Optimization of Geometric Constellation Shaping for Wiener Phase Noise Channels with Varying Channel Parameters

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Abstract We present a novel method to investigate the effects of varying channel parameters on geometrically shaped constellations for communication systems employing the blind phase search algorithm. We show that introduced asymmetries significantly improve performance if adapted to changing channel parameters. ©2022 The Author(s)

Introduction

Blind phase search (BPS) is a state-of-the-art algorithm for blind, feed-forward, carrier phase synchronization for high-rate coherent optical communications receivers. A big advantage over decision-directed, feedback-based carrier phase synchronization algorithms is the possibility for parallel and pipelined implementation [1]. When a classical square quadrature amplitude modulation (QAM) constellation is used in systems employing BPS as their carrier phase synchronization algorithm, a phase ambiguity is introduced by rotational symmetry of the constellation. Additionally, classical square QAM suffers from a penalty in achievable rate and a gap to capacity, which can be overcome with geometrically or probabilistically shaped constellations [3], [4], [6]-[10]. Therefore, we propose to apply geometric constellation shaping to improve spectral efficiency and robustness of optical communication systems employing BPS. In previous works, geometric constellation shaping (GCS) for BPS has been either optimized on a single set of channel parameters [10], or on a range of sets of channel parameters [8]. In [10], performance is only improved for channel parameters which have been used for training, or in better channel conditions. Training on a range of channel parameters results in a constellation which is robust to varying channel parameters, but may be underperforming for good channel conditions [8], [10]. For both approaches, changes in the position of constellation points compared to classical square QAM cannot easily be attributed to either performance improvement of the BPS algorithm, or robustness to channel impairments. Thus we propose to apply GCS with an additional channel condition parameter input at the mapper and demapper to investigate the effect of varying channel parameters on GCS for BPS. A similar approach with parameterizable and trainable neural mapper and demapper has been shown in [6] for additive white Gaussian noise (AWGN) channels. This will show effects of variation in different channel parameters on constellations maximizing the bitwise mutual information (BMI)¹. We will compare the performance of the parameterized GCS constellation with a classical square QAM constellation and a constellation robust to changes in parameters.

Parameterizable Binary Auto-Encoder

The system model in Fig. 1 used for our work is an end-to-end (E2E) model representing a highrate coherent optical communication system with a transmission channel affected by AWGN and laser phase noise. We use a binary auto-encoder to perform the GCS, which uses bit vectors of length m bits per symbol as input and outputs m log-likelikood ratios (LLRs). At the transmitter (Tx), m information bits b_k are converted to a one-hot vector. The AWGN standard deviation σ_n and Wiener phase noise increments' standard deviation σ_{Φ} are fed as inputs to the Txneural network (NN) to generate a constellation \mathcal{M} of size $|\mathcal{M}| = 2^m$. With the output of the Tx-NN and the one-hot vector, one complex constellation point $x_k \in \mathcal{M}$ is selected. This is represented in the system model in Fig. 1 as dot product (\odot) between the one-hot vector and the constellation vector. The complex symbol vector $\boldsymbol{x} = (x_0, \dots, x_{B-1})^{\mathrm{T}}$ for one batch of size B is sent through the auto-encoder channel model. In our system model the auto-encoder channel is

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¹The BMI is often referred to as generalized mutual information (GMI) in the optical communications community. We prefer to use the term BMI due to its easier resemblence with the operational meaning.



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Fig. 1: System model of parameterizable auto-encoder with differentiable BPS

comprised of AWGN, Wiener phase noise and a differentiable BPS. The differentiable BPS differs from the classical, non-differentiable, BPS algorithm in the replacement of the $\arg \max$ operation with a differentiable approximation [10]. At the output of the (differentiable) BPS, complex symbols \hat{x} are sent to a neural demapper with a receiver (Rx)-NN, which returns m LLRs. The Tx-NN consists of an input dimension of 2 and two fully connected layers with output dimension of 2^{m+1} with ReLU activation functions for the input and hidden layer. In contrast to the system model shown in [10], both neural mapper and demapper have additional inputs for σ_n and σ_{Φ} , which allows the GCS to be optimized across a range of channel conditions. At the transmitter (Tx), an optimized constellation for a particular channel condition can be obtained by evaluating the output of the Tx-NN. We implemented our system for simulation and validation in the PyTorch framework [5].

Parameterized GCS Setup

To learn a geometrically shaped constellation which is optimized on a range of channel conditions, defined by a pair of parameters σ_n and σ_{ϕ} , we take the following approach: For every batch in a training epoch, a new σ_n and σ_{Φ} is sampled uniformly in $[\sigma_{n,\min}, \sigma_{n,\max}]$ and $[\sigma_{\phi,\min}, \sigma_{\phi,\max}]$. The BPS algorithm is updated accordingly with the transmit constellation obtained with the new parameters. In this work, we selected σ_n such that the signal to noise ratio (SNR) is between $14 \,\mathrm{dB}$ and $25 \,\mathrm{dB}$ and σ_{Φ} is selected such that the laser linewidth is between $50\,\rm kHz$ and $600\,\rm kHz$ for a symbol rate of 32 GBaud. BPS has been configured with 60 test angles and a window size of 128. Training has been performed for 1000 epochs with a linearly increasing batchsize from 1000 to 10000 samples. Similar to [10], the temperature parameter of the differentiable BPS is decreased from 1.0 to 0.001 during training to approximate the real BPS more closely at the end of the training. The binary cross entropy (BCE) is



Fig. 2: Shaped constellation at SNR of $18\,\mathrm{dB}$ and varying laser linewidth



Fig. 3: Shaped constellation at a laser line width of $100\,\rm kHz$ and varying SNR

used as loss function and the Adam algorithm [2] is used for the optimization. With the binary autoencoder, both geometric shaping and bit labelling are optimized simultaneously. For simplicity, bit labels are omitted in the constellation plots.

Effects of Varying Channel Parameters

To investigate the separate effect of varying AWGN and Wiener phase noise on the shaped constellation for m = 6 bits per symbol, we plot some transmit constellations in Fig. 2 and Fig. 3. In Fig. 2, AWGN is fixed to yield an SNR of 18 dB and the linewidth is varied between 50 kHz, which is shown in blue, and 600 kHz, which is shown in orange. It can be observed that most points in the transmit constellation deviate insignificantly for varying linewidth. The biggest change is observed for a single constellation point in the top right of the constellation, which is shifted out-



Fig. 4: Validation results of parameterizable GCS for different SNRs

wards for a larger laser linewidth. A smaller but notable shift outwards can be observed for three constellation points in the bottom right. This observation can be explained with improved performance of the BPS algorithm if a constellation point is separated by a larger distance radially. In Fig. 3, we display transmit constellations for a fixed laser linewidth of $100 \, \mathrm{kHz}$ and the SNR varying between $14 \,\mathrm{dB}$, in blue color, and $25 \,\mathrm{dB}$, in orange color. For low SNRs, the constellation points are clustered in groups, which shows the effect of reducing the distance between constellation points differing in a single bit and increasing the distance between other constellation points. Similarly to the effects of varying linewidths in Fig. 2, the same constellation points are moved outwards for low SNR, which, again, points to improved BPS performance due to less ambiguities of the constellation impaired by phase rotations.

Performance With Parameter Misestimation

In Fig. 4, we show validation results in terms of BMI for parameterizable geometric constellation shaping (pGCS) constellations. The results are shown in the plot for a range of SNRs and across a range of laser linewidths. The results displayed in solid lines are obtained with matching parameter inputs to the Tx-NN and Rx-NN. This corresponds to transmitter and receiver with perfect channel knowledge. For results with dotted lines, the SNR is overestimated by $2 \, dB$ at the receiver and transmitter. Results in dashed lines are performed with the autoencoder system underestimating the SNR by 2 dB. For high SNRs, estimation errors in the SNR do not incur a performance penalty with the transmitter and matched receiver system. Underestimating the SNR always leads to a better BMI than overestimating.



Fig. 5: Performance comparison of parametrizable GCS with square QAM and robust constellation from [8]

Performance comparison

In Fig. 5, the validation results of the pGCS constellation, a Gray-mapped square QAM and a constellation robust to variance in the channel parameters [8] are compared in terms of the BMI. The Gray-mapped square QAM constellation, which is combined with a matched parameterized neural demapper, and the robust constellation are trained on the same range of channel parameters as the pGCS constellation, but without a parameterized channel condition input. The pGCS constellation outperforms the reference constellations for all channel conditions. The robust constellation matches the performance of the pGCS constellation very closely for $17 \, dB$ and 18 dB, but shows a significantly worse performance for higher and lower SNR. The robust constellation shows a good performance for increasing laser linewidth, with only an insignificant drop in performance compared to the pGCS constellation. The performance of the Gray-mapped QAM constellation is close to the performance of the pGCS constellation for high SNR and low laser linewidth. For increasing laser linewidths, a substantial drop in performance can be observed.

Conclusions

In this work, we have shown the effect of changes in channel parameters for GCS constellations in the presence of the BPS algorithm for carrier phase recovery. We have introduced the novel pGCS constellation. For higher laser linewidths and AWGN, the introduced asymmetry of a small number of constellation points contributes significantly to performance improvement over static reference constellations.

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