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Modulation-Format Dependent Impact of Modal Dispersion on Cross-Phase Modulation in SDM Transmission

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Abstract *We show that the interplay between spatial mode dispersion (SMD) and the modulation format has a substantial impact on cross-phase modulation (XPM) in space-division multiplexed systems with strongly coupled modes. We propose a simple formula to account for SMD in the modulation-format-dependent XPM contribution. ©2022 The Author(s)*

Introduction

The study of the nonlinear cross-talk between the optical paths of a space-division multiplexed (SDM) system[1],[2] is particularly challenging due to the presence of spatial mode dispersion (SMD), that is responsible for introducing a critical layer of complexity in the analysis. While mode dispersion is typically negligible in single-mode transmission systems, it plays a key role for SDM systems where it can reach much larger values[3],[4]. When linear equalization is feasible at the receiver, high values of SMD are beneficial to mitigate nonlinear cross-talk in strongly-coupled SDM systems[1],[5],[6].

As the characterization of the optical fiber nonlinearities becomes non-trivial in SDM systems, simple perturbative models, originally derived for single-mode transmissions, play a pivotal role. Of particular interest is the Gaussian noise (GN) model[7] for the estimation of the nonlinear interference (NLI) variance. This model was extended in[8] to SDM links without accounting for the effects of SMD between strongly-coupled modes.

The conservative prediction of the GN model was improved by including the consideration of the modulation format[9]–[12]. In particular, the SMD impact on the cross-phase modulation (XPM) variance was modeled in[11] for large SMD values while neglecting the effect of SMD within individual channels. Arbitrary values of SMD have been accounted for in[13] for self-phase modulation (SPM) only and in[14] by using Ito's calculus. In[14] we characterized the dependence of the NLI variance on the amount of SMD under the assumption of Gaussian modulation, and showed that an SMD value minimizing the XPM variance in strongly-coupled transmissions exists.

In this work, we extend the investigation of[14] by looking into the role of the modulation format in the interaction between Kerr effect and mode dispersion. We observe a substantial impact and provide a simple formula to estimate it.

Numerical analysis

At first, we investigated the joint impact of SMD and modulation format on the XPM variance via numerical simulations. We performed split-step Fourier method (SSFM) simulations based on a waveplate model of the optical fiber[15]. We considered an optical fiber carrying $N = 2$ spatial modes for a total of 4 strongly-coupled polarization modes. The fiber had dispersion 17 ps/(nm·km), attenuation 0.2 dB/km and nonlinear coefficient[1] $\frac{1.26}{2N}$ $(\mathrm{W}\cdot\mathrm{km})^{-1}$. We transmitted random sequences of 2^{17} symbols on each space and frequency channel.

We estimated the XPM variance for a single-span link of 100 km, where the dependence on the modulation format is stronger[9],[16]. In each spatial mode, we sent two wavelength division multiplexing (WDM) channels spaced away 100 GHz, for a total of 8 independent channels. The data were digitally modulated by complex Gaussian distributed symbols, quadrature shift-keying (QPSK), or 16 quadrature amplitude modulation (16QAM) at a symbol rate of 49 Gbaud.

Figure 1 shows the XPM variance per-polarization as a function of the SMD coefficient[14], where the results are represented in the form of a histogram extracted from 500 independent realizations of the random-mode coupling process. As observed in[14] for the same setup, the XPM variance for the Gaussian case exhibits a minimum around $8\,\mathrm{ps}/\sqrt{\mathrm{km}}$ where the XPM variance reduction due to SMD is approximately 1.5 dB.

Surprisingly, Fig. 1 shows that such a variance reduction grows for 16QAM and QPSK transmissions. In particular, the QPSK curve exhibits up to approximately 4.5 dB of XPM variance reduction

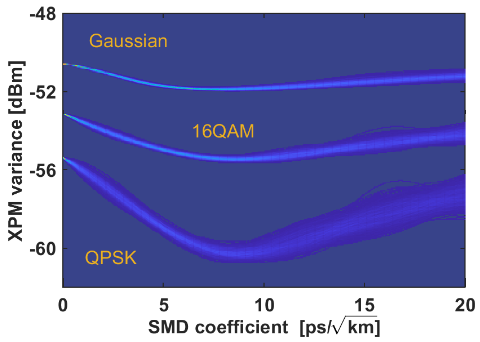

Fig. 1: XPM variance vs SMD coefficient. $N = 2$ strongly coupled modes. Gaussian distributed symbols, 16QAM or QPSK modulation. Single span of 100 km. SSFM results histogram for 500 realizations.

thanks to the beneficial effect of SMD. The numerical results reported in Fig. 1 suggest that the role of the modulation format in setting the NLI is SMD dependent, yielding, in this setup, a relevant extra reduction of up to 3 dB wrt to the case in which SMD is absent.

Motivated by such non-negligible XPM mitigation, we extended the GN theory in[14] to include SMD on the format-dependent contribution to the XPM variance.

Results

According to perturbative theory, the NLI variance can be expressed as

$$\sigma^2_{\mathrm{NLI}} = \sigma^2_{\mathrm{NLI,GN}} - \sigma^2_{\mathrm{NLI,FON}} + \sigma^2_{\mathrm{NLI,HON}} \qquad (1)$$

where $\sigma^2_{\mathrm{NLI,GN}}$ is the contribution to the NLI variance provided by second-order symbol statistics (aka GN contribution) while $\sigma^2_{\mathrm{NLI,FON}}$ and $\sigma^2_{\mathrm{NLI,HON}}$ account for the fourth-order noise (FON) and higher-order noise (HON) contribution, respectively[9],[10],[16]. While the GN term depends on the transmitted symbols only through their average power, the FON and HON terms carry information also on the modulation format. The FON contribution in Eq. (1), which is the dominant format-dependent part, is negative and thus acts as a correction to the over-estimate of the NLI variance given by the GN model[9],[10].

Unfortunately, the inclusion of SMD in the non-GN contributions in Eq. (1) prevents the derivation of analytical solutions. Nevertheless, the analysis of XPM can be significantly simplified, along the lines of[14], by neglecting the effects of SMD within individual WDM channels. This is equivalent to assuming that different WDM channels experience different random-mode coupling processes, whereas random-mode coupling is constant within the bandwidth of each channel. The idea is sketched in Fig. 2 .

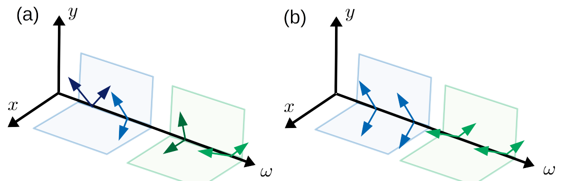

Fig. 2: Sketch of SMD-induced depolarization: (a) different for all frequencies, and (b) approximated as identical for all the frequencies within individual WDM channels.

Under this simplifying assumption, we derived the following expression for the FON contribution to the XPM variance:

$$\sigma^2_{\mathrm{XPM,FON}} = \frac{1}{2N}\Bigg[(2N+1)^2\sigma_1^2(\alpha) + (2N-1)\frac{\left(\alpha+\frac{\Delta\omega^2\mu^2}{N}\right)}{\alpha}\sigma_1^2\left(\alpha+\frac{\Delta\omega^2\mu^2}{N}\right)\Bigg] \qquad (2)$$

where α is the attenuation, $\Delta\omega$ is the spacing between WDM channels, and $\mu = \sqrt{\frac{N^3}{4N^2-1}}\eta_{\mathrm{SMD}}$ with η_{SMD} the SMD coefficient[14],[17]. The term $\sigma_1^2(\alpha)$ represents the per-polarization FON contribution to the XPM variance in the absence of polarization-dependent effects[9]. Since HON is negligible for cross-channel effects[16], we limited the analysis to the derivation of the FON contribution. Equation 2 is the central result of this work.

We then performed SSFM simulations to assess the accuracy of the proposed simplified formula. To this aim, we extracted the FON contribution to the XPM variance from SSFM results as $\sigma^2_{\mathrm{XPM,FON}} \approx \sigma^2_{\mathrm{XPM,QPSK}} - \sigma^2_{\mathrm{XPM,GN}}$, where the last two terms were estimated via QPSK and Gaussian simulations. The SSFM results, averaged over 500 random seeds, are plotted with markers in Fig. 3(top) for the single-span setup of Fig. 1 , and in Fig.3(bottom) for 5 spans.

The theoretical results are also shown, by means of solid curves, in the same figure, where the FON contribution to the XPM variance is a plot of the newly derived expression Eq. (2). We observe that this contribution spans a much smaller range compared to the GN term. This observation supports the use of the simplified formula, which captures the main features of the FON contribution across the relevant range of SMD values, thus yielding a very accurate estimate of the XPM variance for QPSK transmission. In particular, while the GN model predicts a maximum XPM reduction of approximately 1.5 dB at $\eta_{\mathrm{SMD}} = 8\,\mathrm{ps}/\sqrt{\mathrm{km}}$, its correction through the proposed formula allows us to correctly predict a reduction of approximately 4.5 dB after one span

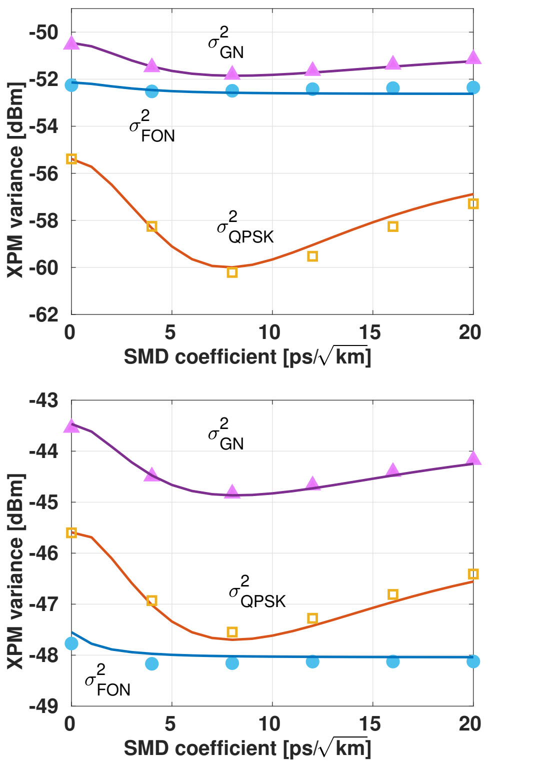

Fig. 3: XPM variance vs SMD coefficient after one (top) and five spans (bottom). Other parameters as in Fig. 1 . Markers: SSFM results for QPSK (squares) and the corresponding GN (triangles) and FON (circles) contributions. Lines: theory.

when QPSK is considered. The XPM variance decrease reduces to 2 dB after 5 spans, as shown in Fig. 3(bottom), due to a smaller FON correction. Figure 3 allows us to note that the larger SMD-induced XPM-reduction for the QPSK is due to the large relative importance of the FON correction around the value of the SMD coefficient at which the GN contribution reaches a minimum.

While we limited SSFM simulations to 5 spans in order to collect results for a statistically meaningful number of random seeds, the observed qualitative behavior persists even for longer links, where the FON correction is less relevant[16]. This is clarified in Fig. 4 , where, by exploiting the novel formula, we report a theoretical estimation of the XPM reduction at $\eta_{\text{SMD}} = 8\,\text{ps}/\sqrt{\text{km}}$ with respect to its value in the absence of SMD, as a function of the number of spans N_{s}. It can be seen that the reduction remains unchanged for a Gaussian transmission, as the XPM variance scales with N_{s} (in linear scale) for any SMD coefficient. On the other hand, the XPM reduction for 16QAM and QPSK decreases without reaching the GN value, thus suggesting the importance of the FON contribution even for long-haul links.

Finally, we investigated the transmission of N_{ch} WDM channels per mode spaced 75 GHz mod-

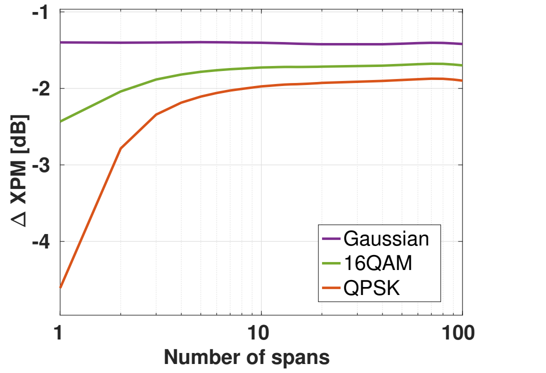

Fig. 4: XPM variance reduction at SMD coefficient 8 ps/$\sqrt{\text{km}}$ wrt the no SMD case vs number of spans.

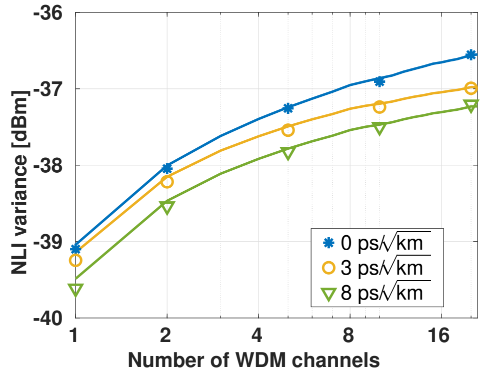

Fig. 5: NLI variance vs number of WDM channels at 75 GHz spacing modulated with QPSK. $N = 2$ strongly-coupled modes propagation along a 5×100 km link with fixed SMD coefficient. Markers: SSFM simulations. Lines: theory.

ulated with QPSK symbols at 64 Gbaud over a 5×100 km link. In Fig. 5 we report the NLI variance resulting from SSFM simulations (markers) and the theory (lines), as a function of N_{ch} for three fixed values of the SMD coefficient. The theory here includes also the SMD impact on the SPM variance of FON and HON contributions through the scaling rule in[13]. The GN contribution, modeled via[14], included all four-wave mixing processes[9]. The excellent agreement between theory and simulations is self-evident and it comes with computation times that are orders of magnitude smaller.

Conclusions

We showed via numerical simulations that the impact of SMD on the NLI variance strongly depends on the modulation format. We thus extended the GN model in[14] to account for the effect of mode dispersion on the format-dependent contributions of the NLI variance. The proposed model shows good accuracy against SSFM simulations and constitutes a useful tool for quickly assessing the performance of SDM systems.

Acknowledgments

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