A Pareto-Optimality Based Multi-Objective Optimisation Approach to Assist Optical Network (Re-)Design Choices

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Abstract *This paper presents a Pareto-optimality based multi-objective optimisation approach to optimise optical network parameters considering multiple performance metrics simultaneously. Resultant Pareto-optimal solutions provide a set of options for practitioners to assist network (re-)design choices considering multiple (conflicting) performance goals.*

Introduction

With ever-growing demand for better optical network performance, it is vital to develop methods to assist performance improvements. Several optimisation studies have been conducted in recent literature using exact[1]–[4] and intelligent[5]–[9] methods considering different network parameters and performance metrics. Most of these studies have considered a single objective/performance goal either maximising throughput, minimising latency, minimising cost or maximising resilience, etc. which could potentially be conflicting with other performance goals in a real world scenario[10]. While it is of research interest to understand these performance goals individually, in a real world network design or operation scenario, its essential to understand the achievability of multiple performance goals simultaneously and their inter-dependencies to assist decision making at network design or operation due to their conflicting nature.

Multi-objective optimisation[11] considers the simultaneous achievability of several performance goals and *Pareto-optimality*[12] provides a method to confirm optimal solutions considering multiple performance goals simultaneously, where the *Pareto-front* represents the set of *non-dominated*[13] solutions for the optimisation problem. Meta-heuristic optimisation methods such as Genetic Algorithms (GA)[14] have been successfully applied to solve a wide range of non-linear non-convex complex engineering optimisation problems[15],[16]. In this paper, a multi-objective genetic algorithm (GA) based on *Pareto-optimality* is proposed to solve the network design optimisation problem for a general real-world optical network considering a set of design parameters and performance metrics including throughput, resilience and cost. Comparison results with single objective GA[8] exhibit the added value from multi-objective GA by generating a set of *Pareto-optimal solutions*[12] that could assist design choices.

Optimisation Problem

The variables of the optimisation problem are the considered network parameters. We consider span length and topology parameters in this study. The objective/fitness functions for fibre installation cost Eq. (1)[8], fibre span cost Eq. (2)† and throughput Eq. (3)[8] optimisation can be formulated as follows:

$$\begin{aligned} \text{Find:} \quad & \boldsymbol{X} = [x_1, x_2, \dots, x_n] \\ \text{Minimise:} \quad & F_{\mathsf{Fibre}}(\boldsymbol{X}) = a \sum_{i=0}^{r} \ell_i \end{aligned} \tag{1}$$

$$\begin{aligned} \text{Find:} \quad & \boldsymbol{X} = [x_1, x_2, \dots, x_n] \\ \text{Minimise:} \quad & F_{\mathsf{Span}}(\boldsymbol{X}) = b \cdot s\,, \end{aligned} \tag{2}$$

$$\begin{aligned} \text{Find:} \quad & \boldsymbol{X} = [x_1, x_2, \dots, x_n] \\ \text{Maximise:} \quad & F_{\mathsf{T}}(\boldsymbol{X}) = \sum_{j=0}^{m} \mathsf{T}_{p_j}\,, \end{aligned} \tag{3}$$

where $\boldsymbol{X}$ is a vector of decision variables containing the parameters for a network, n is the number of parameters, r is the number of links, ℓ_i is the length of link i, s is the number of fibre span segmenting points (fibre span split points), a and b in Dollars USD $[\$]$ are the constants indicating fibre installation cost per kilometre $[\mathrm{km}]$ and cost per a fibre span segmenting point, respectively; m is the number of light-paths and T_{p_j} is the individual

† number of span segmenting points is retrieved from *split fiber* method in GNPy[17]

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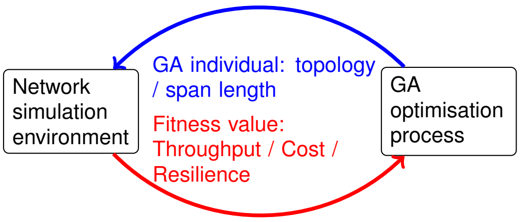

Fig. 1: Optimisation loop describing the iterative information exchange between GA optimisation process and the network simulation environment.

throughput for j^{th} light-path, which is defined as the total Shannon rate[9]:

$$\mathsf{T}_{p_j} = 2R_S \sum_k \log_2\left(1 + \text{SNR}\left[p_j(\lambda_k)\right]\right), \quad (4)$$

where k denotes the channel index in the light-path p_j, R_S is the symbol rate, and $\text{SNR}\left[p_j(\lambda_k)\right]$ stands for the signal-to-noise ratio[2] at the receiver at the end of path p_j of the channel with λ_k centre wavelength. Resilience is modelled as a constraint throughput this study, which is represented by a minimum node degree of 2. Multi-objective optimisation problem considers the above objective functions simultaneously and the *Pareto-optimality* of the solutions is defined as the set of *non-dominated* solutions, where *dominance* relation is formulated as follows[13]:

$$\begin{aligned} &\mathsf{F}_i(x_1) \leq \mathsf{F}_i(x_2)\ \forall\, i \in \{1,\ldots,q\}\ \wedge \\ &\exists\, j \in \{1,\ldots,q\}: \mathsf{F}_j(x_1) \leq \mathsf{F}_j(x_2), \end{aligned} \quad (5)$$

for q number of objective functions F and for the solutions x_1 and x_2.

Framework for Multi-Objective Optimisation in Optical Networks

Algorithm 1 describes the multi objective genetic algorithm (GA)[14] and Fig. 1 outlines the optimisation loop (steps 2-7 in Algorithm 1). We employ the *non-dominated sorting* (step 5) and *crowding distance* calculation (step 6) approach proposed in Deb et al.[18]. *Non-dominated* sorting at each generation enables *Pareto-optimality* by eliminating dominated solutions (5) from the population. The fitness values as described in Eqs (1), (2), and (3) are retrieved from an extended version of the state-of-the-art *GNPy* network simulator[19] for a optical network parameter setting represented by a GA individual. We employ the state-of-the-art $k-$*shortest path routing*[20] and the *first fit spectrum assignment (FFSA)* strategy[21] for routing and spectrum assignment respectively.

Algorithm 1 $(\mu + \lambda)$ - Multi-objective GA

1) Initialise the population $\mathcal{P} = \{X_1, X_2, \ldots, X_{j-1}, X_j, X_{j+1}, \ldots, X_\mu\}$ with μ optical network parameter setting individuals $X_j = [x_1, x_2, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n]$, i.e., a vector of optical network parameters x_i.
2) Select $\mathcal{O} \subseteq \mathcal{P}$, where $|\mathcal{O}| = \lambda$.
3) For each $\{I_1, I_2\} \in \mathcal{O}$, produce offspring I_1' I_2' by crossover and mutation. Add each offspring to $\mathcal{P}$.
4) Fitness evaluation of all $I \in \mathcal{P}$.
5) Non-dominated sort $\mathcal{P}$[18].
6) Calculate crowding distance for $I \in \mathcal{P}$[18].
7) Select $\mathcal{S} \subseteq \mathcal{P}$ where $|\mathcal{S}| = \mu$. $\mathcal{P} := \mathcal{S}$.
8) Repeat step 2 to 8 until termination criterion is reached.

Multi-Objective Optimisation for Fibre Spans

The objective and the constraints for the optimisation simulations are as defined in Eq. (2) and (3). The system parameters are kept fixed: SSMF, the carrier wavelength of 1550 nm, symbol rate of 100 GBd, number of WDM channels of 51 with the roll-off factor of 0.001, and the EDFA noise figure of 4.5 dB. For demand simulation, we consider the state-of-the-art *uniform all-to-all*[22] traffic matrix. For a network graph $\mathscr{G} = (\mathcal{V}, \mathcal{E})$ with a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_z\}$ and set of edges $\mathcal{E}$, source nodes v_i, destination nodes v_j $i \neq j$ and a constant c, uniform traffic matrix $\hat{T}$ is defined as[1]

$$\hat{T}: \quad \forall\, \{v_i, v_j\} \in \mathcal{V}: \quad \hat{T}_{ij} = \frac{c}{z\,(z-1)}. \quad (6)$$

As shown in Fig. 2, optimisation simulations for DTAG network resulted in a set of *non-dominated* solutions generated by the multi-objective GA (A to J) and the two extreme solutions (A and J) generated by single objective GA running two separate runs for throughput F_{T} Eq. (3) and fibre span cost F_{Span} Eq. (2) optimisation. Throughput exhibits a positive correlation with fibre span cost where throughput varies in the range of 25.1 Tbps to 29.5 Tbps for the range of 92 to 190 of span cost units (a). This *Pareto-optimal* set of solutions (A to J) represent the optimal fibre span length choices considering both throughput and fibre span cost.

Multi-Objective Optimisation for Topology Design

The network topology is represented by the respective adjacency matrix of the network graph. A GA individual is extended by a bit-wise vector representing the adjacencies. Bit-wise variation operators are employed including the inversion mutation, where bit `1` is flipped to bit `0`

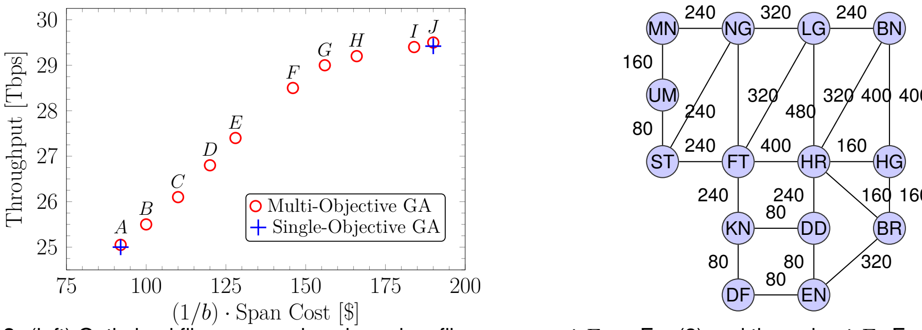

Fig. 2: (left) Optimised fibre span values based on fibre span cost F_{Span} Eq. (2) and throughput F_{T} Eq. (3) optimisation using multi-objective GA (A to J in red) and single objective GA (A and J in blue) for DTAG network (right). The resultant *Pareto-optimal* fibre span length values referred by points (A to J) in km are $A = 104.6$, $B = 102.3$, $C = 96.4$, $D = 91.3$, $E = 86.5$, $F = 71.6$, $G = 67.5$, $H = 63.7$, $I = 55.7$, $J = 50.1$. b is the cost per span segmenting point in Eq. (2). The edge weights of the graph correspond to the distances in km.

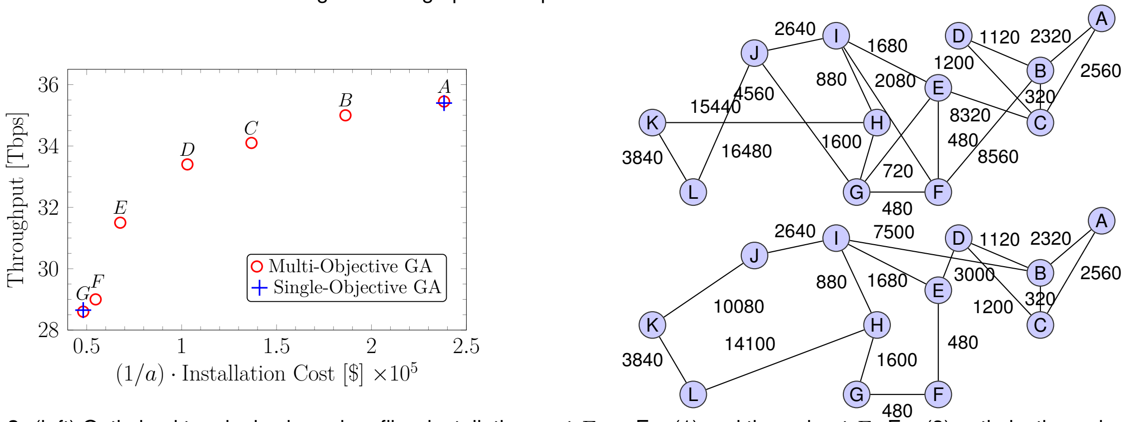

Fig. 3: (left) Optimised topologies based on fibre installation cost F_{Fibre} Eq. (1) and throughput F_{T} Eq. (3) optimisation using multi-objective GA (A to G in red) and single objective GA (A and G in blue) for GB network. Bottom right topology describes the *Pareto-optimal* counterpart (represented by the point $F(53800, 29.1)$ in the left plot) of the original GB topology $(64080, 29.1)$ in top right which is *dominated* (Eq. 5) by higher fitness values. a is the cost per 1km in Eq. (1). The edge weights of the graphs correspond to the distances in km.

and vise versa and the crossover operators inspired by the bit-wise operators `OR`, `AND` and `XOR` as explained in the work by Lima et al.[23]. Additionally, resilience is considered as a constraint on the minimum node degree of 2. Fig. 3 describes the *Pareto-optimal* set of points (A to G) obtained by multi objective GA and the single optimal points (A and G) obtained by single objective GA by optimising for a single objective cost or throughput separately. Each point (A to G) corresponds to a topology that is *non-dominated* by any other topology based on the fitness values for throughput F_{T} Eq. (3) and fibre installation cost F_{Fibre} Eq. (1). Based on throughput and cost requirements network designers can choose from these *Pareto-optimal* topologies. It is noted that the original GB topology (top right in Fig. 3) has been *dominated* Eq. (5) by a topology (bottom right in Fig. 3) which corresponds to the point $F(53800, 29.1)$ in the *Pareto-optimal* set (Fig. 3 left). Hence, it can be suggested that, at a network re-design phase original GB topology can be improved by modifying to be topology F.

Conclusion and Future Work

A multi-objective optimisation approach based on *Pareto-optimality*[18] is proposed to optimise network design parameters considering multiple performance metrics. The effectiveness of the approach is evaluated with two example scenarios of firstly, optimisation of fibre parameters based on throughput and fibre span cost and secondly, topology parameters considering throughput, fibre installation cost and resilience for two benchmark core networks DTAG and GB. In both cases, the *Pareto-optimal* set of solutions generated by the multi-objective GA exhibits the potential to provide more informative guidance to network designers to choose between various options considering multiple performance metrics compared to the single extreme solutions generated from a single-objective GA considering only one performance metric. Future work will extend this approach to optimise operational parameters.

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