space by a fully connected layer. The outputs are then processed by several FNO blocks. The outputs are finally mapped back to four channels by a fully connected layer and reconstructed to the waveform after the propagation. The samples at both ends are eliminated due to the circular convolution caused by FFT. When the training is done, the FNO can learn the complex impairments in the fibre channel.

Simulation Setup

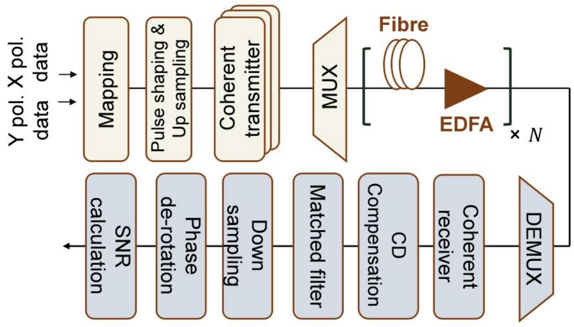

Fig. 2: The diagram of the simulation system.

The diagram of the simulation system is shown in Fig. 2. The channel number of the wavelength-division multiplexing (WDM) system is set to 5. The symbol rate is 30GBaud per channel and the channel spacing is 50GHz. The symbol length is set to 16384. The modulation format is dual polarization 16 QAM. A root raised cosine (RRC) filter with a roll-off factor of 0.02 is used for pulse shaping at the transmitter. The launch power ranges from -2 dBm to 2 dBm with a step size of 1 dB. For the optical fibre channel, the fibre type is standard single mode fibre (SSMF). The span number is set from 1 to 10. Erbium-doped fibre amplifiers (EDFAs) with a noise figure of 5 dB are adopted for amplification. SSFM with a step size of 10 m is applied to obtain the waveform after transmission. The central channel is filtered out at the receiver and CD compensation, matched filter, down sampling and phase de-rotation are used for signal processing. The SNR is calculated in the end.

The proposed neural network contains four FNOs. The input length of the network is 8192 samples. GELU, a high-performing activation

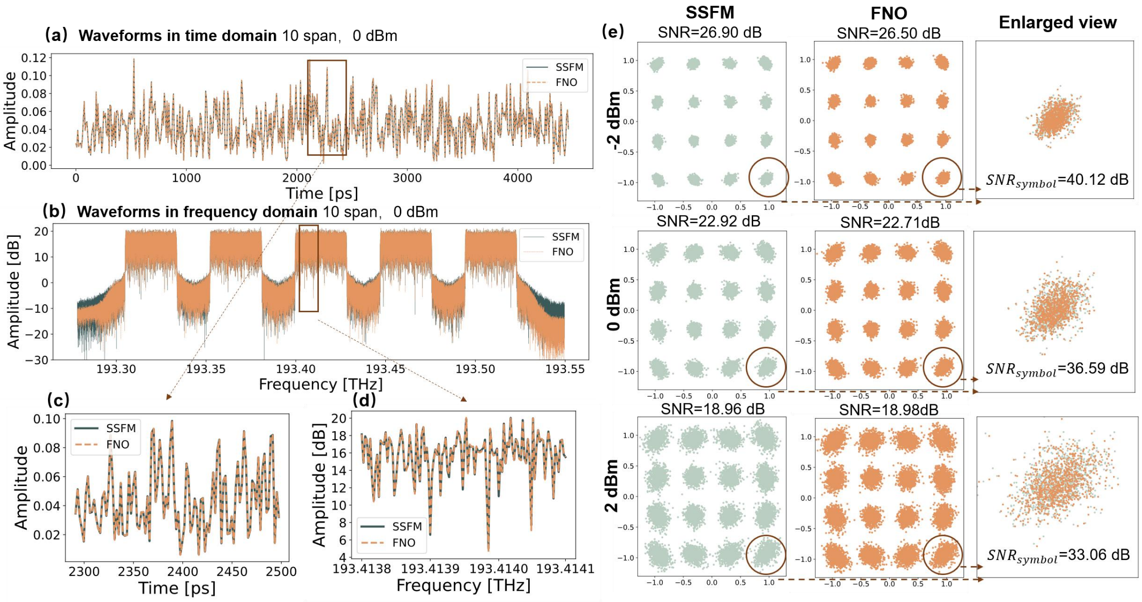

Fig. 3: (a) (b) The waveforms modelled by SSFM and FNO in time and frequency domain for 10 spans without ASE noise with the launch power of 0 dBm. **(c) (d)** The details of the waveforms in (a) and (b). **(e)** The constellations of waveforms modelled by SSFM and FNO for 800 km without ASE noise with launch power of -2, 0, 2 dBm.

function [9] is utilized for nonlinear activation. Adam is used for learning optimization. Based on the structure, we trained two models for different launch power ranges. One is for the -2 dBm to 0 dBm range, and the other is for the 0 dBm to 2 dBm range.

Results and discussions

To intuitively show the accuracy of the proposed structure, we plot the waveforms modelled by SSFM and FNO in time and frequency domain in Fig. 3 (a) and (b), respectively, for 800 km transmission with the launch power of 0 dBm. Fig. 3 (c) and (d) show the details of the waveforms, in which the results of SSFM and FNO are almost perfectly overlapped. The constellations at the receiver are shown in Fig. 3 (e), under the condition of transmitting over 800 km (10 spans) without ASE noise, and the launch power is -2, 0, 2 dBm. There is little difference between the constellations from SSFM and FNO. In the enlarged view, the constellations show a great deal of overlap. In order to quantitatively measure the accuracy, we defined the SNR_{symbol} as

$$SNR_{symbol} = \frac{\sum_1^{N_{symbol}} (Rx_{SSFM})^2}{\sum_1^{N_{symbol}} (Rx_{SSFM} - Rx_{FNO})^2}, \quad (7)$$

where Rx_{SSFM} and Rx_{FNO} represent the symbols received from SSFM and FNO, and N_{symbol} is the number of symbols. The SNR_{symbol} is up to 40 dB in the simulation, which means the estimation error is much smaller compared with the signal power.

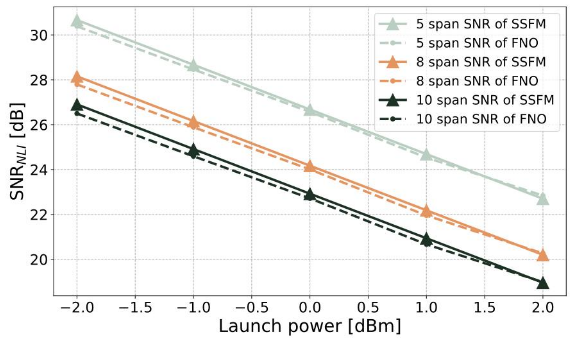

Fig. 4: The SNR without ASE noise for different launch

In the simulation, the SNR without ASE noise is defined as:

$$SNR_{NLI} = 10log_{10}\left(\frac{P_{sig}}{P_{NLI}}\right), \quad (8)$$

where P_{sig} and P_{NLI} represent the signal power and nonlinear noise power, respectively. The SNR_{NLI} estimation error is less than 0.1 dB for 80km (1 span). For long-haul transmission simulation, the error is less than 0.45 dB for 800km (10 spans). The SNR_{NLI} for 5, 8 and 10 spans with different launch power from -2 to 2 dBm are plotted in Fig. 4. The curve of the proposed method has the same trend as SSFM that, 1-dB launch power increase incurs 2-dB reduction of SNR_{NLI} [10], which is consistent with the physical theory. The small estimation error of SNR_{NLI} demonstrates the accuracy of the proposed method.

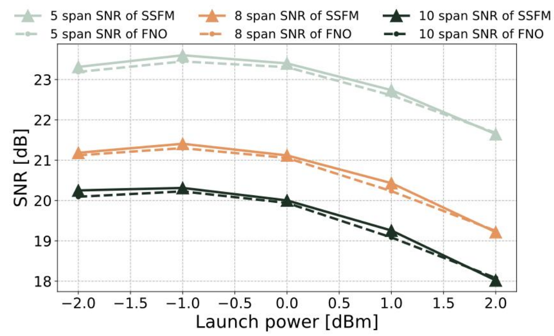

Fig. 5: The SNR with ASE noise for different launch power and transmission distance.

Next, the ASE noise induced by EDFA is considered as the Gaussian noise. The updated curves are plotted in Fig. 5, indicating that the optimal launch power of both models is around -1 dBm. All of the SNR estimation errors are less than 0.2 dB in the simulation.

Conclusions

We proposed a Fourier neural operator based optical channel modelling structure, with both the time-domain and frequency-domain operator. The model trained by the structure demonstrates high accuracy in WDM long-haul waveform modelling.

Acknowledgements

This work was supported by National Key R&D Program of China (2018YFB1800902), and National Natural Science Foundation of China (62175145).

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