

# Model for Nonlinear Interference Noise in Raman-amplified WDM Systems

F. Lorenzi<sup>(1,\*)</sup>, G. Marcon<sup>(1)</sup>, A. Galtarossa<sup>(1)</sup>, L. Palmieri<sup>(1)</sup>, A. Mecozzi<sup>(2)</sup>, C. Antonelli<sup>(2)</sup>, M. Santagiustina<sup>(1)</sup>

<sup>(1)</sup> Department of Information Engineering, University of Padova, Italy,

<sup>(\*)</sup> [francesco.lorenzi.2@studenti.unipd.it](mailto:francesco.lorenzi.2@studenti.unipd.it),

<sup>(2)</sup> Department of Physical and Chemical Sciences, University of L'Aquila, Italy

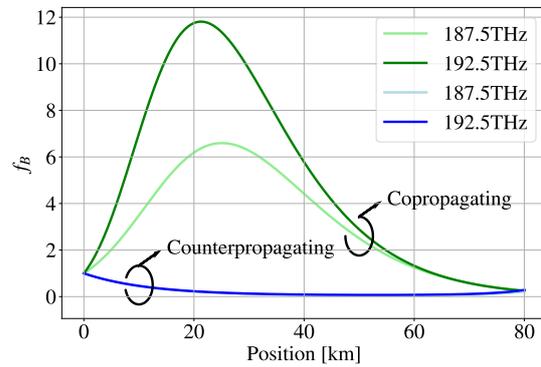
**Abstract** An extension of the model of NLIN to Raman amplified links is presented, in the context of WDM systems. Noise estimation is obtained for an 80km amplified link with optimized pump placement, in co- and counterpropagating regime, for a C+L band configuration. ©2022 The Author(s)

## Introduction

Nonlinear Interference Noise (NLIN) represents a major limitation to the throughput of Wavelength Division Multiplexing (WDM) systems<sup>[1]</sup>. In previous works, a model for the interaction between channels is derived from a first-order perturbation analysis in time-domain<sup>[2]</sup>. This was done assuming the same attenuation-gain properties for all the channels.

However, the proposed NLIN model assumed uniform attenuation-gain over the WDM spectrum, and did not take into account different propagation properties for different channels. While the nonlinear coefficient and dispersion remain constant, the signal power evolutions may depend on channel frequencies. Two main factors contribute to this dependence, the first being fiber attenuation spectrum, and most importantly, Raman gain spectrum, which depends on fiber position. On a separate research track, a deep learning technique was proposed to solve the optimal Raman multiple pump configuration problem given a target amplification among the WDM spectrum<sup>[3]</sup>. Excellent on-off gain fluctuations were obtained (0.04dB). The present development of the NLIN model address the scenario of Raman amplifiers in which Raman interaction spectrum is strongly dependent on channel-pump spacing<sup>[4]</sup>. Moreover, while the NLIN model<sup>[2]</sup> was oriented toward multi-span long-haul communications, for a typical Raman amplified link fiber the dispersion length is on the order of a few hundreds km, well above the typical length of the link itself, and the proposed approximation, which uses Fresnel transform - Fourier transform approximation<sup>[2],[5]</sup> is not easily exploitable.

This work aims to obtain a method for estimating NLIN in those Raman amplified systems, and to obtain numerical estimates with usual param-



**Fig. 1:**  $f_B(z)$  profile along the fiber for channel 1 and 50, in co- and counter-pumping. Values for -10dBm average input power.

eters, and optimized Raman pumping. This approach is tested on a common amplifier configuration, and metrics of interest are derived.

## Proposed model

Consider two WDM channels: A is the channel of interest and B the interfering channel. Let us consider the transmission, at  $z = 0$  of the pulse trains complex envelopes

$$\begin{aligned} u_A(0, t) &= \sum_k a_k g(0, t - kT) \\ u_B(0, t) &= \sum_k b_k g(0, t - kT) \end{aligned} \quad (1)$$

channel spectral spacing is implicit in the complex envelope notation, so the physical field contains different time-harmonic components in different channels. The definition is such that

$$\int_{-\infty}^{+\infty} |g(z, t)|^2 = 1. \quad (2)$$

The problem of NLIN estimation can be addressed<sup>[2]</sup> by introducing a correction term in the generic received symbol  $\tilde{a}_0$ . Such correction will in general be dependent on the symbol energies

of both the channel of interest and the interfering one. By considering an optimal receiver setting and dispersion compensation, the general correction term can be written as<sup>[2]</sup>

$$\Delta\tilde{a}_0 = i\gamma \sum_{h,k,m} a_h a_k^* a_m S_{h,k,m} + 2a_h b_k^* b_m X_{h,k,m}. \quad (3)$$

These terms, and in particular  $X_{h,k,m}$ , whose units are [m/s], are independent on modulation format and average power, and summarize the fiber and channel properties. Moreover, the terms with  $h = 0$  and  $k = m$  dominate the sum, as they correspond to full pulse overlapping. The target quantity to be estimated, is  $X_{0,m,m}$ , whose contribution to the estimation error is

$$\Delta\tilde{a}_0 \approx i2\gamma a_0 \sum_m |b_m|^2 X_{0,m,m}. \quad (4)$$

In order to obtain  $X_{0,m,m}$  in Raman amplified link, we consider a single-channel normalized-field NLSE, similar to the following attenuation-normalized one

$$\frac{\partial}{\partial z} u = -i\frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} u + i\gamma f(z) |u|^2 u. \quad (5)$$

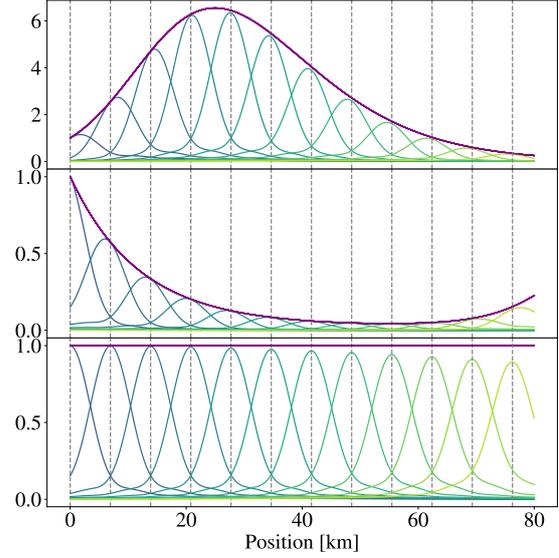
The original approach considered the field  $u$  as the superposition of two channels. That approach is unsatisfactory in our case because of the dependence of the attenuation-gain term  $f(z)$  on the channel center frequency. A novel formulation of the problem requires the use of a pair of NLSE, one per channel, in which the term  $f(z)$  splits in  $f_A(z)$  and  $f_B(z)$ . By gathering the coupling coefficient<sup>[6]</sup>, and considering spectral spacing, we obtain

$$\frac{\partial}{\partial z} u_A = -i\frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} u_A + i\gamma \left( f_A(z) |u_A|^2 + 2\frac{f_A(z)}{f_B(z)} |u_B|^2 \right) u_A, \quad (6)$$

$$\frac{\partial}{\partial z} u_B = -\Delta\beta_1 \frac{\partial}{\partial t} u_B - i\frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} u_B + i\gamma \left( f_B(z) |u_B|^2 + 2\frac{f_B(z)}{f_A(z)} |u_A|^2 \right) u_B. \quad (7)$$

We get a first-order correction similar to the original one, with the desired attenuation-gain coefficient, from which it is possible to write

$$X_{0,m,m} = \int_0^L dz f_B(z) \int_{-\infty}^{+\infty} dt \times |g^{(0)}(z,t)|^2 |g^{(0)}(z,t - mT - \beta_2\Omega z)|^2. \quad (8)$$



**Fig. 2:** Normalized values of the  $z$ -dependent integrand of Eq. 8 for train of interfering symbols, superimposed to  $f_B$  (purple). Top: co-pumping scheme, middle: counter-pumping scheme – -3dB target Raman gain — bottom: perfect amplification. Values for -10dBm average input power, adjacent channels.

The resulting noise variance can be evaluated: as can be inferred from Eq. 4 noise consists in a random phase perturbation. The expression of phase noise variance is solely dependent on the interaction term  $X_{0,m,m}$  and the statistics of the chosen transmission modulation

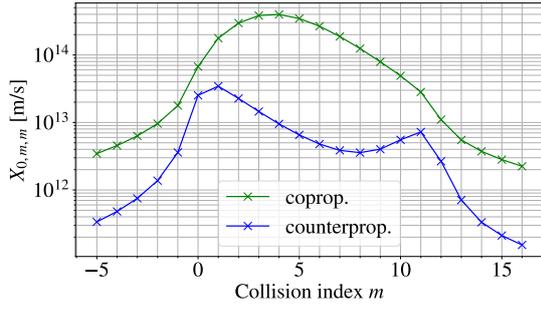
$$\Delta\theta^2 = 4\gamma^2 (\mathbb{E}[|b_0|^4] - \mathbb{E}[|b_0|^2]^2) \sum_m X_{0,m,m}^2. \quad (9)$$

where  $\mathbb{E}$  is the expectation operator, evaluated over constellation symbol distribution. Assuming statistical independence between noise contributions, we sum their variances.

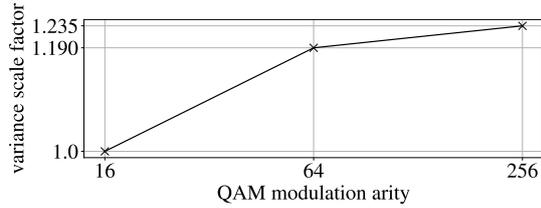
### Numerical results

The system considered is an 80km standard fiber link (with  $\beta_2=23\text{ps}^2\text{km}^{-1}$ ,  $\gamma=1.3\text{W}^{-1}\text{km}^{-1}$ ). Transmission consists of a WDM grid of 50 channels at 10Gbaud, with Nyquist pulses, spaced by 100GHz. WDM center frequency is 190THz, so channel center frequency spans from about 187.5THz to 192.5THz. Raman amplifier consists in 10 pumps in the counterpropagating scenario, and 8 pumps in the copropagating one. Average signal power is considered in the range from -20dBm to 0dBm at the input.

For every chosen signal power, a pump placement optimizer similar to the one proposed<sup>[3]</sup> was executed, and we found pump wavelengths and powers using a flat target on-off gain of -3dB. Moreover, the optimization procedure provided the signal amplitude profile along the fiber span,



**Fig. 3:**  $X_{0,m,m}$  coefficient. At the left and right side of the plot is the contribution of partial collision, which fall outside of the fiber span. Values for -10dBm average input power.



**Fig. 4:** Normalized noise variance vs QAM modulation arity

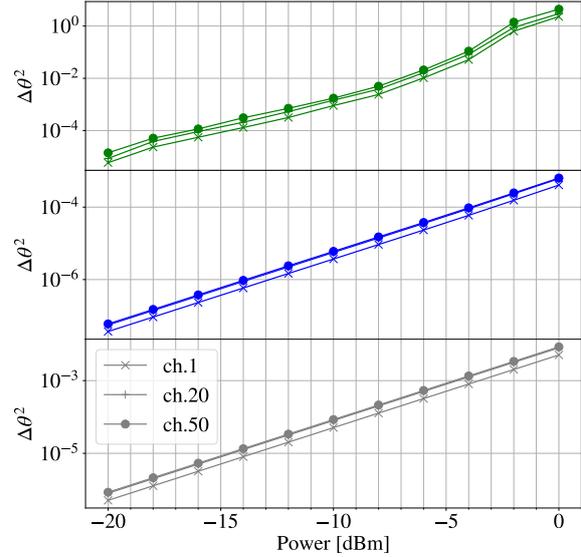
for every channel. From this profile we derived the  $f_B(z)$ ; two examples are shown in Fig. 1.

Collision shapes, i.e. the integrand of Eq. 8, are shown in Fig. 2, in the case of adjacent channels, with various amplifications. The corresponding  $X_{0,m,m}$  is shown in Fig. 3.

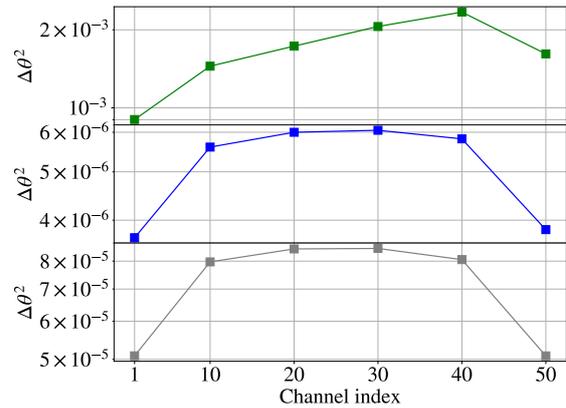
From Eq. 9 we deduce that QAM modulation arity only impacts the noise variance through a multiplicative factor. This factor, normalized to the 16-QAM case, is represented in Fig. 4 for common modulations. The noise variance with respect to signal power is shown in Fig. 5 for 16-QAM, and various channels of interest. Fig. 6 shows the dependence from channel position: notice the strong symmetry breaking in copropagating case due to variation in Raman amplification profile (Fig. 1).

## Conclusions

We presented an extension of the analytical model for NLIN, flexible and able to account for channel-dependent attenuation and gain. Predictions in the case of Raman co- and counter-pumping show that the counter-pumping scheme presents a phase noise variance nearly one order of magnitude less than the co-pumping scheme. Moreover, NLIN noise in copropagating amplification is asymmetric with respect to the channel frequency position, due to the larger differences in the evolution of the channel signal powers. Future development may involve the integration of NLIN estimation in the Raman amplifier design



**Fig. 5:** Phase noise dependence on per-channel average signal power. 16-QAM constellation. Top: copropagating, middle: counterpropagating, bottom: perfect amplification



**Fig. 6:** Phase noise dependence on channel position in WDM grid. 16-QAM constellation, -10dBm average power. Top: copropagating, middle: counterpropagating, bottom: perfect amplification

optimization, along with ASE noise.

## Acknowledgements

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