A Model of the Nonlinear Interference in Space-Division Multiplexed Systems with Arbitrary Modal Dispersion

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Abstract *We show how to include modal dispersion in the Gaussian noise model extended to space-division multiplexed systems with strongly coupled modes. The proposed model enables fast and accurate design of SDM links. Here we use it to reveal a considerable dependence of cross-nonlinearities on modal dispersion.*

Introduction

Strong random mode coupling in space division multiplexed (SDM) systems is a promising approach to increasing the capacity of long-haul communication links[1–3]. Besides reducing the accumulation of the modal dispersion (MD) along the distance and thus the multiple-input multiple-output (MIMO) requirements for the receiver, strong coupling (SC) mitigates also the accumulation of nonlinear effects[1]. However, the numerical simulation of such systems is particularly heavy both in computational time and in memory requirements of typical RAM and GPUs[4].

Approximate models are thus mandatory for a system analysis with minimal effort. Most of the interest in the literature has been captured by perturbative models, for which four-wave mixing (FWM) takes a closed-form expression. In this framework, the variance of the received nonlinear interference (NLI) can be computed, with expressions particularly simple with Gaussian distributed input signals, as per the Gaussian noise (GN) model[5]. The basic GN model, first introduced for single-mode fibers (SMF), was extended to SDM in[6,7], however without including MD in the theory. While the interaction of MD with Kerr effects is typically small in SMF (where MD is referred to as polarization-mode dispersion (PMD)), justifying its neglection, it may be relevant in SDM due to the higher values of MD in both multi-core and multi-mode fibers[3,8]. The effect of MD has been characterized in some limit cases in[9], where a generalization of the enhanced GN model of[10] to SDM transmission was introduced.

In this work, we extend for the first time the SDM-GN model to account for arbitrary values of MD in systems operating in SC regime, thus describing the models in[6,9] as special cases of our theory. The proposed model can be used for a fast performance evaluation of SDM-based networks.

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Theory

Let $a_{\mathbf{k}}$ be a data symbol at discrete-time k_1, frequency channel k_2, and spatial mode k_3, that we collected in $\mathbf{k} = (k_1, k_2, k_3)$ following[11]. In bra-ket notation, the transmitted signal is thus $|A\rangle = \sum_{\mathbf{k}} a_{\mathbf{k}} |G_{\mathbf{k}}(0,t)\rangle$, i.e., a symbol-weighted combination of the shaping functions $|G_{\mathbf{k}}(0,t)\rangle \triangleq p(t-k_1T)e^{j\Omega_{k_2}t}|k_3\rangle$, with p supporting pulse, Ω_{k_2} carrier frequency of the k_2-th channel, T symbol time. In this framework, the discrete-time channel model under perturbative assumptions is:

$$y_{\mathbf{i}} = a_{\mathbf{i}} + n_{\mathbf{i}} = a_{\mathbf{i}} - j\sum_{\mathbf{k,m,n}} a^*_{\mathbf{k}} a_{\mathbf{m}} a_{\mathbf{n}} \mathcal{X}_{\mathbf{kmni}}$$

where the second term on the rhs is the zero-mean NLI, weighted by the tensor:

$$\mathcal{X}_{\mathbf{kmni}} = \gamma\kappa \int_0^z f(\xi)\, \langle G_{\mathbf{k}}(\xi,t)|G_{\mathbf{m}}(\xi,t)\rangle \times \langle G_{\mathbf{i}}(\xi,t)|G_{\mathbf{n}}(\xi,t)\rangle\, \mathrm{d}\xi \quad (1)$$

where γ is the nonlinear coefficient, κ a mode-dependent weighting function[1], $f(z)$ the loss profile, and $|G_{\mathbf{k}}(z,t)\rangle$ is the shaping function at coordinate z after linear impairments only. In SMF, eq. (1) reduces to the expression analyzed in[12,13]. The main result of the GN model theory[11] yields the following variance of the NLI $n_{\mathbf{i}}$:

$$\mathrm{var}(n_{\mathbf{i}}) = \sum_{\mathbf{k,m,n}} \underbrace{\mathcal{X}_{\mathbf{kmni}}\mathcal{X}^*_{\mathbf{kmni}}}_{\mathcal{I}_1} + \underbrace{\mathcal{X}_{\mathbf{kmni}}\mathcal{X}^*_{\mathbf{knmi}}}_{\mathcal{I}_2}. \quad (2)$$

The main differences between the presence/absence of MD are the following. First, without MD the tensor is symmetric in the inner indexes[11], i.e., $\mathcal{X}_{\mathbf{kmni}} = \mathcal{X}_{\mathbf{knmi}}$, which, in particular, yields a degenerate factor 2 for cross-phase modulation (XPM). This is not the case with MD, thus breaking the degeneracy. Second, from (1-2) it is worth noting that the product of two tensors yields the product of two integrals in z. Likely, such integrals can be closed in scalar propagation after moving in the frequency domain, yielding the well-known FWM kernel, with great simplifications for the GN model theory[5,6,10]. This property is no longer valid in SDM with MD, since the unitary matrix $\mathbf{T}(z,\omega)$ describing linear crosstalk and MD is a random matrix with a complex evolution along z. Although $\mathbf{T}$ can be constructed by the wave-plate model, the resulting numerical evaluation of (2) remains extremely complex because of the double z-integrals. To simplify the analysis, we focused on the expectation of (2) with respect to the random-mode coupling realizations to see if some simplification is possible. Since each $|G_{\mathbf{k}}(z,t)\rangle$ implies a convolution with the transfer matrix $\mathbf{T}$, the expectation calls for the averaging of the product of 8 matrices at different coordinates and frequencies. By using Ito's calculus[14,15] we evaluated such an expectation for SPM-like and XPM-like contributions[5], corresponding to the indexing $\mathbf{iiii}$ for the first and $\mathbf{knmi} \equiv \mathbf{kiki}$ or $\mathbf{kkii}$ for the second. As a main result, we found that to evaluate a generic XPM term, the FWM kernel of a fiber supporting SDM of $2N$ polarizations, much longer than the attenuation length, changes from the classical SMF result $|\eta_0(\alpha)|^2 = \frac{1}{\alpha^2+\Delta\beta^2}$, with $\Delta\beta = \beta(\omega+\omega_1)+\beta(\omega+\omega_2)-\beta(\omega+\omega_1+\omega_2)-\beta(\omega)$ the phase-matching coefficient, into:

$$|\eta|^2 = \frac{m}{\alpha}\left(\alpha_1(1+c)\left|\eta_0\left(\alpha_1\right)\right|^2+\alpha_2(1-c)\left|\eta_0\left(\alpha_2\right)\right|^2\right) \tag{3}$$

where $\alpha_{1,2} \triangleq \alpha + (p \mp q)\frac{N^2\mathsf{SMD}^2}{4N^2-1}$, SMD being the spatial mode dispersion parameter defined in[16], are two MD-dependent attenuation factors with:

$$p = \frac{\omega_1^2+\omega_2^2}{2}, \quad q = \sqrt{p^2 - \omega_1^2\omega_2^2\left(1-\frac{1}{4N^2}\right)}$$

and, with reference to the notation in (2):

$$c \triangleq \begin{cases} \frac{p}{q} - \frac{\omega_1^2}{q}\left(1-\frac{1}{4N^2}\right), & \mathcal{I}_1\text{-term} \\ \frac{p}{q}\,. & \mathcal{I}_2\text{-term} \end{cases}$$

The factor m is equal to N for the $\mathcal{I}_1$ XPM mixing, and $1/2$ for $\mathcal{I}_2$. With the novel FWM kernel it is possible to evaluate the GN model integrals as per[11] in the frequency domain, for instance by using the Monte Carlo method of[10]. The increase in complexity with respect to the SMF case is negligible for any number of modes N. Note also that the SMF case with PMD is a special case of the proposed theory with $N=1$.

The multi-span case follows the same rules of SMF, hence with an extra phased-array factor[5].

Simplified formula

If we neglect intra-channel MD[17], i.e., MD acts only by an inter-channel effect through phase shifts among channels, the model can be greatly simplified. Since MD is absent within a channel, the inter-mode FWM process underpinning the XPM effect can be reduced to the interaction of the carrier frequencies rather than all the frequencies making up the channels. This case, which is a special case of the one in (3), yields the following relation between the scalar-case XPM variance $\sigma^2_{\mathsf{X,s}}$ and its SDM counterpart σ^2_{XPM}:

$$\sigma^2_{\mathsf{XPM}} = \frac{2N+1}{2N}\Bigg((2N+1)\sigma^2_{\mathsf{X,s}}(\alpha)+ \frac{(2N-1)\left(\alpha+\frac{\Delta\omega^2\mu^2}{N}\right)}{\alpha}\sigma^2_{\mathsf{X,s}}\left(\alpha+\frac{\Delta\omega^2\mu^2}{N}\right)\Bigg) \tag{4}$$

where $\Delta\omega$ is the frequency spacing of the two channels generating XPM. Apart from its elegance, (4) enables to use closed-form approximations of $\sigma^2_{\mathsf{X,s}}$ developed for SMF[5] for the case of SDM links with MD. As a sanity check, we observe that (4) accounts for the limit cases analyzed in[9], namely:

$$\sigma^2_{\mathsf{XPM}} = \begin{cases} 2(2N+1)\sigma^2_{\mathsf{X,s}}\,, & \mathsf{MD} \to 0 \\ \frac{(2N+1)^2}{2N}\sigma^2_{\mathsf{X,s}}\,. & \mathsf{MD} \to \infty \end{cases}$$

The ratio of the no-MD to the infinite-MD case is $\frac{4N}{2N+1}$, as first found in[9].

Results

We focused on the XPM contribution to the NLI variance, which dominates FWM in modern optical links at high symbol rates[5]. Since the superposition principle works for the XPM variance, it is sufficient to focus on a two-channel scenario at a given channel spacing. We analyzed a 100km SDM fiber working in the SC regime, while varying the magnitude of MD. We tested the analytical results of our model by comparison with split-step Fourier method (SSFM) simulations. The SSFM had variable step sizes with the nonlinear phase

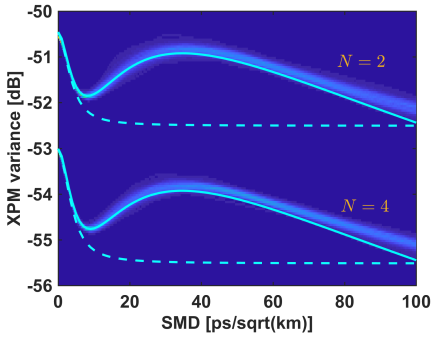

Fig. 1: Power-normalized XPM variance vs SMD for $N = 2$ and 4 modes in a strongly-coupled SDM fiber. The shaded lines are a scatter diagram of SSFM results with respect to the random waveplates. The proposed full GN model based on (3) is in solid lines, while approximation (4) in dashed lines.

criterion, set up in the worst SMD scenario until observing convergence of the SSFM results. We used Gaussian distributed symbols at 49 Gbd with variable frequency spacing between the two interfering channels. The channel under test had power -30 dBm to avoid SPM, while the other had 0 dBm. We used 65536 symbols, and repeated the simulations for a total of 500 random seeds. We considered the transmission of $N = 2$ and 4 spatial modes in a fiber with dispersion 17 ps/nm/km, attenuation 0.2 dB/km, nonlinear coefficient[1] $\gamma\kappa = \frac{4}{3}\frac{2}{2N+1}\overline{\gamma}$ with $\overline{\gamma} = 1.26$ 1/(W·km), and used 10000 random waveplates. No amplified spontaneous emission was considered since NLI was the goal of the investigation. At the receiver, we ideally removed all the accumulated linear effects, and after matched filter detection and carrier phase recovery we estimated the noise variance of the received symbol.

Figure 1 shows the XPM variance vs. the SMD coefficient. Solid lines are the complete theory from (3), dashed-lines are from (4). The shaded stripes are scatter diagrams of SSFM results binned in a 2D histogram, with brightest colors indicating frequent events. The two channels were spaced 100 GHz. The solid line is the prediction of our GN model based on the kernel (3). We observe an excellent agreement between the full theory and SSFM. Moreover, the small random deviations are an indicator that the average kernel is a good metric to estimate the system performance, notwithstanding the 10000 waveplates. The plot shows a considerable NLI power reduction with moderate MD values. The approximated formula (4) works well up to ~ 5 ps/$\sqrt{\text{km}}$, which covers several SDM fibers analyzed in the literature[3,8,18]. We observed a similarly excellent agreement even for SPM.

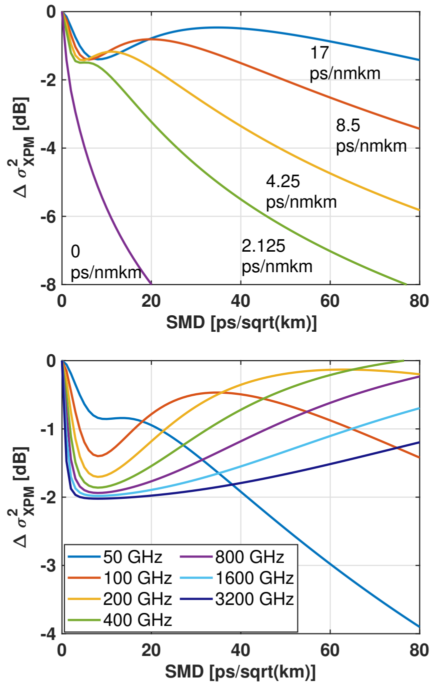

Fig. 2: XPM variance normalized to the no-MD case vs SMD for $\Delta f = 100$ GHz (top) and at $D = 17$ ps/nm/km (bottom).

Having tested the validity of the model in a representative case, we focused on the impact of the system parameters. In Fig. 2 we show the impact of the fiber dispersion at a fixed spacing of 100 GHz (top), and the contrary at fixed dispersion of 17 ps/nm/km (bottom). The results use the complete GN model based on (3). Note that in absence of dispersion the curve decreases monotonically for increasing MD because the pulse collision length[13], i.e., the length over which two specific symbols interact by XPM, is infinite, so that MD can effectively prevent the coherent build-up of the NLI. This is not the case for finite collision lengths, with the unexpected result that the XPM variance increases beyond some MD value. One plausible, yet not final, interpretation is that the smaller decorrelation experienced by the closer band edges of the two channels creates a resonance in the variance. For large SMD, the decorrelation length gets much smaller than the collision length and the variance restarts to decrease as in the dispersion-less case.

Finally, note an intriguing minimum at $\Delta f > 100$ GHz, which broadens as Δf increases.

Conclusions

We extended the GN model to strongly-coupled SDM with arbitrary modal dispersion. We derived a simple expression of the XPM variance that is highly accurate for MD values that are relevant in practical systems.

Acknowledgments

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