Analytical SNR Prediction in Long-Haul Optical Transmission using General Dual-Polarization 4D Formats

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Abstract *Nonlinear interference models for dual-polarization 4D (DP-4D) modulation have only been used so far to predict signal-signal nonlinear interference. We show that including the signal-noise term in the prediction of the effective signal-to-noise ratio in long distance DP-4D transmission improves the accuracy by up to 0.2 dB. ©2022 The Author(s)*

Introduction

Nonlinear interference (NLI) modeling in optical fiber transmission is a key tool to analyze the performance of optical communication systems and to optimize modulation formats. Various analytical models for nonlinear fibre propagation have been proposed in the literature [1]–[3]. Among these models the enhanced Gaussian noise (EGN) model enables an accurate estimatation of the NLI induced by polarization-multiplexed 2D (PM-2D) formats, where two identical 2D formats are used to transmit information independently over two orthogonal polarization modes. However, PM-2D formats are only a subset of all the possible dual-polarization four-dimensional (DP-4D) modulation formats, e.g., geometrically-shaped 4D formats [4], [5].

Multidimensional modulation formats have been considered as an effective approach to harvest shaping gains [6], especially in nonlinear optical fiber channel [7]. In order to fully explore the potential of DP-4D modulation formats in the nonlinear fiber channel, 4D NLI models have been introduced in [8], [9] as a tool to efficiently find a trade-off between linear and nonlinear shaping gains [10], [11].

Under the additive NLI noise assumption, the effective signal-to-noise ratio (the SNR after fiber propagation and the receiver digital signal processing including chromatic dispersion compensation and phase compensation) for multi-span systems can be approximated as

$$\mathrm{SNR}_{\mathrm{eff}} \triangleq \frac{P}{N_s\sigma^2_{ASE}+\sigma^2_{ss}+\sigma^2_{sn}}, \qquad (1)$$

where P denotes the transmitted signal power per channel, N_s is the number of spans. The total noise power consists of three parts: i) amplified spontaneous emission (ASE) noise over one span denoted as σ^2_{ASE}, ii) signal-signal (S-S) NLI power denoted as σ^2_{ss} and iii) signal-ASE noise (S-N) NLI power denoted as σ^2_{sn}.

In previous works, 4D NLI models have been validated and used only in terms of σ^2_{ss} prediction. The impact of σ^2_{sn} in the total effective SNR was thus neglected for general DP-4D formats.

In this work, by assessing the contribution of signal-ASE noise interaction in the total NLI power, we analytically study the effective SNR in multispan amplified optical fiber transmission systems using general DP-4D formats. This study is validated via split-step Fourier method (SSFM) simulations using various DP-4D modulation formats. Our results show that including the S-N term can reduce the estimation error of the effective SNR by 0.2 dB, which can be translated into a 4% prediction accuracy improvement in terms of transmission reach.

Improving the Accuracy for 4D NLI Model

To improve the accuracy of the effective SNR prediction, we study the impact of signal-ASE interaction for optimized 4D modulation formats based on the NLI model, which is built on the fact that the x- and y-polarization could be dependent of one another [9].

For dual-polarized signals over single-channel transmission, the signal-signal NLI power σ^2_{ss} in Eq. (1) can be approximated as [12, Eq. (1)]

$$\sigma^2_{ss} \approx \eta_{ss} N_s^{1+\varepsilon} P^3, \qquad (2)$$

where ε is a coherence factor for self channel interference which is a function of fiber link parameters (attenuation, dispersion, span length, etc) [1, Eq.(40)]. The η_{ss} denotes the signal-signal NLI power coefficient over one span. Here we denote the accumulated signal-signal NLI power coefficient over N_s spans as $\eta_{ss}^{(N_s)} = \eta_{ss} N_s^{1+\varepsilon}$. For gen-

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eral DP-4D formats, the modulation-dependent coefficient $\eta_{ss}^{(N_s)}$ for multi-span system can be calculated using Eq. (1) in [13].

As we discussed in the introduction, the ASE noise generated by erbium-doped fibre amplifier (EDFA) leads not only to an additive white Gaussian noise (AWGN) but also to a nonlinear interference that produced by ASE noise and transmitted signal interaction [14]. Under the assumption of flat transmitted signal spectrum and same propagated signal and ASE noise bandwidth, the signal-ASE NLI power coefficient can be estimated as $\eta_{sn} = 3\eta_{ss}$ [12], [15]. Thus, by following [15, Eq. (8)], the NLI power of signal-ASE interaction for DP-4D modulation can be derived as

$$\sigma_{sn}^2 = \xi\eta_{sn}\sigma_{ASE}^2 P^2 = 3\xi\eta_{ss}\sigma_{ASE}^2 P^2, \qquad (3)$$

where σ_{ASE}^2 is the power of ASE noise over one span, $\xi \approx \frac{N_s^{2+\varepsilon}}{2+\varepsilon} + \frac{N_s^{1+\varepsilon}}{2}$ is the signal-ASE NLI accumulation coefficient.

Therefore, by considering both signal-signal and signal-ASE interaction, the NLI power can be estimated via Eq. (2) and (3), where we can obtain η_{ss} as $\eta_{ss}^{(N_s)}/N_s^{1+\varepsilon}$. Note that $\eta_{ss}^{(N_s)}$ is a constant value (for a given system configuration) linked to the contributions of both modulation-independent and modulation-dependent nonlinearities, thus NLI power is also a function of the given 4D modulation format.

The optical system we consider in this work is a single channel, multi-span transmission system with a symbol rate of 45 GBaud and a root-raised-cosine filter with roll-off factor of 0.01%. The fiber link has the following parameters: attenuation coefficient $\alpha = 0.2$ dB/km, dispersion parameter $\beta_2 = -21.7$ ps^2/km and nonlinear coefficient $\gamma = 1.3$ (W km)$^{-1}$. Each span consists of an 80 km single-mode fiber followed by an EDFA with a noise figure of 5 dB.

Fig. 1 shows the noise power, i.e., $\sigma_{ASEtot}^2 = N_s\sigma_{ASE}^2$, σ_{ss}^2, σ_{sn}^2, against transmission distance. Considering for example 4D-PRS64 at a distance of 1600 km, σ_{sn}^2 differs from σ_{ss}^2 by a factor of 17.2 dB, while the difference is reduced to 10.6 dB for that of 7500 km. The proportion of σ_{sn}^2 in NLI power keeps increasing as the number of fiber span increases.

To investigate the dependence of signal-ASE NLI on the modulation format, uniform square PM-256QAM is chosen as a baseline format and the NLI power is shown as dashed lines in Fig. 1. A 0.3 dB gap can be found when comparing these two modulation formats. It is also shown

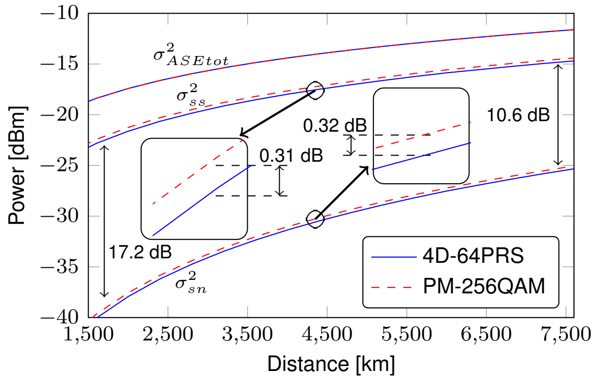

Fig. 1: Noise power versus transmission distance at launch power of 0.5 dBm. Noise is shown separately, as ASE noise, signal-signal NLI and signal-ASE NLI.

that the gap between σ_{sn}^2 and σ_{ss}^2 decreases as the transmission distance increases. In particular, this gap reduces from 17.2 dB at 1,500 km to 10.6 dB at 7,500 km. This indicates that the effect of signal-ASE NLI can not be fully neglected in very long-distance transmission. More results of modulation formats are shown in the next section.

Simulation Results and Analysis

In this section, the accuracy of 4D model with S-S and S-N is validated via comparing with SSFM for different 4D modulation formats. The SSFM simulates the nonlinear Manakov equation with an uniform step size of 0.1 km.

In Fig. 2 (a) and (b), the estimation of NLI power are evaluated by using i) the 4D model with S-S only (blue bars)[1], ii) 4D model with S-S and S-N (red bars), iii) SSFM (yellow bars) for different distances and modulation formats, respectively. To target on a practical SD-FEC with 25% overhead, 4D modulation formats are selected at required minimum SNR in which GMI $=$ $0.8m$ bit/4D for different spectral efficiencies with $m \in \{3, 4, 5, ..., 10\}$ from the existing 4D formats, which include the sphere packing database in [16], some recently proposed 4D formats such as 4D-64PRS [5] and a family of 4D orthant-symmetric (OS) formats [11].[2]

Fig. 2 (a) shows that the gap between our analytical predictions and SSFM becomes larger as distance increases for 4D-OS128 format [17]. For a distance of 8000 km, the 4D model with S-S underestimates NLI power by 15% compared to SSFM, which can be halved by considering the S-N term. In order to translate this gap into effective SNR, we define the deviation of the effec-

[1]Note that the 4D model is equivalent to EGN model for conventional PM-2D formats.

[2]The coordinates and labeling of these 4D modulation formats can be also found online at https://github.com/TUe-ICTLab/Binary-Labeling-for-2D-and-4D-constellations.

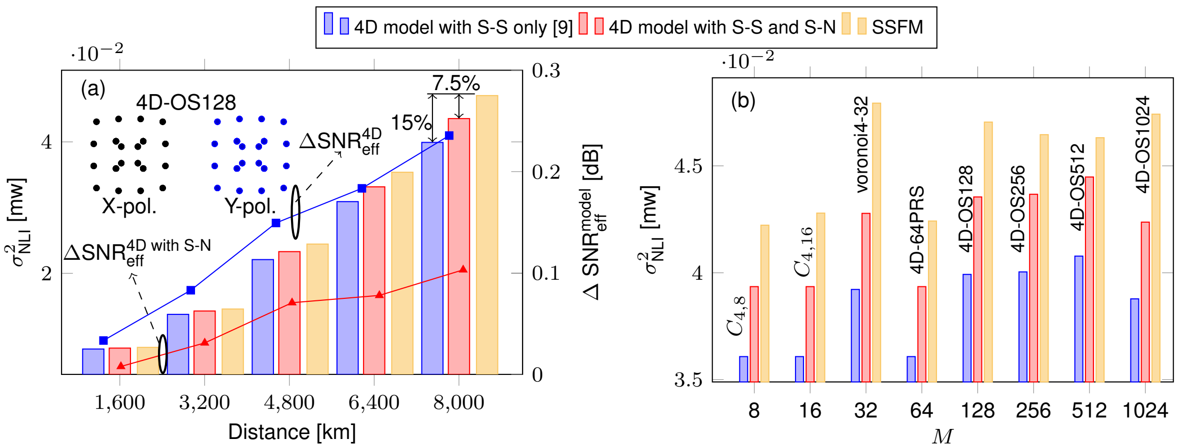

Fig. 2: Simulation results of multi-span optical fiber transmission with single channel: (a) NLI power and $\Delta\mathrm{SNR}_{\mathrm{eff}}^{\mathrm{model}}$ vs. transmission distance for 4D-OS128 (inset); (b) NLI power for 4D various modulation formats at distance of 8000 km.

tive SNR between a NLI model (4D or 4D model with S-N) estimation and the SSFM simulation as $\Delta\mathrm{SNR}_{\mathrm{eff}}^{\mathrm{model}} \triangleq \mathrm{SNR}_{\mathrm{eff}}^{\mathrm{model}} - \mathrm{SNR}_{\mathrm{eff}}^{\mathrm{SSFM}}$. For all distances shown, the deviation of 4D model with considering S-N interaction (red line in Fig. 2 (a)) is within 0.1 dB.

Fig. 2 (b) shows the NLI power estimation for various modulation formats with different cardinalities M over a distance of 8000 km. For all models shown, the tolerance of different 4D modulation formats to NLI is different. For example, the 4D-64PRS with constant modulus property has better nonlinear tolerance. In addition, for all 4D modulation formats shown, the 4D model with S-N can improve the prediction accuracy of NLI power.

Fig. 3 shows the transmission performance estimation in terms of normalized generalized mutual information (NGMI) for the 4D models. It can be found that the 4D model with S-N can reduce the transmission reach prediction error by 2% and 4%, when compared to the 4D model with S-S only at NGMI of 0.8 for 4D-OS512 and 4D-OS128, respectively. The prediction accuracy gains come from reducing the 4D model overestimation of $\mathrm{SNR}_{\mathrm{eff}}$ compared to the 4D model with S-N. As shown in the insets (a) and (b) of Fig. 3, the 4D model with S-N reduces the gap from SSFM by 0.1 dB at 6000 km for 4D-OS512 and by 0.2 dB at 10000 km for 4D-OS128 compared to accounting only for the S-S term. Therefore, the 4D model with S-N could provide a better accuracy on performance prediction than 4D model, especially in long-distance transmission.

Conclusion

In this paper, we evaluated the weight of signal-ASE noise interaction in the prediction of the effective SNR of general DP-4D constellations. Our results show that when signal-ASE noise interactions are considered the accuracy of SNR estimation is improved by 0.2 dB with respect to using existing 4D NLI models to compute only the signal-signal NLI contribution. Providing an analytical expression for the signal-ASE noise interaction may improve the design of nonlinear-tolerant 4D modulation formats in long-haul systems. Future work will focus on the design of DP-4D formats minimising the joint contribution of signal-signal and signal-noise NLI.

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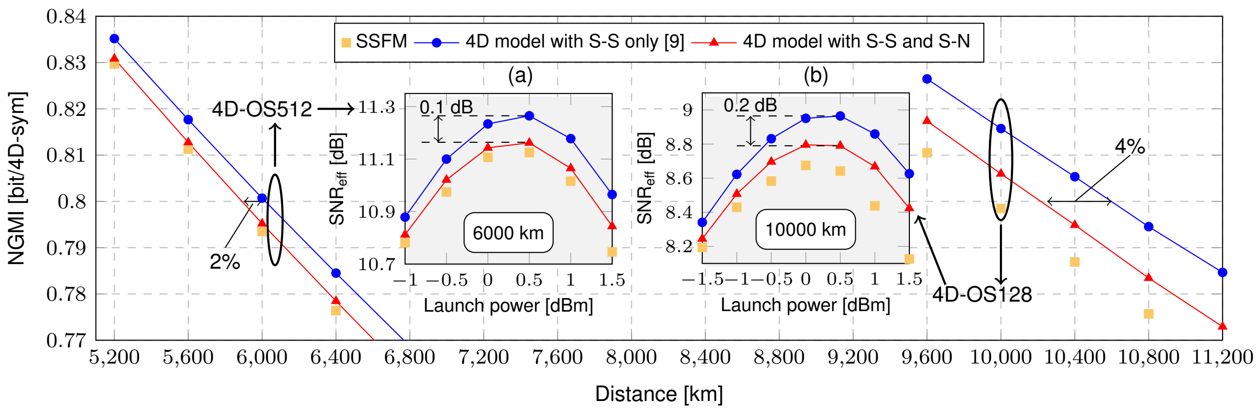

Fig. 3: NGMI vs. transmission distance at optimal launch power for 4D-OS128 and 4D-OS512. Insets: $\mathrm{SNR}_{\mathrm{eff}}$ vs. launch power.

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