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Nonlinearity Tolerant Shaping with Sequence Selection

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Abstract *We introduce a new metric for sequence selection to achieve nonlinearity tolerant probabilistic amplitude shaping (PAS). The new metric provides an about 0.5 dB higher effective signal-to-noise ratio for PAS with short-length constant composition distribution matching in a dual-polarized 256 QAM transmission over a long-haul fiber link. ©2022 The Author(s)*

Introduction

Probabilistic amplitude shaping (PAS) has established itself as a popular approach for integrating shaping and forward error correction (FEC) coding[1]. In the context of transmission over optical fiber, the interplay between shaping and nonlinear interference (NLI) is of particular interest[2]–[4]. For PAS with constant composition distribution matching (CCDM), it has been demonstrated that the effective signal-to-noise ratio (SNR) at the output of the fiber link decreases as the CCDM blocklength increases[5]. Since this trend is not observed if a symbol interleaver is applied to shaped sequences, it can be concluded that the temporal properties of shaped sequences play a role in the severity of NLI. Similarly, the mapping of symbols of a shaping block to the quadrature and polarization components of an optical fiber transmission affects nonlinearity tolerance[6].

The work in[7] tried to capture the effect of shaping blocklength by introducing the energy dispersion index (EDI) for shaped sequences. The EDI is proportional to the empirical variance of the windowed symbol-energy sequence. This has been extended to the exponentially weighted EDI (EEDI)[8]. For PAS with CCDM, (E)EDI have been demonstrated to be good predictors of the effective SNR[7],[8]. This has been utilized in the recently proposed list-encoding CCDM (L-CCDM), in which the generation of CCDM candidate sequences is followed by a selection module that uses EDI as a metric[9]. Candidate generation is made possible by inserting redundant flipping bits into each block of information bits.

In this paper, we introduce a new sequence-selection criterion that (a) does not make the simplifying assumption of an (exponentially) windowed symbol-energy sequence and (b) is systematically extended to cross-polarization and cross-phase modulation NLI. Our approach is inspired by the lowpass filtering model used to explain the benefits of a 4-dimensional mapping in the nonlinear regime in[10]. In particular, we derive our lowpass filtered symbol-amplitude sequence (LSAS) metric from the first-order perturbation-based modeling of NLI. We also show that LSAS is related to (E)EDI when applying simplifying assumptions. Besides these insights, we demonstrate the gains of sequence selection with LSAS for achievable information rate (AIR)[11], effective SNR, and Q-factor for system settings as in[9].

PAS with Sequence Selection

We consider PAS using a distribution matcher (DM) followed by sequence selection for NLI tolerance, similar to[9]. At the transmitter, v flipping bits are concatenated with $k - v$ information bits to form the DM input block of k bits. The DM maps the k bits to a sequence of L amplitude symbols with the desired distribution. Due to the flipping bits, this results in 2^v candidate sequences per quadrature and polarization component of the transmit signal. A sequence selection module chooses the best candidate based on the LSAS metric introduced in the next section. We note that selection is made jointly over signal components, i.e., over 2^{2dv} candidates, where $d = 1$ for single and $d = 2$ for dual-polarization transmission. Then, for each component, L amplitude symbols are mapped to $L \times (m - 1)$ bits, where 2^{m-1} is the number of possible amplitudes. A systematic FEC generates the sign bits to obtain the quadrature amplitude modulation (QAM) transmission symbols.

The Proposed Sequence Selection Metric

The proposed LSAS metric is based on the first-order perturbation based model for NLI. As such, it directly permits the consideration of inter-polarization and inter-channel effects in dual-

$$b_p^{(s)}(n) \approx a_p^{(s)}(n)\left[1+i\gamma\left(\sum_{p'\in\mathcal{P}}\sum_{s'\in\mathcal{S}}\sum_{m\in\mathbb{Z}} E_{p'}^{(s')}(n+m)h_{p,p'}^{(s,s')}(m)\right)\right] \quad (1)$$

$$\approx a_p^{(s)}(n)\exp\left[i\gamma\left(\sum_{p'\in\mathcal{P}}\sum_{s'\in\mathcal{S}}\underbrace{\sum_{m\in\mathbb{Z}}(E_{p'}^{(s')}(n+m)-\bar{E}_{p'}^{(s')})h_{p,p'}^{(s,s')}(m)}_{\triangleq \Delta E_{p,p'}^{(s,s')}(n)}+c_p^{(s)}\right)\right] \quad (2)$$

polarization and multi-channel systems with a set $\mathcal{S}$ of channels. We start with the expression of nonlinear signal-signal distortion from Eq. (102) in[12], to obtain (1) as an approximation of the nth received symbol $b_p^{(s)}(n)$ after linear equalization in polarization p and channel s as a function of the transmitted data symbols $a_{p'}^{(s')}(m)$, $m \in \mathbb{Z}$, in polarization $p' \in \mathcal{P} \triangleq \{x, y\}$ and channels $s' \in \mathcal{S}$. In (1), γ is the fiber nonlinearity parameter, $h_{p,p'}^{(s,s')}(n)$ is the perturbation coefficient, representing intra- ($p = p'$) and cross-polarization ($p \neq p'$) interferences of self-phase modulation (SPM) ($s = s'$) and cross-phase modulation (XPM) ($s \neq s'$), respectively. Furthermore, we have defined the signal energy terms

$$E_p^{(s)}(n) \triangleq |a_p^{(s)}(n)|^2, \quad \bar{E}_p^{(s)} \triangleq \mathbb{E}\left[|a_p^{(s)}(n)|^2\right], \quad (3)$$

where $\mathbb{E}$ is the statistical expectation operator, and the latter is used in (2). In (2), we applied $\exp(it) \approx 1 + it$ for $t \ll 1$, and we chose to separate the term

$$c_p^{(s)} = \sum_{p'\in\mathcal{P}}\sum_{s'\in\mathcal{S}} \bar{E}_{p'}^{(s')} \sum_{m\in\mathbb{Z}} h_{p,p'}^{(s,s')}(m). \quad (4)$$

As it has been illustrated in[10], the signal $h_{p,p'}^{(s,s')}$ can be interpreted as a filter with a lowpass characteristic. Hence, the approximation of NLI in (1) is in the form of a lowpass filtered symbol-energy signal $E_{p'}^{(s')}$, i.e., the convolution of $E_{p'}^{(s')}$ and $h_{p,p'}^{(s,s')}$ accounts for the NLI from channel s' and polarization p' into channel s and polarization p. Approximation (2) shows that this NLI manifests itself as phase noise, which thus can in part be compensated by a carrier phase recovery. We account for the latter by extracting the data-independent term $c_p^{(s)}$, so that it is not considered in the LSAS metric.

From the expression in (2) and the discussion above, we consider $\Delta E_{p,p'}^{(s,s')}(n)$ as defined in (2) as the term that is pertinent for sequence selection. Accordingly, we propose the LSAS metric

$$\lambda_{\mathrm{LSAS}} = \sum_{n=0}^{L-1}\sum_{p\in\mathcal{P}}\left|a_p^{(s)}(n)\sum_{p'\in\mathcal{P}}\sum_{s'\in\mathcal{S}}\Delta E_{p,p'}^{(s,s')}(n)\right|^2 \quad (5)$$

for the joint selection of $2dL$ amplitude symbols for the quadrature and polarization components within one shaping block of channel s. We note that the sums in (5) collapse for the case of single-polarization, i.e., $d = 1$ and $\mathcal{P} = \{x\}$, and single channel transmission, i.e., $\mathcal{S} = \{s\}$.

The derivation of the LSAS metric allows us to relate it to the EDI criterion from[7]. In particular, when applied for sequence selection, the estimated EDI using empirical averages as given in Eq. (53) in[7] needs to be used, as it has been done for L-CCDM in[9]. If we further assume that $\bar{E}_p^{(s)}$ is close to the empirical mean $\frac{1}{L-W}\sum_{n=1+W/2}^{L-W/2}|a_p^{(s)}(n)|^2$, where W is the EDI window length, then the EDI metric can be written as

$$\lambda_{\mathrm{EDI}} = \frac{1}{L-W-1}\sum_{n=1+W/2}^{L-W/2}\left(\Delta\tilde{E}_{p,p}^{(s,s)}(n)\right)^2, \quad (6)$$

where $\Delta\tilde{E}_{p,p}^{(s,s)}$ is the special case of $\Delta E_{p,p}^{(s,s)}$ when $h_{p,p}^{(s,s)}(m) = 1$ for $m \in \{-\frac{W}{2}, \ldots, \frac{W}{2}\}$ and 0 otherwise. In addition to this simplification, λ_{EDI} does not include the amplitude terms $a_p^{(s)}(n)$ used in λ_{LSAS}, and it has been defined only for single-polarization and single-channel transmission.

We believe that our derivation of (6) from symbol-energy related signal-to-signal distortion (1) provides further credence to the EDI criterion for quantifying NLI. At the same time, it suggests that the LSAS metric is a more accurate measure, and that it extends more naturally to dual-polarization and multi-channel transmission.

Numerical Results

We adopt the settings from[9] for ease of comparability but consider both single and dual-polarized transmission. The system uses root-raised cosine (roll-off 0.1) transmission at 32 GBaud in 11 WDM channels with 50 GHz spacing. The fiber link consists of 20 spans with 80 km span length, fiber loss 0.2 dB/km, chromatic dispersion parameter 17 ps/nm/km, and nonlinearity parameter 1.37 $W^{-1}km^{-1}$. At the end of each span, an erbium-doped fiber amplifier (EDFA) with a noise

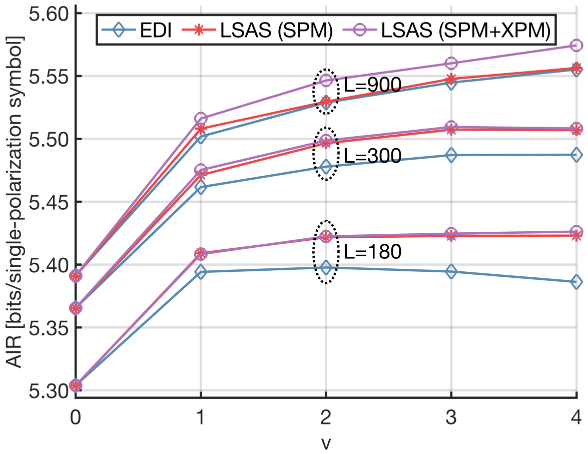

Fig. 1: AIR vs. v for different CCDM blocklengths L.

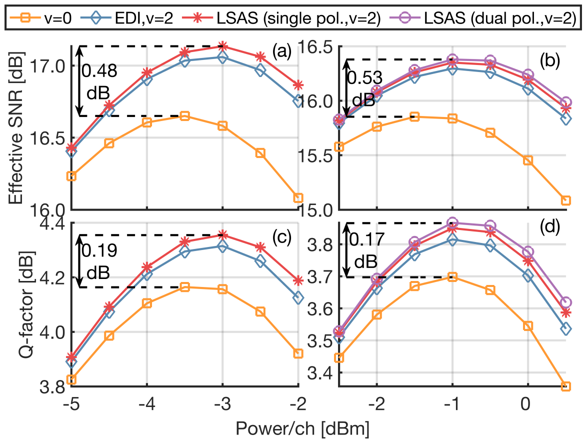

Fig. 2: Effective SNR (a,b) and Q-factor (c,d) vs. launch power. (a,c): Single pol. (b,d) dual pol. CCDM with $L = 180$.

figure of 6 dB is deployed. At the receiver, chromatic dispersion and a constant phase shift are compensated. The constellation is 256-QAM, and we apply CCDM with a Maxwell Boltzmann target distribution for an effective shaping rate of 2.4 bits/amplitude, i.e., the rateloss due to flipping bits for $v > 0$ is accounted for. The EDI metric uses $W = 100$ as in[9]. The performance results for the WDM center channel will be shown.

Fig. 1 shows the AIR for single-polarized transmission (as in[9]), as a function of v for CCDM with different blocklengths L and sequence selection using EDI and LSAS. CCDM without sequence selection corresponds to $v = 0$. For LSAS, we differentiate between LSAS only considering SPM terms ("LSAS (SPM)") and also considering XPM terms from adjacent channels ("LSAS (SPM+XPM)"). For the latter, to retain low complexity, sequence selection is done for one channel at a time, proceeding from WDM outer channels towards center channels, so that only 2^{2v} candidates are considered for each channel. The figure shows that LSAS outperforms EDI as a selection criterion, with an AIR gain of about 0.2 bit/pol compared to CCDM without selection at the blocklength of $L = 900$. EDI suffers from windowing amplitude sequences for shorter blocklengths, which discards $\frac{W}{2}$ symbols at the beginning and end of each block to get meaningful empirical variance measures. Such windowing is not required for LSAS. For the relatively long blocklength of $L = 900$, LSAS (SPM) and EDI perform similarly, which means windowing and the exact shape of the SPM lowpass filter $h_{p,p'}^{(s,s')}$ are not critical. However, LSAS can integrate XPM terms, which again leads to gains over EDI also for $L = 900$. We note that the lowpass filter $h_{p,p'}^{(s,s')}$ is narrower for XPM ($s \neq s'$), than for SPM ($s = s'$). Therefore, we do not see benefits for LSAS with XPM terms for short blocklengths, for which the spectrum of CCDM sequences is high-pass[10], as XPM is suppressed in the metric.

Next, we focus on the short CCDM blocklength case of $L = 180$, suitable for practical implementation. Figs. 2 (a,b) show effective SNR and Figs. 2 (c,d) show Q-factor as functions of launch power for single (a,c) and dual (b,d) polarization. Based on the results in Fig. 1, we choose $v = 2$. We observe SNR and Q-factor gains due to selection with LSAS over shaping without sequence selection of about 0.5 dB and 0.2 dB, respectively. Similarly, the benefits of selection with LSAS over EDI are also noticeable in these performance indicators. For the case of dual-polarization, the LSAS metric that includes cross-polarization terms yields slightly higher effective SNR and Q-factor. However, the extra gains are small. We thus conclude that the cross-polarization terms do not significantly influence the decision of the sequence selector for the chosen transmission settings.

Conclusions

We derived the LSAS metric for nonlinearity tolerant PAS. LSAS can directly be applied to dual-polarization transmission and incorporate XPM NLI terms, which is an advantage over, for example, the EDI criterion. The XPM terms naturally require access to neighbor channel data. We thus foresee their use in digital subcarrier modulation systems. We demonstrated notable AIR, SNR, and Q-factor gains with LSAS-based sequence selection for CCDM-based PAS with practical blocklengths and the optical link settings from[9]. Equally important, our derivation provides insights into the interplay between amplitude sequences and link characteristics, and a relation to the previously proposed EDI metric.

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