Real-Time Gapless Analog Time Frequency Analysis for Bandwidths above 20 GHz with Nanosecond Resolution

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Abstract *We propose a scheme for real-time, joint time-frequency analysis of arbitrary continuous optical signals. Through phase-only linear operations, we demonstrate the recovery of ns-duration transient events, 22-GHz bandwidth operation, and sub-GHz resolution. Billions of Fourier transforms per second are computed without information loss.*

Introduction

A spectrogram is a joint time-frequency representation (JTFR) of a signal. The amplitude of the signal over time and the frequency content in each time section account for all the ascertainable information of a waveform. Thus, JTFRs are amongst the most complete ways to describe a waveform[1]. For slowly varying signals we can use digital signal processing (DSP) to compute the spectrogram (e.g., Short Time Fourier Transform (STFT)). This algorithm Fourier transforms (FT) each small time section, and these spectra are subsequently plotted consecutively in the vertical direction. For a faithful representation of the signal under test (SUT), it is required that no sections of the signal be missed (i.e., that the JTFR is gapless) so that transient events be properly detected. Processing the spectrogram in real time is especially suited to applications which register, and must act on, a large amount of nonperiodic data as quick as possible. One such application is real-time target identification of unmanned aerial vehicles (UAVs). For instance, complex artificial intelligence schemes are being developed to quickly analyze the spectrograms resulting from radar and lidar Doppler velocimetry to classify UAVs[2]. For applications involving physical speeds greater than 100km/s (e.g., implosions within inertial confinement fusion facilities), the resulting spectrograms can have above 10 GHz bandwidths and features changing on the scale of tens of nanoseconds[3]. Real-time spectral sensing over broad bandwidths is also required for cognitive communication systems. Specifically for secure transmission, frequency hopping signals are selected for their ability to rebuff nefarious interference. Currently, the hopping rate is limited by device switching speeds for detection and digitization[4].

To this day, JTFRs are habitually computed through DSP approaches which are limited in the number of spectra per second. For this reason, crucial developments in the above applications are hindered. Tektronix currently claims the fastest FT engine (<10M FTs/s) and shortest minimum detectable signal length (sub-microsecond) for real time spectrum analyzers[5],[6]. Additionally, DSP approaches are restrictive for broadband waveforms, as real-time analyzers are generally limited to signals below hundreds of MHz. Although there is intensive research in advancements for computation schemes, DSP approaches are facing challenges arising from physical limitations of the electronic circuits they operate on[7].

S. Reddy *et al.* recently reported a completely analog, real-time, and gapless method for JTFRs[8]. Using a fast pulse train for temporal sampling, followed by photonics based passive transformations, a rate of 5 billion FTs/s was achieved. They also recorded transient events as short as 5 ns and up to 5 GHz bandwidth. The performance of this analog system is far beyond reach of DSP methods. However, there are two main difficulties in this method. Firstly, the SUT must be sampled by a fast optical pulse train. This increases system complexity and reduces energy efficiency. Secondly, the detector bandwidth must be much higher than the SUT bandwidth (i.e., comparable to the employed pulse train). This is a significant issue for prospective applications involving bandwidths of tens of GHz or higher.

In this communication, we present a novel analog, real-time, gapless spectrogram by means of passive, phase only transformations. We refer to this as the time lens spectrogram (TLS). Our proposed method is theoretically lossless and does not require intensity sampling of the SUT with a high speed pulse train. Furthermore, the mathematical derivations indicate that the maximum bandwidth is not limited by the detector speed. We demonstrate billions of FTs/s with sub-GHz resolution and tens of GHz bandwidths. For the first time, we show a time lens system can be configured for gapless, real-time analysis of continuous non-periodic signals.

Operation Principles

A well-known concept in spatial optics is the FT property of a parabolic lens[9]. An object placed at the focal length of a lens will create an image which is the FT of the object in the paraxial approximation. Just as a spatial lens focuses light in space, a time lens focuses light in time[10]. A parabolic time lens is a quadratic phase modulation over time, which can be applied with an electro-optic modulator. Following this quadratic phase modulation with a dispersive element to provide the quadratic phase filtering over frequency, we get the spectrum of the waveform mapped into the time domain[10],[11]. Time-lens FTs have so far been constrained to operate on time-limited waveforms since the output image will be stretched in time. If objects are too close, complex interferences will distort the output. Here we demonstrate an extension to the capabilities of time lenses from JTFR for time-limited events, to JTFR for continuous and arbitrary signals without information loss.

To do this we employ an array of consecutive time lenses as indicated in Fig. 1. Such arrangements (also known as lenticular lenses[12],[13]) have been used for sampling optical waveforms. Following this, the signal travels through an amount of group velocity dispersion. This produces an array of spectra displaying the frequency content of the waveform during that time segment. To enable this extension to continuous signals, the lenticular time-lens system must be designed to ensure that there is no interference among consecutive time-mapped spectra.

We briefly recall that the effects of a time lens and of second order chromatic dispersion are modelled as:

$$\varphi(t) = \frac{C_L}{2}t^2 \quad , \quad \phi(\omega) = \frac{\beta_2 L}{2}\omega^2$$

where $\varphi(t)$ and $\phi(\omega)$ are the phase functions along time (t) and radial frequency (ω) domains imposed by the time lens and dispersive elements, respectively. C_L determines the strength of the time lens, and β_2 is the group velocity dispersion per length (L) of the dispersive element. The imaging conditions for the phase manipulations, such that the maximum full bandwidth $2f_{max}$ of the SUT is mapped to a temporal section that is no larger than T_L (the time aperture), are:

$$C_L = \frac{1}{L\beta_2} \quad , \quad |\beta_2 L| = \frac{T_L}{4\pi f_{max}}$$

A frequency component f_i will be mapped at a time $t_i = 2\pi\beta_2 L f_i$ away from the center of the time lens. The number of analysis points N_f is $2f_{max}/\delta f$. The first contribution to the broadening of the frequency resolution δf is due to the rectangular shape of the lens aperture. The model of the time lens array is a sum of shifted parabolas multiplied by a rectangular window function. The output spectrum will thus be convolved with the sinc function corresponding to the FT of the lens aperture ($\delta f \approx \pi/(4T_L)$). The detector does not need to exceed the bandwidth of the SUT, only fast enough to resolve pulse widths on the order of T_L/N_f.

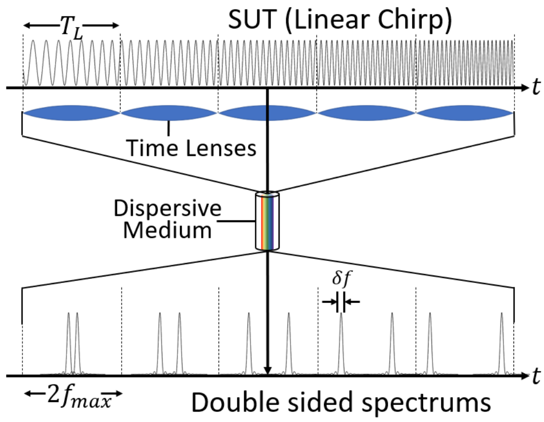

Fig.1: Concept for the time lens spectrogram (TLS). The blue shapes are a representation of the time lenses. The result for a linear chirp is spreading sinc pulses of width δf with a temporal location t corresponding to the frequency content of the SUT during each time lens aperture T_L. Upon proper design, the maximum frequency of the signal f_{max} will be mapped near to the edge of each time lens.

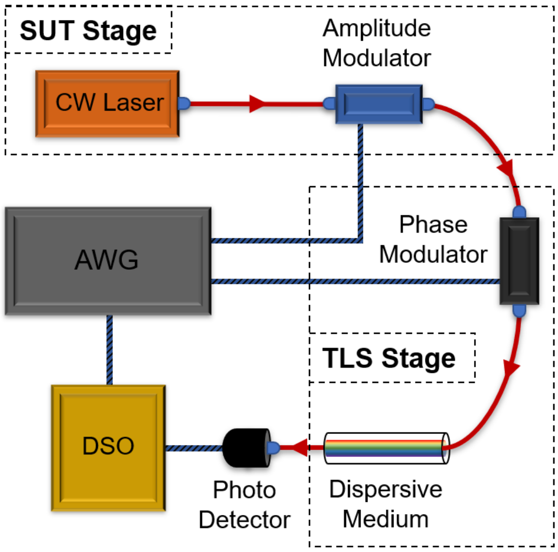

Fig.2: Experimental setup for the time lens based spectrogram (TLS). CW: continuous wave. AWG: arbitrary waveform generator. DSO: Digital sampling oscilloscope.

Methods

The setup employed for experimental demonstration is shown in Fig. 2. We prepared the SUT by carving out an infrared continuous wave (CW) laser light using a radio frequency (RF) signal. The CW laser by NKT photonics was centered at 1549.84 nm and sent to a 40-GHz intensity modulator by Optilab which was driven by a 65-GSa/s Arbitrary Waveform Generator (AWG), model M8195A by Keysight. The SUT is then measured to provide the data for the digitally processed STFTs in Figs. 3 and 4 and sent through the TLS setup. The same AWG is also used to drive the 40-GHz electro-optic phase modulator (EOspace). The phase modulated SUT then travels through a Linearly Chirped Fiber Bragg Grating (LCFBG) and is detected by a 50-GHz photodetector connected to a 70-GHz bandwidth digital sampling oscilloscope.

Results

We demonstrate two configurations of the TLS system. In both cases the maximum phase shift was approximately 2π. This corresponds to 8 analysis points per side of each spectrum. The maximum dispersion available was equivalent to ~700 km of single mode fiber ($\beta_2 L = 15{,}268\ \mathrm{ps}^2$) which was used for our first system. In the second, an LCFBG equivalent to 115 km ($2{,}524.4\ \mathrm{ps}^2$) was used. The time apertures for the two cases were 1.08 ns and 353 ps, while the maximum full bandwidths were 11.2 GHz and 22.4 GHz, respectively. We first test a SUT comprising two simultaneous linear frequency chirps, one increasing and one decreasing, shown in Fig. 3. For both cases, the linear chirp along the positive frequency side extends from 0 up to the maximum frequency for the chosen time lens, with a mirrored spectrum on the negative

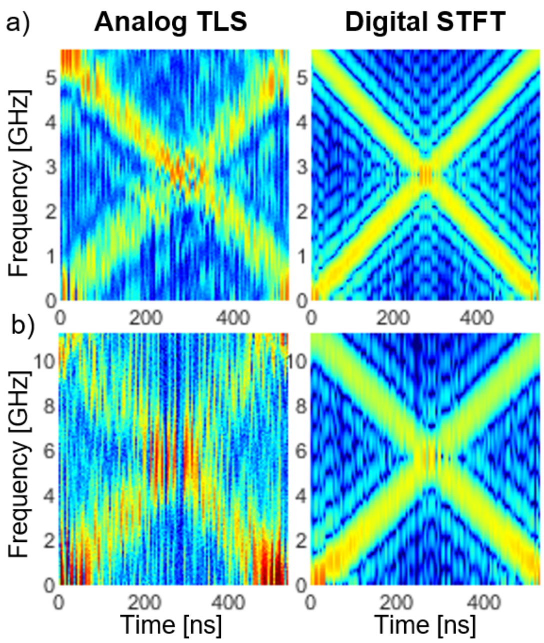

Fig.3: Crossing linear chirp signals with frequencies extending from 0 up to the maximum frequency of that lens, for time apertures a) 1.08 ns and b) 356 ps. The left plots depict the output of the TLS setup, where each vertical slice is a consecutive temporal output. The STFT with no overlap is calculated using DSP on the measured SUT. All colour scales are logarithmic.

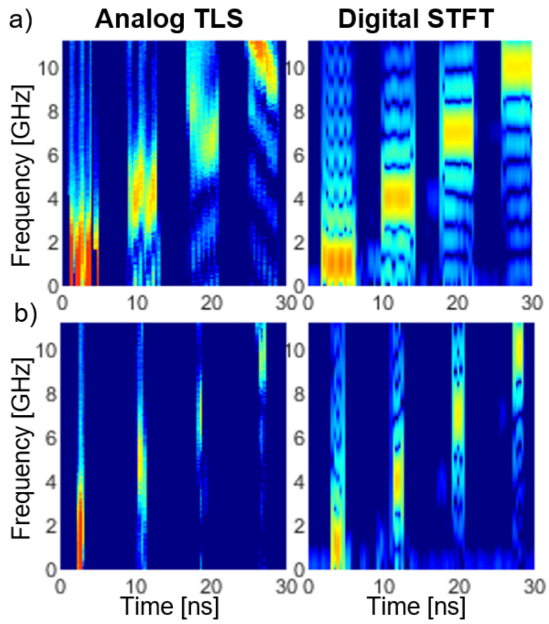

Fig. 4: Frequency hop signal. In 4.a) the hops are 4 ns in duration, in 4.b) the hops are 1 ns. The left plots show the output of the TLS system with T_L=353 ps. The STFT is calculated using digital processing on the measure SUT. All colour scales are logarithmic.

side. These figures confirm the expected sub-GHz frequency resolution ($\delta f{\sim}701$ MHz) for a 5.61 GHz window with 8 analysis points. For the shorter time lens (T_L=353 ps), the sinc function that convolves with the spectrum is larger, producing a worsened frequency resolution and a broader trace of the chirps. In the following trial, it was found events lasting ~$3T_L$ could be accurately detected, allowing recovery of transients at nanosecond scales. Fig 4 depicts a series of frequency hops for the T_L=353 ps time lens at frequencies (1, 4, 7, and 10 GHz). In 4 a) the duration of these hops is 4 ns while in 4 b) the hops are 1 ns. The full time-frequency information of these events is clearly recovered through our TLS method, closely matching the digital STFT calculation.

Conclusions

A time lens based analog spectrogram is demonstrated on continuous signals in real-time without information loss. The TLS system results in a time mapped spectrogram using phase-only manipulations, in a real-time fashion allowing up to 2.8× 10^9 FTs/s. We demonstrate detection of 1 ns transients, sub-GHz frequency resolution, and bandwidths up to 22.4 GHz, far beyond the capabilities of modern DSP and previous real-time analog spectrogram methods. The method should prove useful for applications such as radar and lidar ranging and classification[2],[3],[14], cognitive and secure communications[4], reaction chemistry[15], biomedical signals[16], and radio astronomy[17].

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